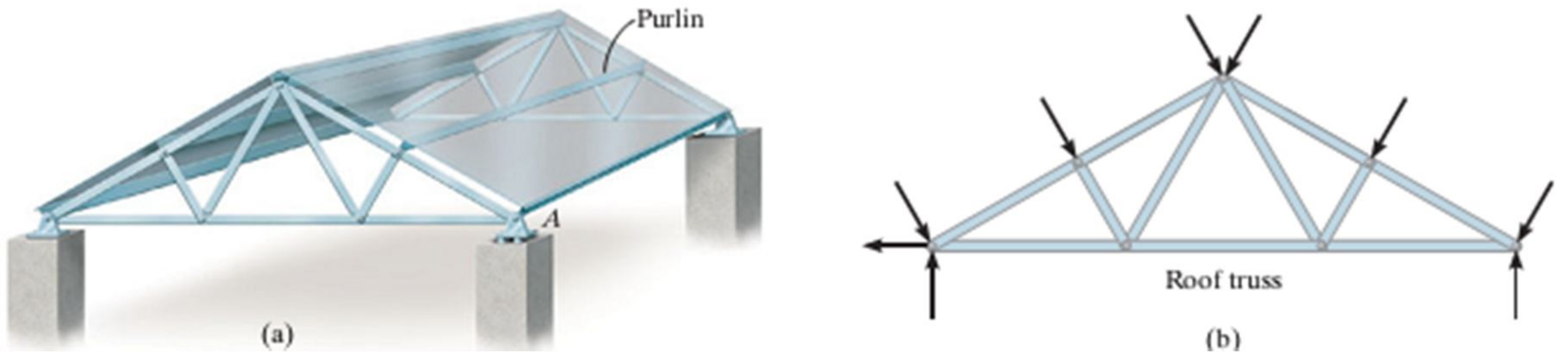
Engineering Mechanics

ME-205F

Second year B.Tech Degree

Truss & Frame

A *truss* is a structure composed of slender members joined together at their end points. The members commonly used in construction consist of wooden struts or metal bars. In particular, *planar* trusses lie in a single plane and are often used to support roofs and bridges. The truss shown in Fig. 6–1*a* is an example of a typical roof-supporting truss. In this figure, the roof load is transmitted to the truss *at the joints* by means of a series of *purlins*. Since this loading acts in the same plane as the truss, Fig. 6–1*b*, the analysis of the forces developed in the truss members will be two-dimensional.



Assumptions in truss analysis

Assumptions for Design. To design both the members and the connections of a truss, it is necessary first to determine the *force* developed in each member when the truss is subjected to a given loading. To do this we will make two important assumptions:

- ***All loadings are applied at the joints.*** In most situations, such as for bridge and roof trusses, this assumption is true. Frequently the weight of the members is neglected because the force supported by each member is usually much larger than its weight. However, if the weight is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, with half of its magnitude applied at each end of the member.
- ***The members are joined together by smooth pins.*** The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a *gusset plate*, as shown in Fig. 6–3*a*, or by simply passing a large bolt or pin through each of the members, Fig. 6–3*b*. We can assume these connections act as pins provided the center lines of the joining members are *concurrent*, as in Fig. 6–3.

(b)

Fig. 6–3

Simple plane truss

Simple Truss. If three members are pin connected at their ends, they form a *triangular truss* that will be *rigid*, Fig. 6–5. Attaching two more members and connecting these members to a new joint *D* forms a larger truss, Fig. 6–6. This procedure can be repeated as many times as desired to form an even larger truss. If a truss can be constructed by expanding the basic triangular truss in this way, it is called a *simple truss*.

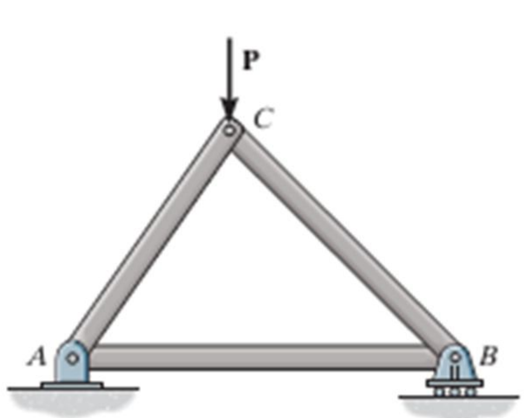


Fig. 6–5

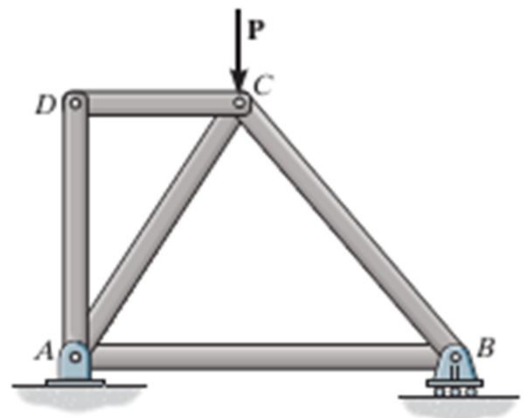


Fig. 6–6

Truss analysis-Method of joints

In order to analyze or design a truss, it is necessary to determine the force in each of its members. One way to do this is to use the method of joints. This method is based on the fact that if the entire truss is in equilibrium, then each of its joints is also in equilibrium. Therefore, if the free-body diagram of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint. Since the members of a *plane truss* are straight two-force members lying in a single plane, each joint is subjected to a force system that is *coplanar and concurrent*. As a result, only $\Sigma F_x = 0$ and $\Sigma F_y = 0$ need to be satisfied for equilibrium.

When using the method of joints, always start at a joint having at least one known force and at most two unknown forces, as in Fig. 6–7*b*. In this way, application of $\Sigma F_x = 0$ and $\Sigma F_y = 0$ yields two algebraic equations which can be solved for the two unknowns. When applying these equations, the correct sense of an unknown member force can be determined using one of two possible methods.

Truss analysis-Method of joints

Procedure for Analysis

The following procedure provides a means for analyzing a truss using the method of joints.

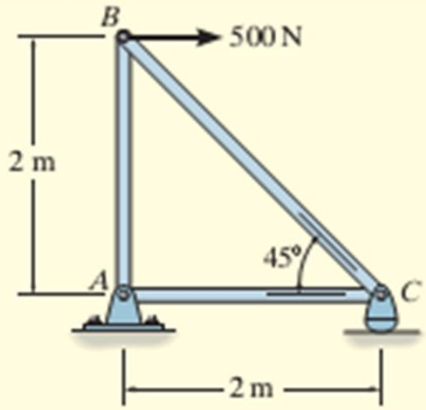
- Draw the free-body diagram of a joint having at least one known force and at most two unknown forces. (If this joint is at one of the supports, then it may be necessary first to calculate the external reactions at the support.)

- Use one of the two methods described above for establishing the sense of an unknown force.

- Orient the x and y axes such that the forces on the free-body diagram can be easily resolved into their x and y components and then apply the two force equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Solve for the two unknown member forces and verify their correct sense.

- Using the calculated results, continue to analyze each of the other joints. Remember that a member in *compression* "pushes" on the joint and a member in *tension* "pulls" on the joint. Also, be sure to choose a joint having at most two unknowns and at least one known force.

Method of joints



(a)

(b)

(c)

Determine the force in each member of the truss shown in Fig. 6–8*a* and indicate whether the members are in tension or compression.

SOLUTION

Since we should have no more than two unknown forces at the joint and at least one known force acting there, we will begin our analysis at joint *B*.

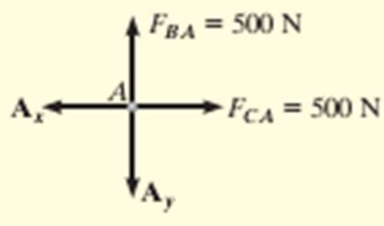
Joint *B*. The free-body diagram of the joint at *B* is shown in Fig. 6–8*b*. Applying the equations of equilibrium, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 500\text{ N} - F_{BC}\sin 45° = 0 \quad F_{BC} = 707.1\text{ N (C)} \quad \textit{Ans.}$$

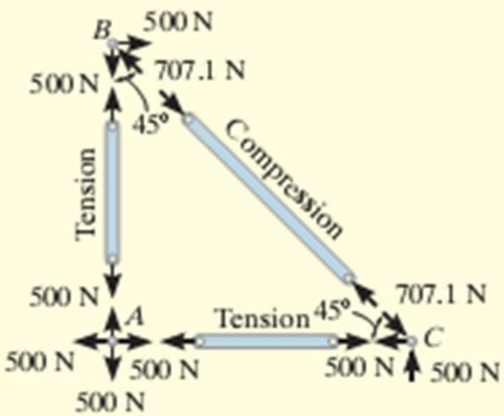
$$+\uparrow \Sigma F_y = 0; \quad F_{BC}\cos 45° - F_{BA} = 0 \quad F_{BA} = 500\text{ N (T)} \quad \textit{Ans.}$$

Since the force in member *BC* has been calculated, we can proceed to analyze joint *C* to determine the force in member *CA* and the support reaction at the rocker.

Method of joints



(d)



(e)

Fig. 6–8

Joint C. From the free-body diagram of joint C, Fig. 6–8c, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{CA} + 707.1 \cos 45° \text{ N} = 0 \quad F_{CA} = 500 \text{ N (T)} \quad \textit{Ans.}$$
$$+\uparrow \Sigma F_y = 0; \quad C_y - 707.1 \sin 45° \text{ N} = 0 \quad C_y = 500 \text{ N} \quad \textit{Ans.}$$

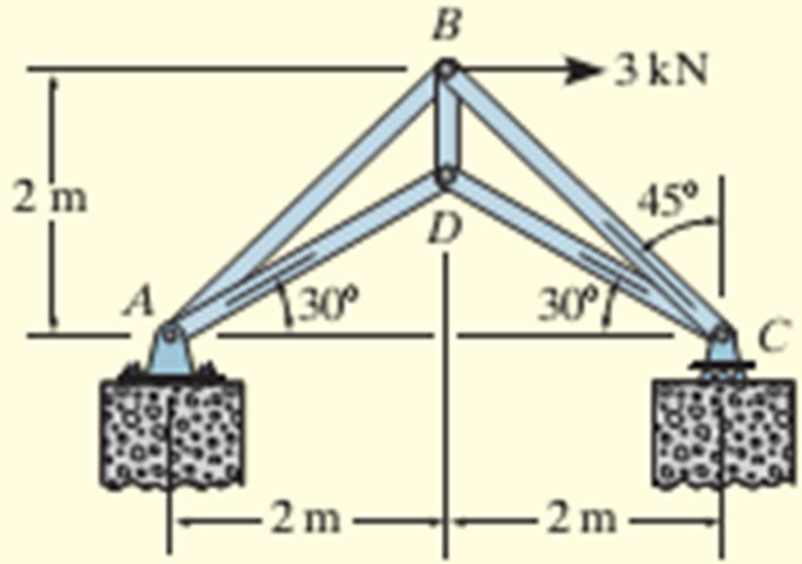
Joint A. Although it is not necessary, we can determine the components of the support reactions at joint A using the results of F_{CA} and F_{BA}. From the free-body diagram, Fig. 6–8d, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 500 \text{ N} - A_x = 0 \quad A_x = 500 \text{ N}$$
$$+\uparrow \Sigma F_y = 0; \quad 500 \text{ N} - A_y = 0 \quad A_y = 500 \text{ N}$$

NOTE: The results of the analysis are summarized in Fig. 6–8e. Note that the free-body diagram of each joint (or pin) shows the effects of all the connected members and external forces applied to the joint, whereas the free-body diagram of each member shows only the effects of the end joints on the member.

Truss, Method of joints

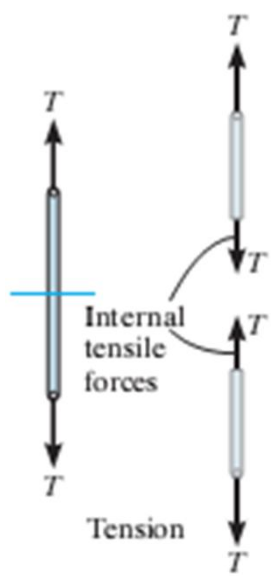
Determine the forces acting in all the members of the truss shown in Fig.



(a)

Truss analysis-Method of Sections

When we need to find the force in only a few members of a truss, we can analyze the truss using the *method of sections*. It is based on the principle that if the truss is in equilibrium then any segment of the truss is also in equilibrium. For example, consider the two truss members shown on the left in Fig. 6–14. If the forces within the members are to be determined, then an imaginary section, indicated by the blue line, can be used to cut each member into two parts and thereby "expose" each internal force as "external" to the free-body diagrams shown on the right. Clearly, it can be seen that equilibrium requires that the member in tension (T) be subjected to a "pull," whereas the member in compression (C) is subjected to a "push."



Truss analysis-Method of Sections

Method of Sections

- If the forces in only a few members of a truss are to be determined, the ***method of sections*** is generally the most appropriate analysis procedure.
- The method of sections consists of passing an *imaginary line* through the truss, cutting it into sections.
- Each imaginary section must be in equilibrium if the entire truss is in equilibrium.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$$

Truss analysis-Method of Sections

Method of Sections

Procedure for analysis - the following is a procedure for analyzing a truss using the method of sections:

1. First, if necessary, determine the support reactions for the entire truss.

2. Next, make a decision on how the truss should be "cut" into sections and draw the corresponding free–body diagrams.

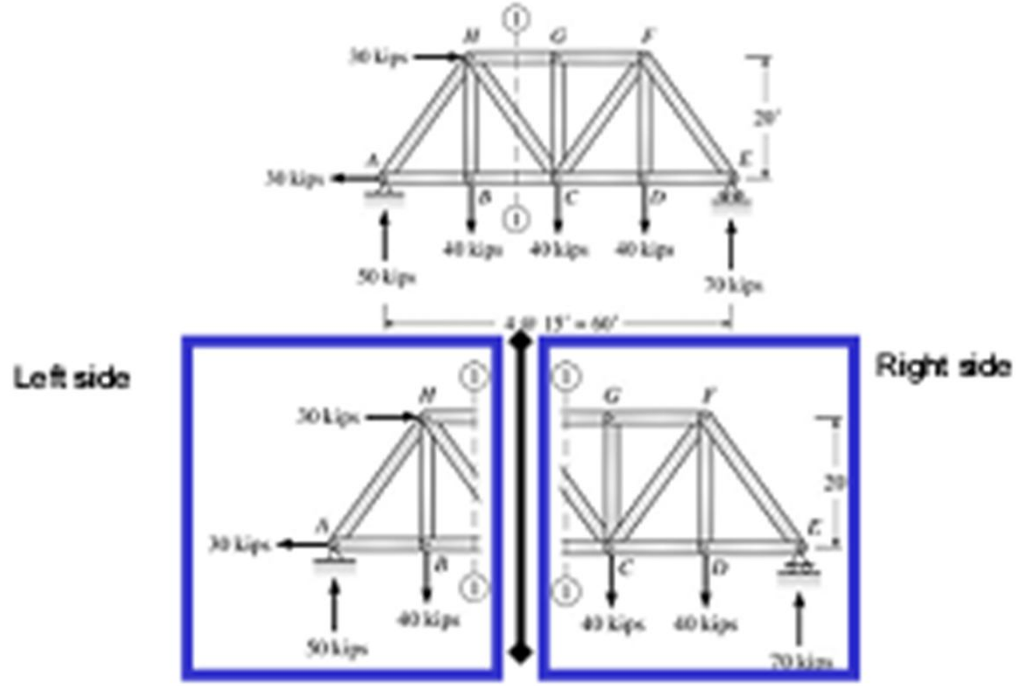
3. Try to apply the three equations of equilibrium such that simultaneous solution is not required.

   Moments should be summed about points that lie at the intersection of the lines of action of two unknown forces, so that the remaining force may be determined.

Truss analysis-Method of Sections

Method of Sections

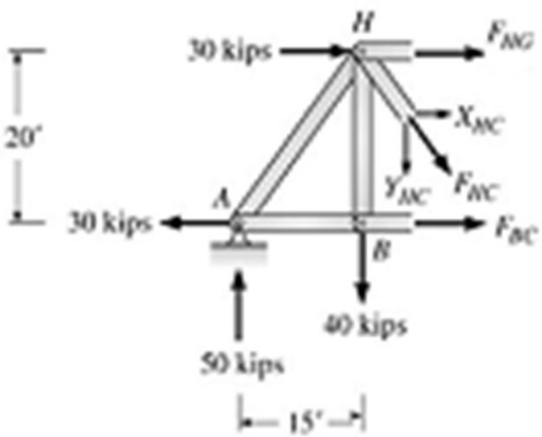
- Imagine cutting a structure into two sections about line 11



Truss analysis-Method of Sections

Method of Sections

- Typically the section with the fewest forces or with section with the most convenient geometry is selected.
- In this example the left-hand side.



- Apply the three equations of equilibrium to the section.
- If possible, attempt to develop an equation in just one unknown.
- Look for points where the lines of action of several forces are concurrent.

Truss analysis-Method of Sections

Procedure for Analysis

The forces in the members of a truss may be determined by the method of sections using the following procedure.

Free-Body Diagram.

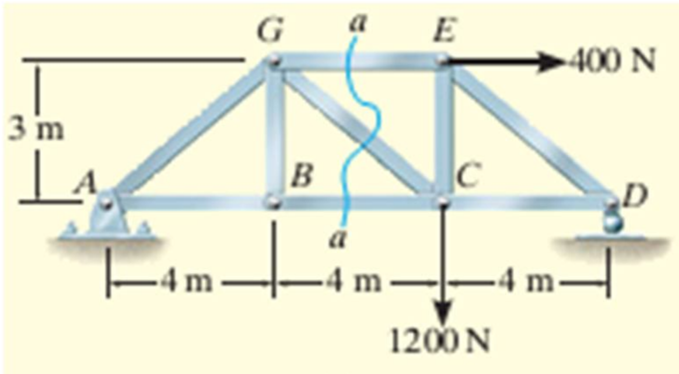
- Make a decision on how to "cut" or section the truss through the members where forces are to be determined.
- Before isolating the appropriate section, it may first be necessary to determine the truss's support reactions. If this is done then the three equilibrium equations will be available to solve for member forces at the section.
- Draw the free-body diagram of that segment of the sectioned truss which has the least number of forces acting on it.
- Use one of the two methods described above for establishing the sense of the unknown member forces.

Equations of Equilibrium.

- Moments should be summed about a point that lies at the intersection of the lines of action of two unknown forces, so that the third unknown force can be determined directly from the moment equation.
- If two of the unknown forces are *parallel*, forces may be summed *perpendicular* to the direction of these unknowns to determine *directly* the third unknown force.

Truss analysis-Method of Sections

Determine the force in members GE, GC, and BC of the truss shown in Fig. 6–16a. Indicate whether the members are in tension or compression.



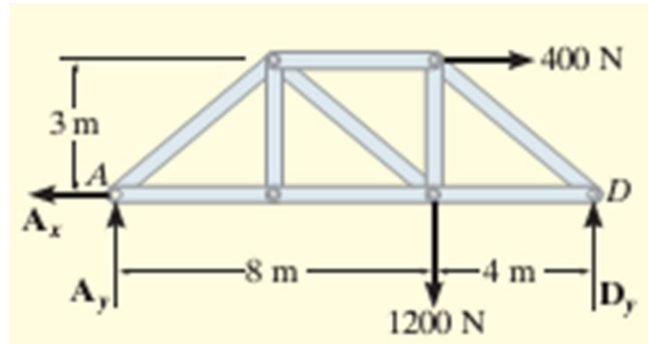
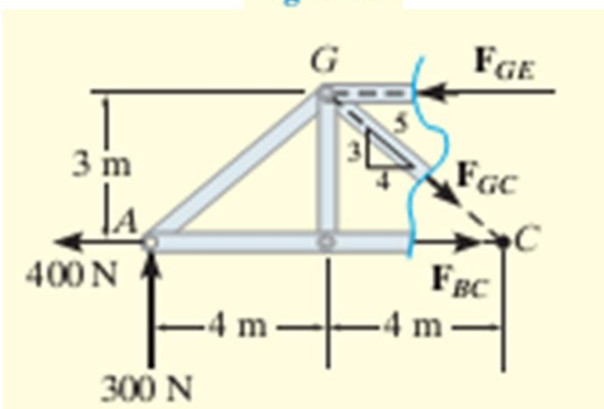
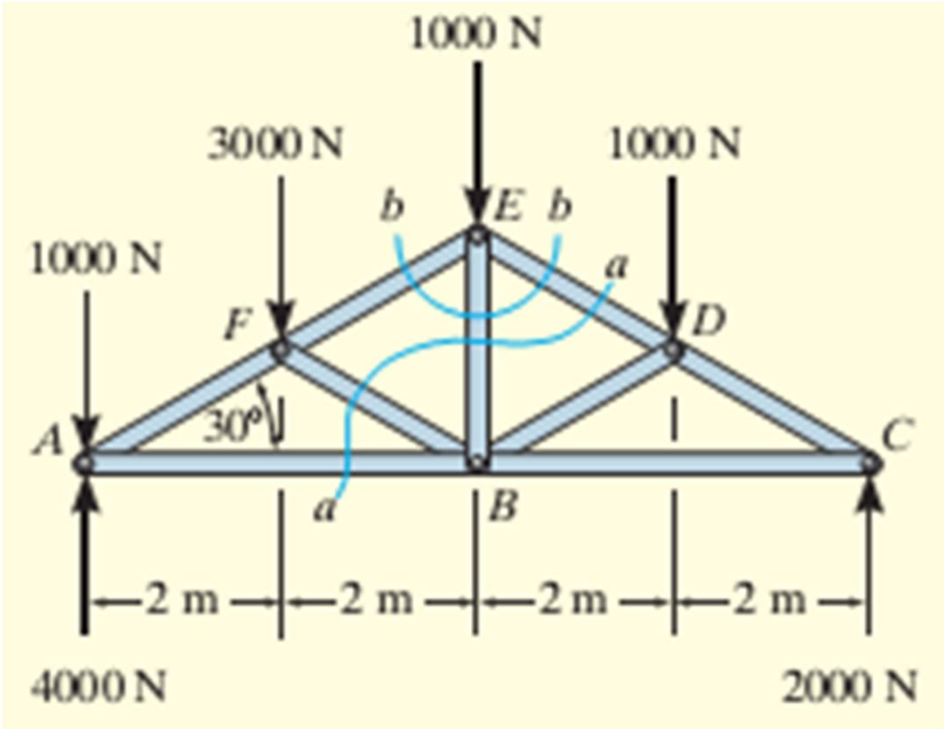


Fig. 6–16



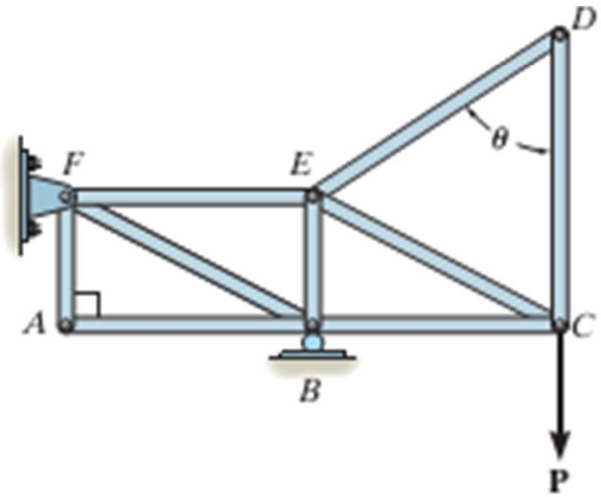
Truss analysis-Method of sections

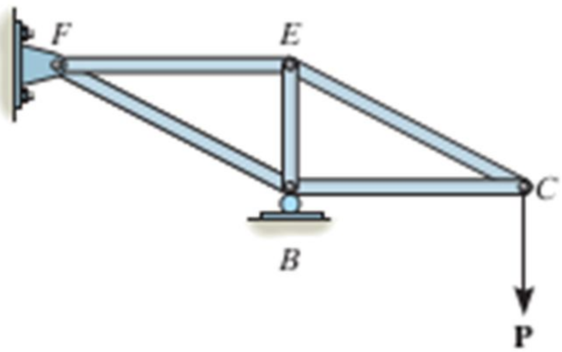


Zero force members:

Truss analysis using the method of joints is greatly simplified if we can first identify those members which support *no loading*.

The zero-force members of a truss can generally be found *by inspection* of each of the joints. For example, consider the truss shown in Fig. 6–11*a*. If a free-body diagram of the pin at joint *A* is drawn, Fig. 6–11*b*, it is seen that members *AB* and *AF* are zero-force members.

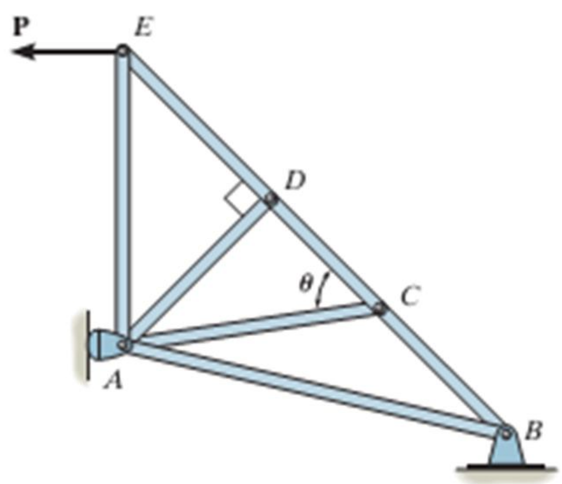


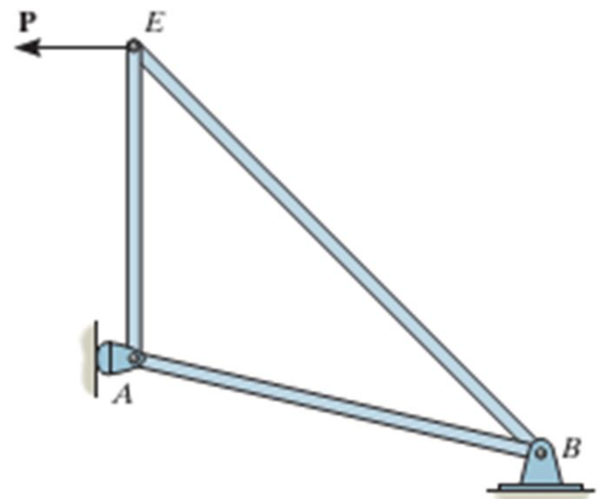


only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero-force members.

Zero force members:

if three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint.





Centre of Gravity

Center of Gravity. A body is composed of an infinite number of particles of differential size, and so if the body is located within a gravitational field, then each of these particles will have a weight dW, Fig. 9–1*a*. These weights will form an approximately parallel force system, and the resultant of this system is the total weight of the body, which passes through a single point called the *center of gravity*, G, Fig. 9–1*b*.*

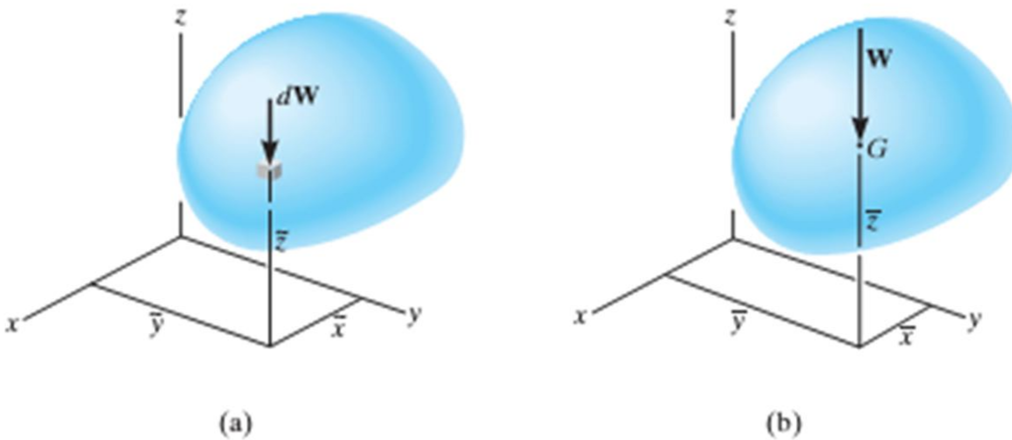


Fig. 9–1

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

Centre of Mass

Center of Mass of a Body. In order to study the *dynamic response* or accelerated motion of a body, it becomes important to locate the body's center of mass C_m, Fig. 9–2. This location can be determined by substituting $dW = g\,dm$ into Eqs. 9–1. Since g is constant, it cancels out, and so

$$\bar{x} = \frac{\int \tilde{x}\,dm}{\int dm} \quad \bar{y} = \frac{\int \tilde{y}\,dm}{\int dm} \quad \bar{z} = \frac{\int \tilde{z}\,dm}{\int dm} \qquad (9\text{–}2)$$

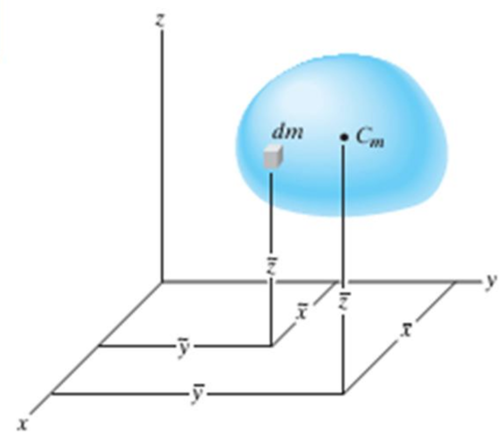


Fig. 9–2

Center of mass & Center of gravity

--CM and CG are not synonymous.
--CG of a body is the point where the vector sum of gravitational forces on all its particles acts
--CM of a body is that point along which when an external force is applied than the body will react to the force in the same manner as a point object of same mass would behave(if point mass is possible)
--CM and CG coincides only if the acceleration due to gravity acting on all the particles of the body have same value.
--for a very large object, at every point of which acceleration due to gravity is not exactly same, than CM and CG does not coincide.
--Both CM as well as CG gravity may lie inside or outside the body depending upon the distribution of masses in the body.

Centroid

Centroid of an Area. If an area lies in the x–y plane and is bounded by the curve $y = f(x)$, as shown in Fig. 9–5a, then its centroid will be in this plane and can be determined from integrals similar to Eqs. 9–3, namely,

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} \tag{9–4}$$

These integrals can be evaluated by performing a *single integration* if we use a *rectangular strip* for the differential area element. For example, if a vertical strip is used, Fig. 9–5b, the area of the element is $dA = y\,dx$, and its centroid is located at $\tilde{x} = x$ and $\tilde{y} = y/2$. If we consider a horizontal strip, Fig. 9–5c, then $dA = x\,dy$, and its centroid is located at $\tilde{x} = x/2$ and $\tilde{y} = y$.

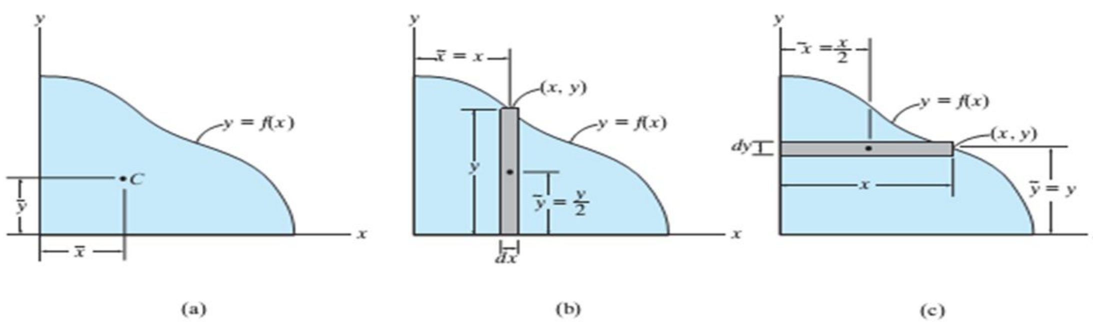


Fig. 9–5

Centroid

.....contd....

Centroid of a Volume. If the body in Fig. 9–3 is made from a homogeneous material, then its density ρ (rho) will be constant. Therefore, a differential element of volume dV has a mass $dm = \rho\, dV$. Substituting this into Eqs. 9–2 and canceling out ρ, we obtain formulas that locate the *centroid* C or geometric center of the body; namely

$$\bar{x} = \frac{\int_V \tilde{x}\, dV}{\int_V dV} \quad \bar{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV} \quad \bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV} \qquad (9\text{–}3)$$

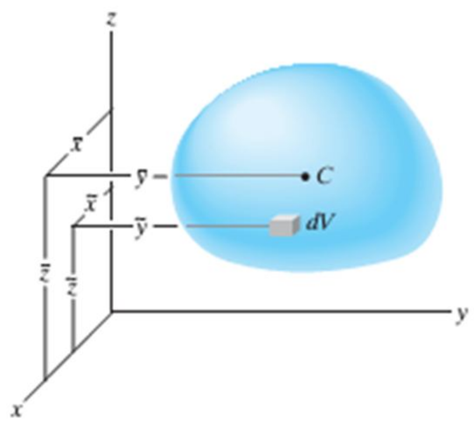


Fig. 9–3