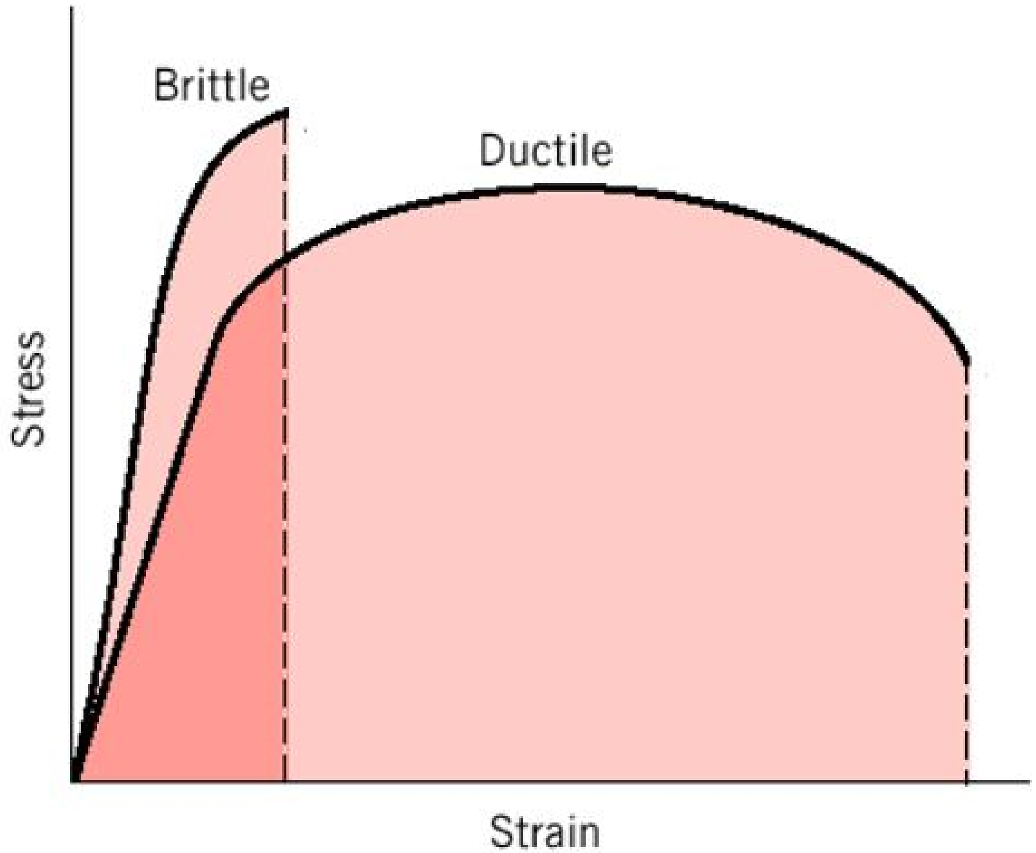
Failure Modes

Ductility and Percent Elongation

- *Ductility* is the degree to which a material will deform before ultimate fracture.
- Percent elongation is used as a measure of ductility.
- Ductile Materials have %E ≥ 5%
- Brittle Materials have %E < 5%
- For machine members subject to repeated or shock or impact loads, materials with %E > 12% are recommended.

Ductile materials - extensive plastic deformation and energy absorption (toughness) before fracture

Brittle materials - little plastic deformation and low energy absorption before failure



DUCTILE VS BRITTLE FAILURE

- Classification:

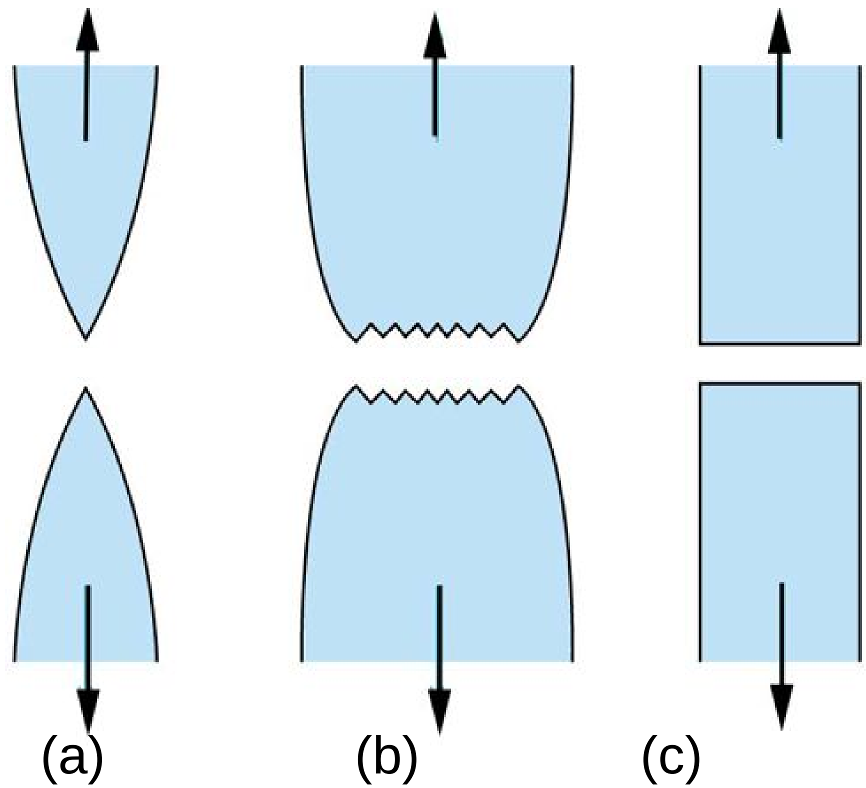
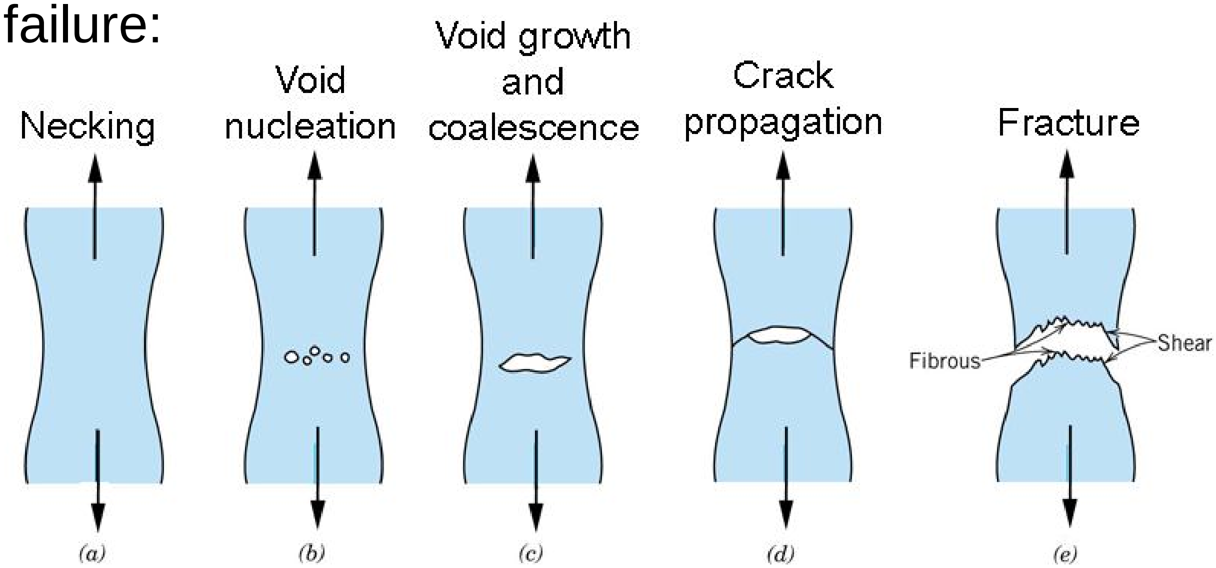


Figure 8.1 (*a*) Highly ductile fracture in which the specimen necks down to a point. (*b*) Moderately ductile fracture after some necking. (*c*) Brittle fracture without any plastic deformation.

• Ductile fracture is desirable!	Ductile: warning before fracture	Brittle: No warning

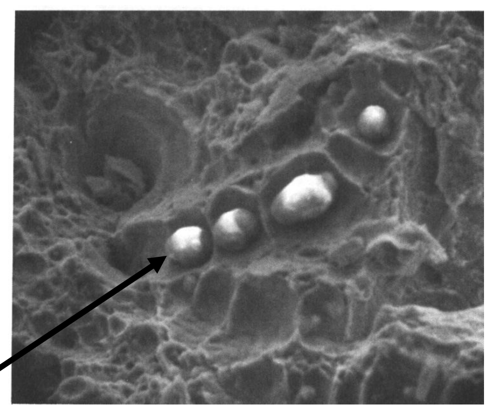
DUCTILE FAILURE

- Evolution to failure:



“cup and cone” fracture

- Resulting fracture surfaces (steel)



50 μm

particles serve as void nucleation sites.

1 μm = 1 X 10^{-6} m = 0.001 mm

Failure Prediction Methods
Static Loads

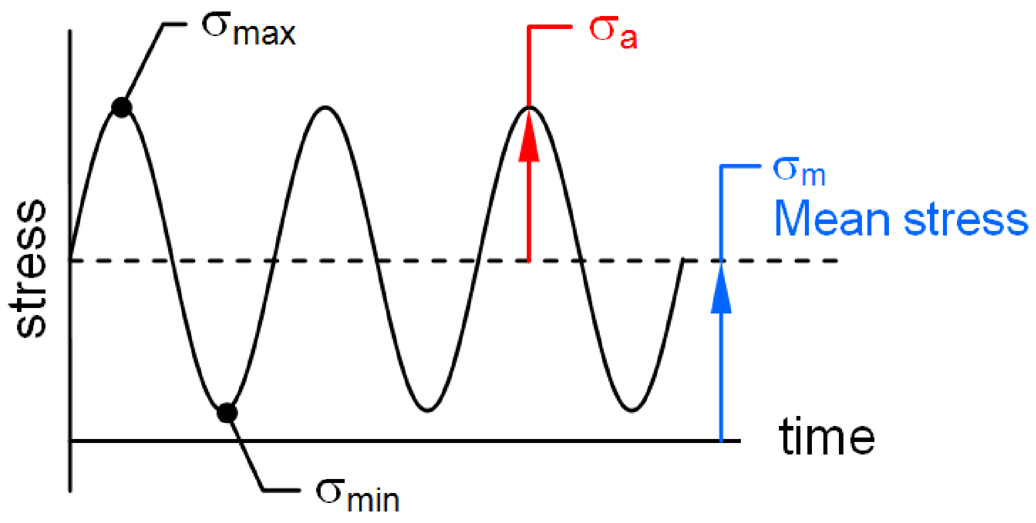
- Brittle Materials - FT:
  - ***Maximum Normal Stress*** - Uniaxial stress
  - ***Modified Mohr*** - Biaxial stress

- Ductile Materials - FT:
  - ***Yield Strength*** - Uniaxial stress
  - ***Maximum Shear Strength*** - Biaxial stress
  - ***Distortion Energy*** - Biaxial or Triaxial

Predictions of Failure
Fluctuating Loads

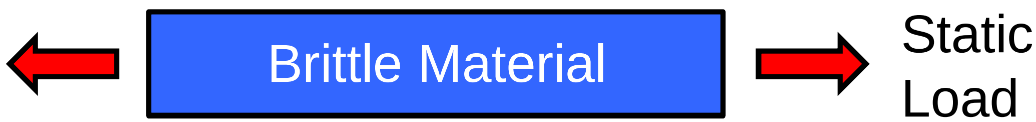
- Brittle Materials:
  - Not recommended

- Ductile Materials:
  - Goodman
  - Gerber
  - Soderberg



Maximum Normal Stress

- Uniaxial Static Loads on Brittle Material:



– In tension:

DESIGN:

$$\sigma_{\max} = K_t \sigma \leq \sigma_d = \frac{S_{ut}}{N}$$

ANALYSIS:

$$N = \frac{S_{ut}}{\sigma_{\max}}$$

– In compression:

DESIGN:

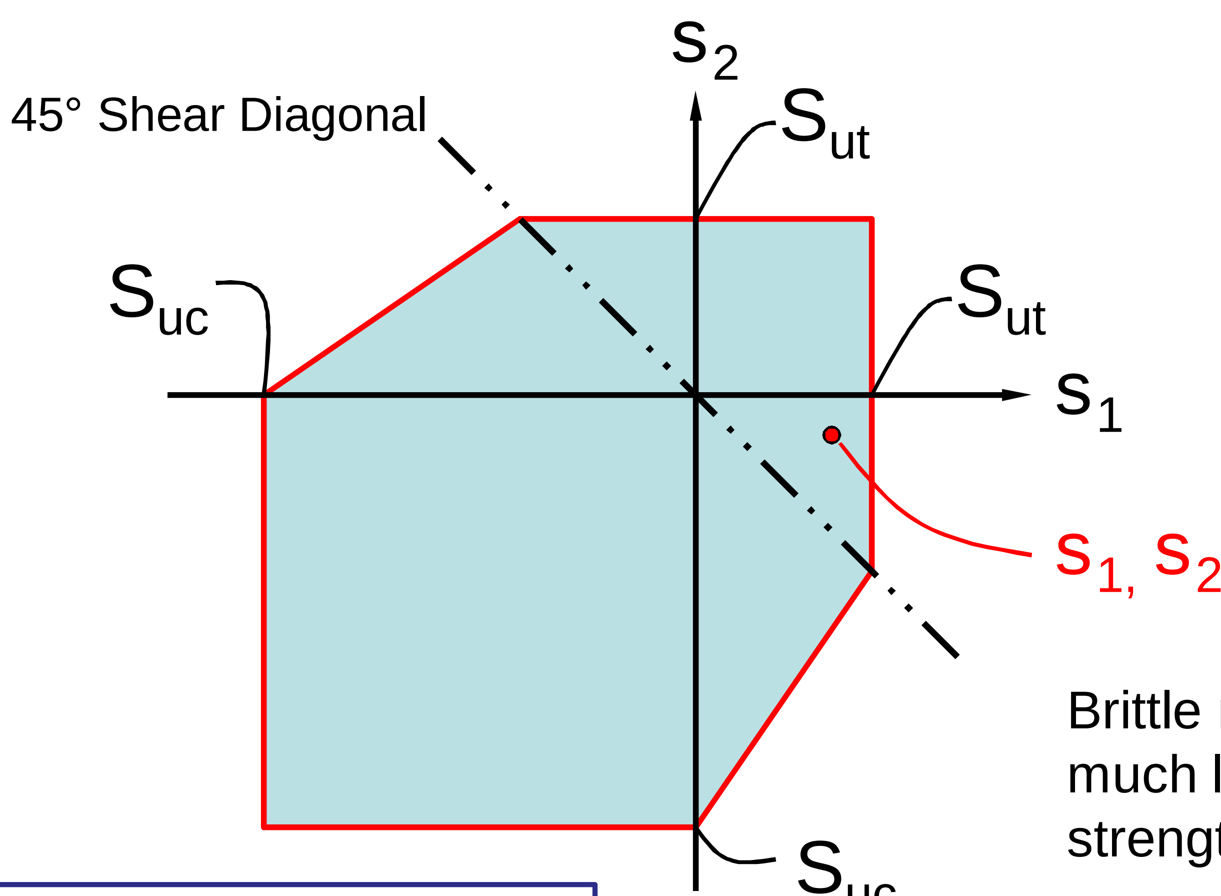
$$\sigma_{\max} = K_t \sigma \leq \sigma_d = \frac{S_{uc}}{N}$$

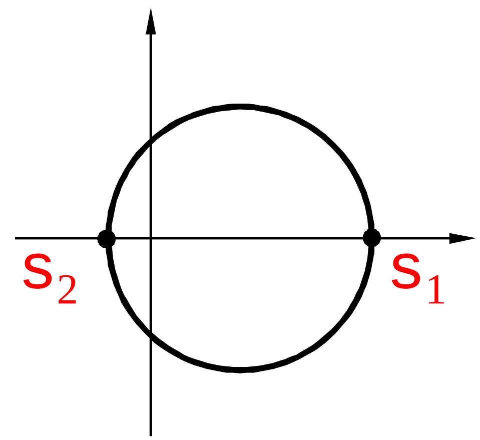
ANALYSIS:

$$N = \frac{S_{ut}}{\sigma_{\max}}$$

Modified Mohr Method

- Biaxial Static Stress on Brittle Materials





Stress concentrations applied to stresses before making the circle

Brittle materials often have a much larger compressive strength than tensile strength

Failure when outside of shaded area

Yield Strength Method

- Uniaxial Static Stress on Ductile Materials

– In tension:

DESIGN:

$$\sigma_{\max} \leq \sigma_d = \frac{S_{yt}}{N}$$

ANALYSIS:

$$N = \frac{S_{yt}}{\sigma_{\max}}$$

– In compression:

DESIGN:

$$\sigma_{\max} \leq \sigma_d = \frac{S_{yc}}{N}$$

ANALYSIS:

$$N = \frac{S_{yc}}{\sigma_{\max}}$$

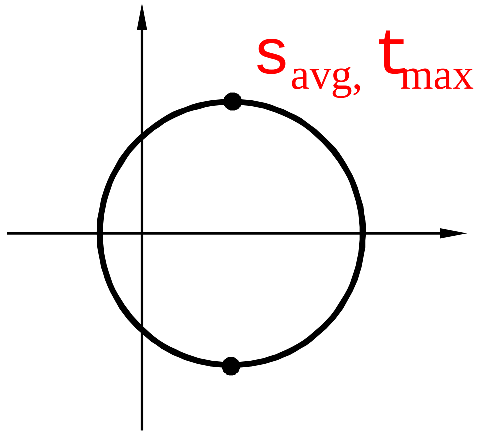
For most ductile materials, $S_{yt} = S_{yc}$

Maximum Shear Stress

- Biaxial Static Stress on Ductile Materials

DESIGN:

$$\tau_{max} \leq \tau_d = \frac{S_{ys}}{N} = \frac{S_y}{2N}$$



ANALYSIS:

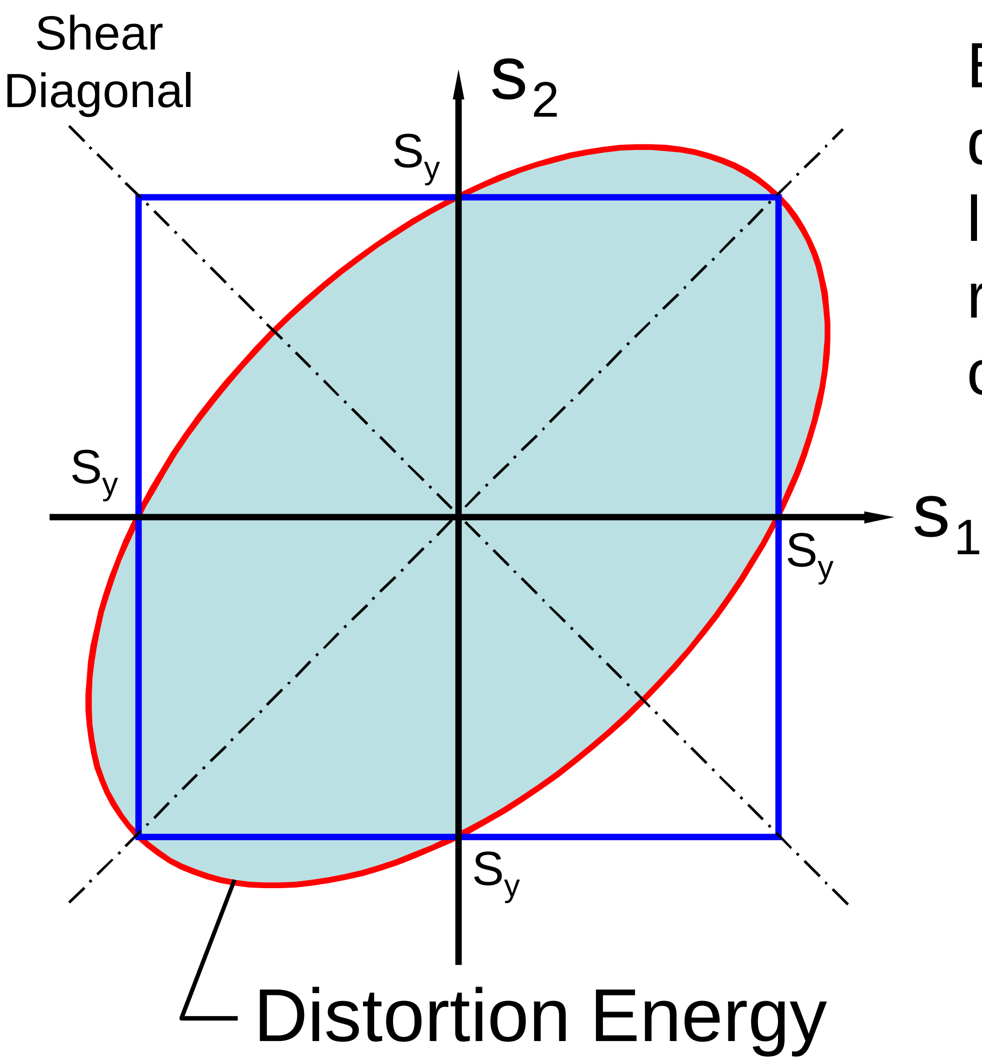
$$N = \frac{S_{ys}}{\tau_{max}}$$

Ductile materials begin to yield when the maximum shear stress in a load-carrying component exceeds that in a tensile-test specimen when yielding begins.

Somewhat conservative approach – use the Distortion Energy Method for a more precise failure estimate

Distortion Energy

- Static Biaxial or Triaxial Stress on Ductile Materials



Best predictor of failure for ductile materials under static loads or under completely reversed normal, shear or combined stresses.

$$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$$

s’ = *von Mises* stress

Failure: s’ > S_y

Design: s’ ≤ $s_d = S_y/N$

ANALYSIS: $N = S_y / s'$

von Mises Stress

- Alternate Form

$$\sigma' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2}$$

For uniaxial stress when $s_y = 0$, $\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$

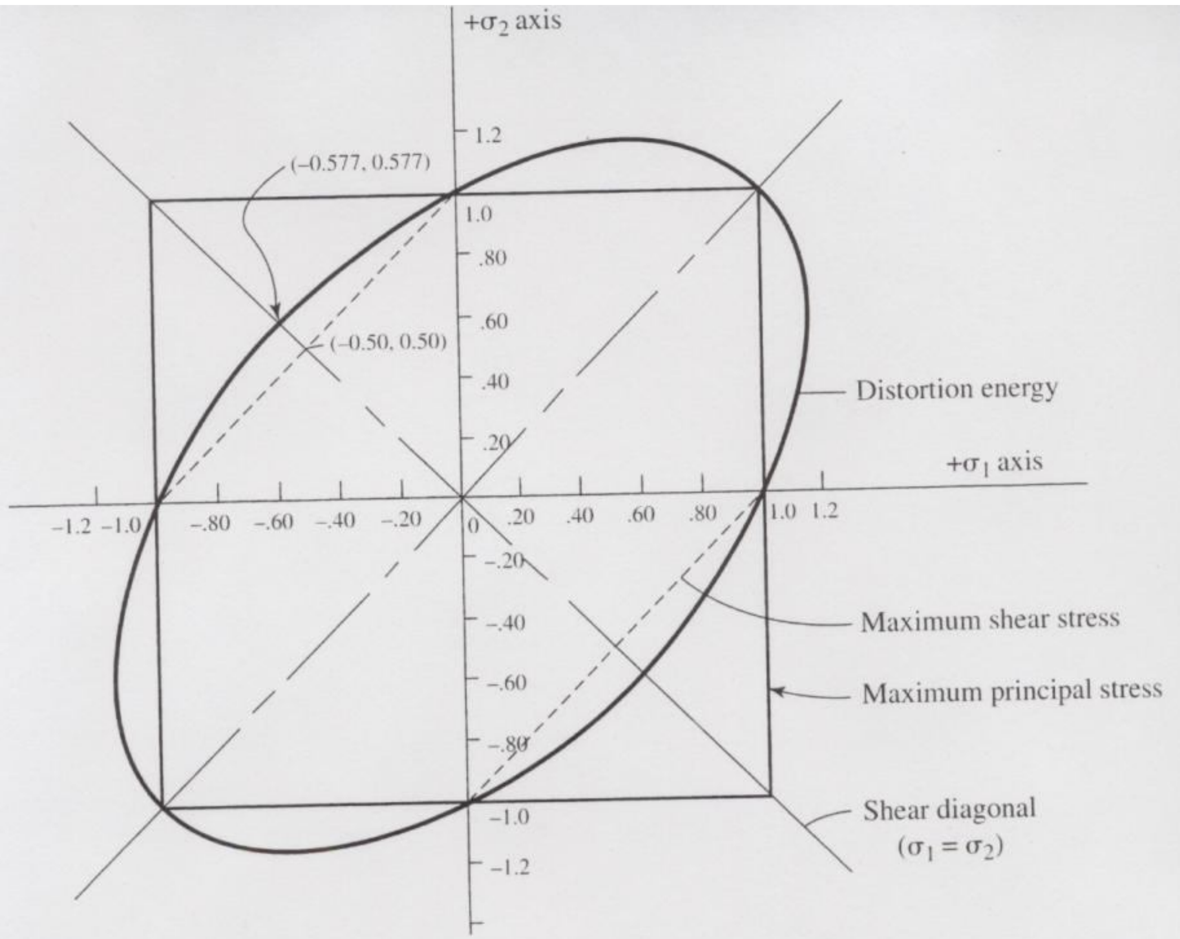
- Triaxial Distortion Energy $(s_1 > s_2 > s_3)$

$$\sigma' = \left(\frac{\sqrt{2}}{2}\right)\sqrt{(\sigma_2 - \sigma_1)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_3 - \sigma_2)^2}$$

Comparison of Static Failure Theories:

Shows “no failure” zones

FIGURE 5–13
Distortion energy method compared with maximum shear stress and maximum principal stress methods



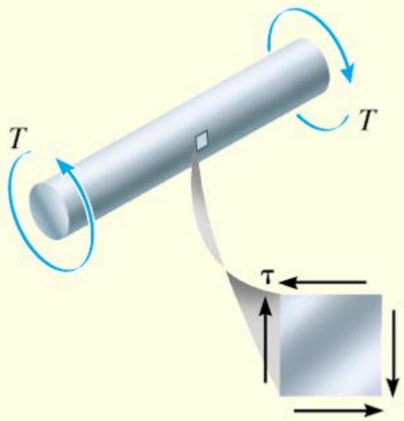
Maximum Shear – most conservative

Summary Static Failure Theories:

- Brittle materials fail on planes of max normal stress:
  - Max Normal Stress Theory
  - Modified Mohr Theory
- Ductile materials fail on planes of max shear stress:
  - Max shear stress theory
  - Distortion energy theory
- See summary table!
- Do example problems for static loading!

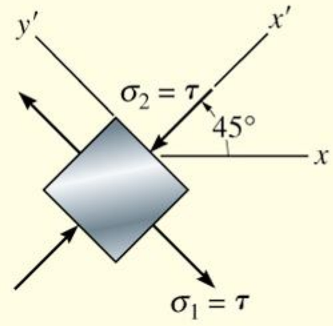
Brittle failure or ductile failure? Key: is the fracture surface on a plane of max shear or max normal stress.

TORQUE:



(a)

DUCTILE

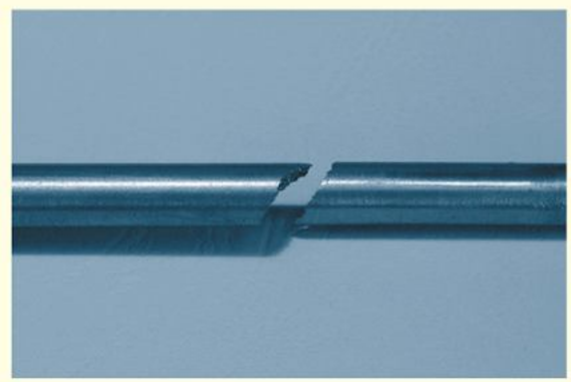


Ans.

(b)

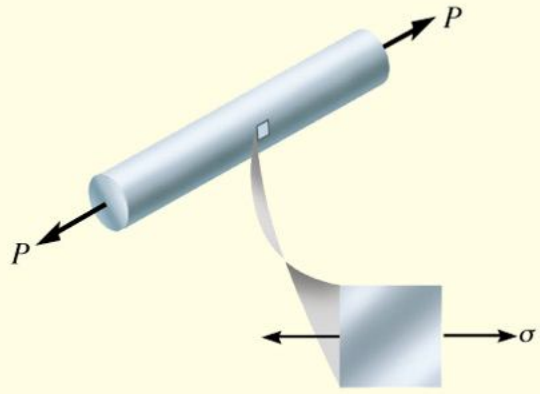
Fig. 9–11

$-\tau$

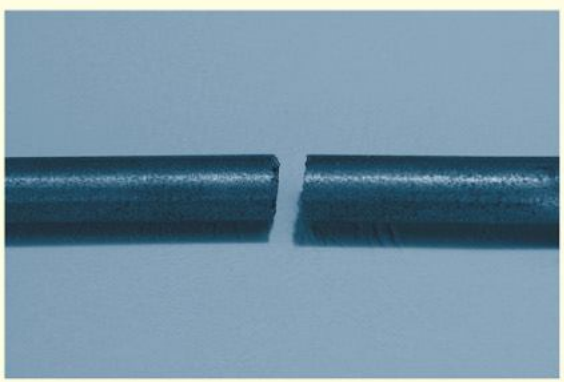


BRITTLE

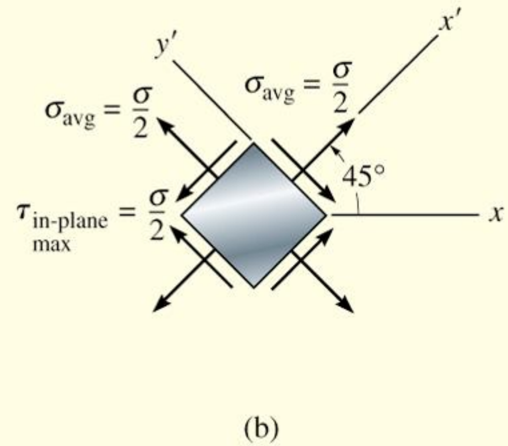
AXIAL



(a)

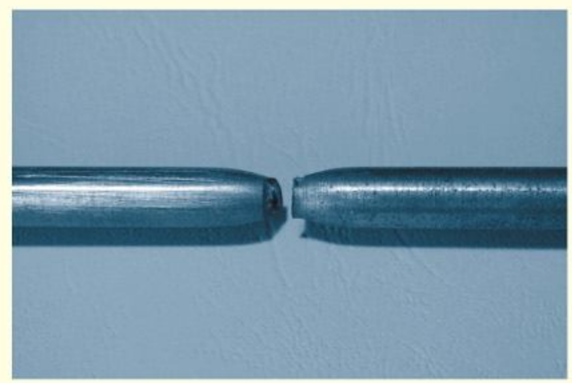


Brittle



(b)

Fig. 9–12

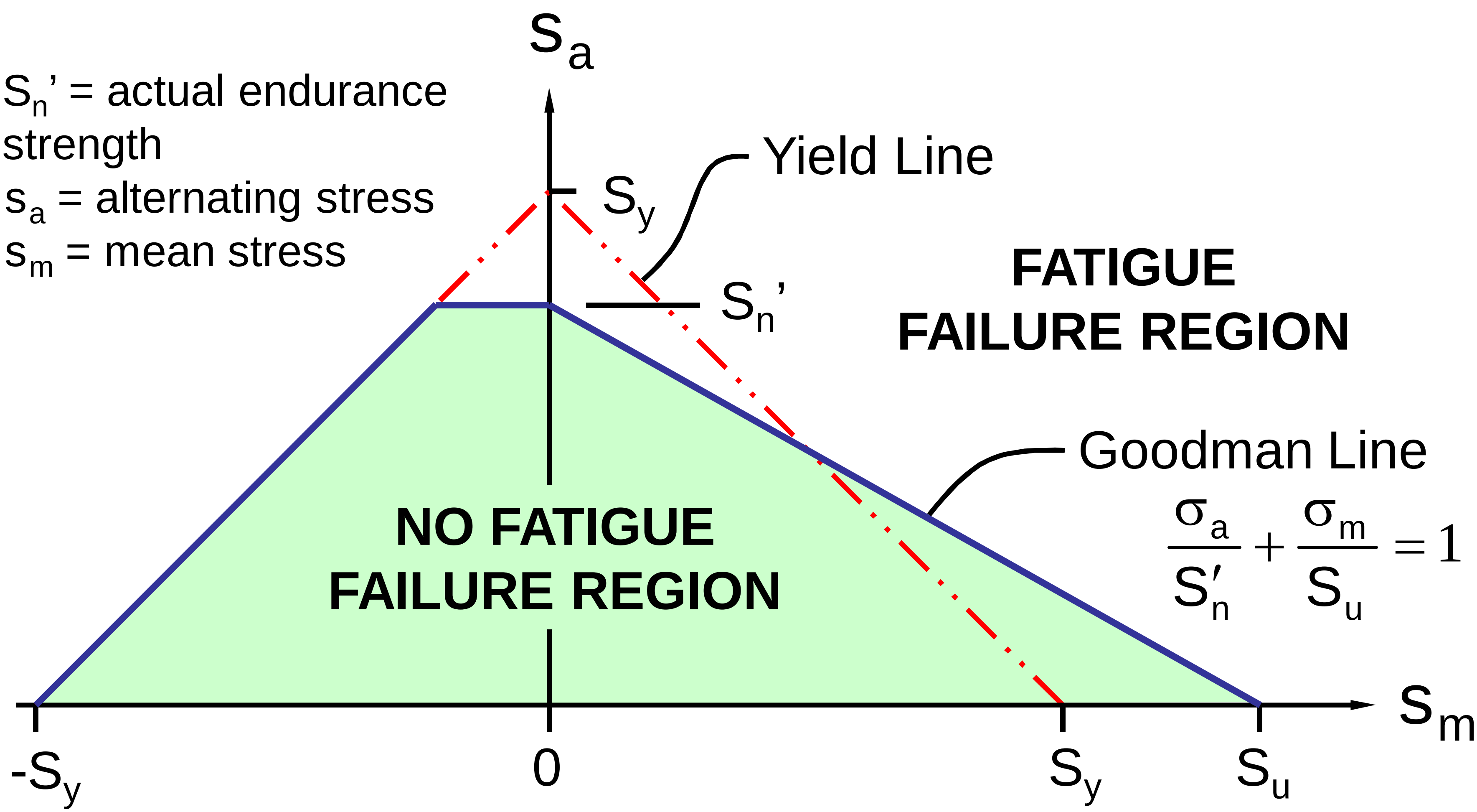


Ductile

Goodman Method

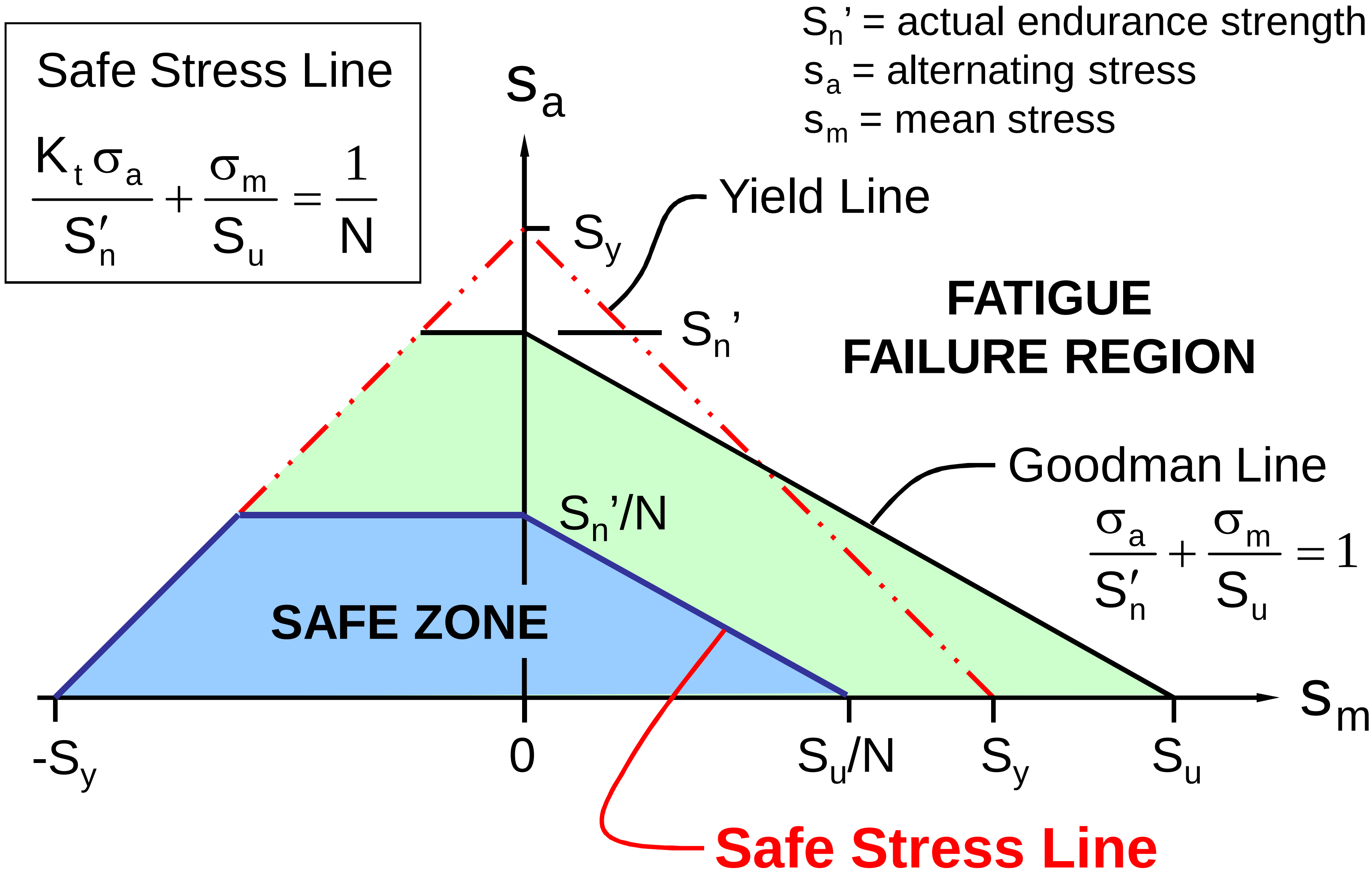
Good predictor of failure in ductile materials experiencing fluctuating stress

S_n' = actual endurance strength
s_a = alternating stress
s_m = mean stress



$$\frac{\sigma_a}{S_n'} + \frac{\sigma_m}{S_u} = 1$$

Goodman Diagram



Actual Endurance Strength

$$S_n' = S_n(C_m)(C_{st})(C_R)(C_S)$$

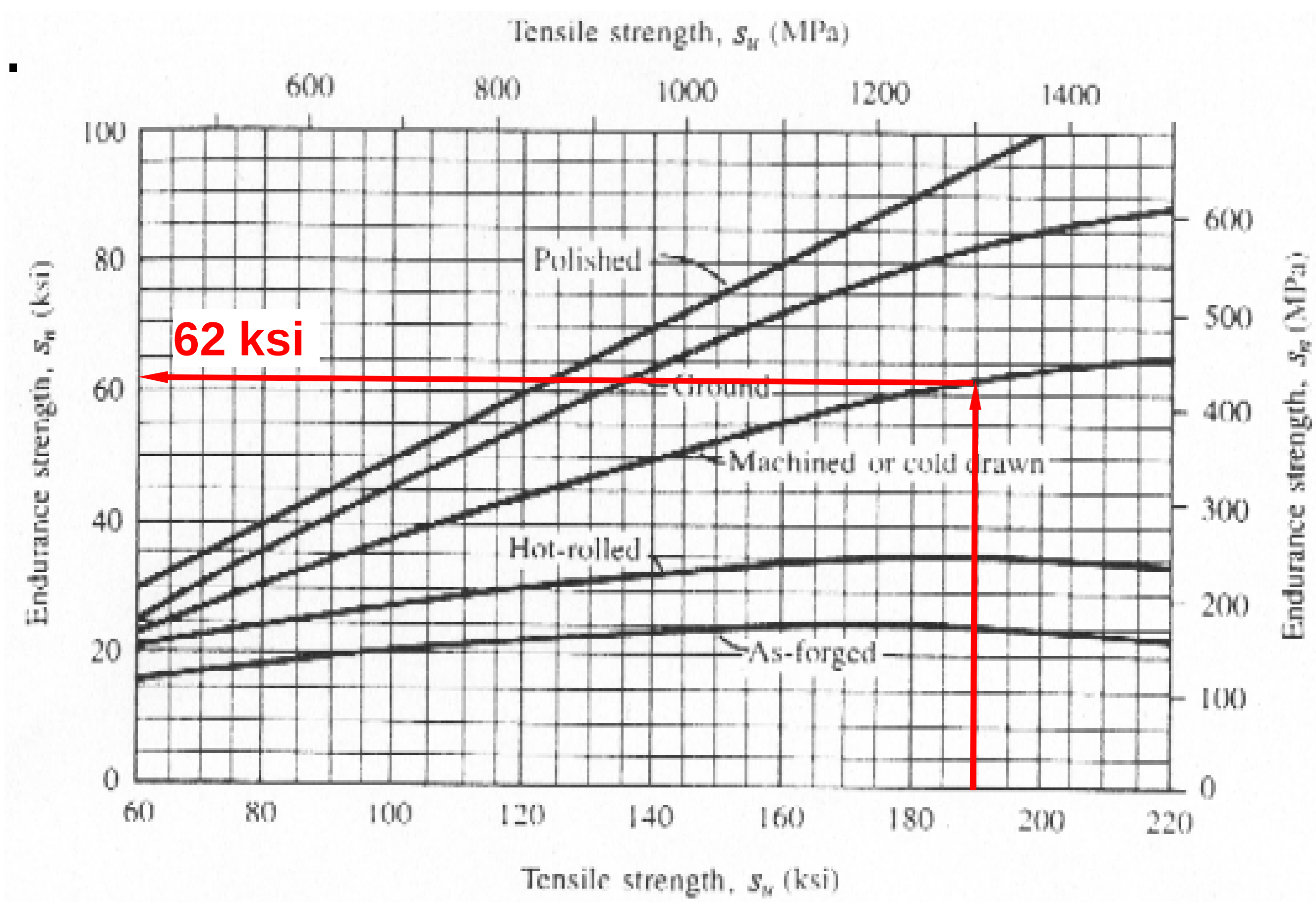
S_n'	= actual endurance strength (ESTIMATE)	
S_n	= endurance strength from Fig. 5-8	
C_m	= material factor (pg. 174)	
C_{st}	= stress type:	1.0 for bending
		0.8 for axial tension
		0.577 for shear
C_R	= reliability factor	
C_S	= size factor	

Actual S_n Example

- Find the endurance strength for a valve stem made of AISI 4340 OQT 900°F steel.

From Fig. A4-5.
S_u = 190 ksi

From Fig. 5-8.
S_n = 62 ksi
(machined)



Actual S_n Example Continued

$$S_n' = S_n(C_m)(C_{st})(C_R)(C_S)$$

$$= 62\text{ ksi}(1.0)(.8)(.81)(.94) = \mathbf{37.8\text{ ksi}}$$

S_n, Table 5-8

Wrought Steel

Axial Tension

Reliability, Table 5-1

99% Probability S_n' is at or above the calculated value

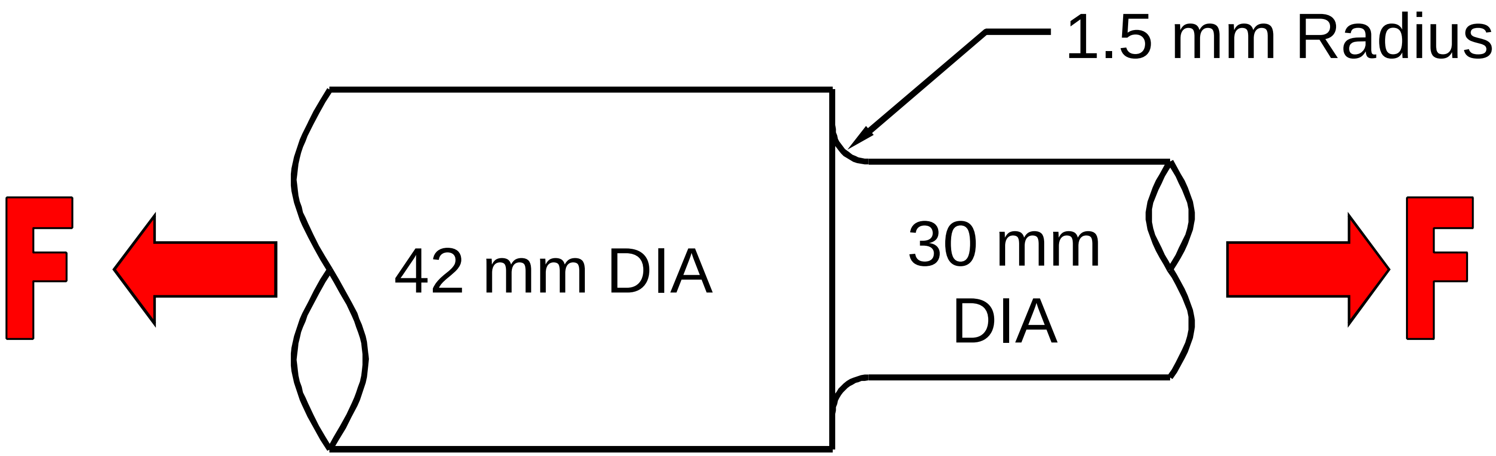
Size Factor, Fig. 5-9

Guessing: diameter ≈ .5”

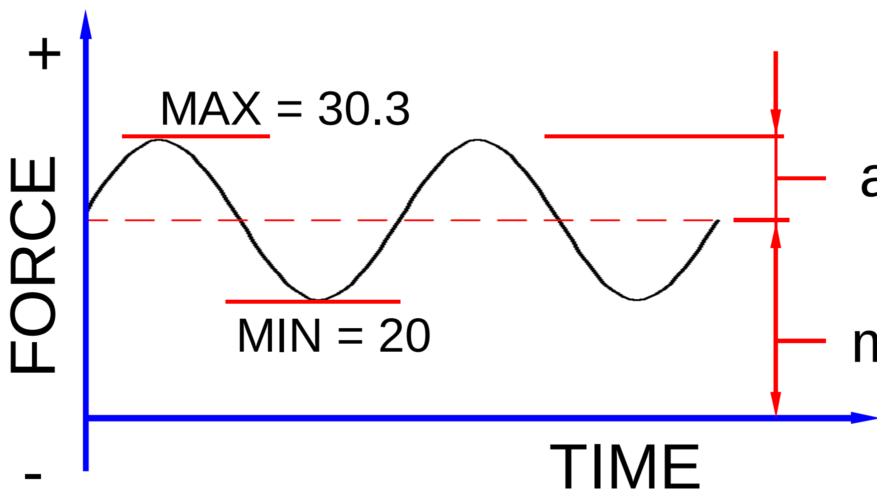
Actual S_n' Estimate

Example: Problem 5-53.

Find a suitable titanium alloy. N = 3



F varies from 20 to 30.3 kN



$$\text{alt} = \frac{30.3 - 20}{2} = 5.15 \text{ kN}$$

$$\text{mean} = \frac{30.3 + 20}{2} = 25.15 \text{ kN}$$

Example: Problem 5-53 continued.

- Find the mean stress:

$$\sigma_m = \frac{25{,}150\ \text{N}}{\frac{\pi}{4}(30\ \text{mm})^2} = \mathbf{35.6\ MPa}$$

- Find the alternating stress:

$$\sigma_a = \frac{5{,}150\ \text{N}}{\frac{\pi}{4}(30\ \text{mm})^2} = \mathbf{7.3\ MPa}$$

- Stress concentration from App. A15-1:

$$\frac{D}{d} = \frac{42\ \text{mm}}{30\ \text{mm}} = 1.4; \quad \frac{r}{d} = \frac{1.5\ \text{mm}}{30\ \text{mm}} = .05 \quad \Rightarrow \mathbf{K_t} = \mathbf{2.3}$$

Example: Problem 5-53 continued.

- S_n data not available for titanium so we will guess! Assume $S_n = S_u/4$ for extra safety factor.
- **TRY** T2-65A, S_u = 448 MPa, S_y = 379 MPa

$$\frac{K_t \sigma_a}{S'_n} + \frac{\sigma_m}{S_u} = \frac{1}{N}$$ (Eqn 5-20)

$$\frac{2.3(7.3\,MPa)}{.8(.86)(448\,MPa/4)} + \frac{35.6\,MPa}{448\,MPa} = \frac{1}{N} = .297$$

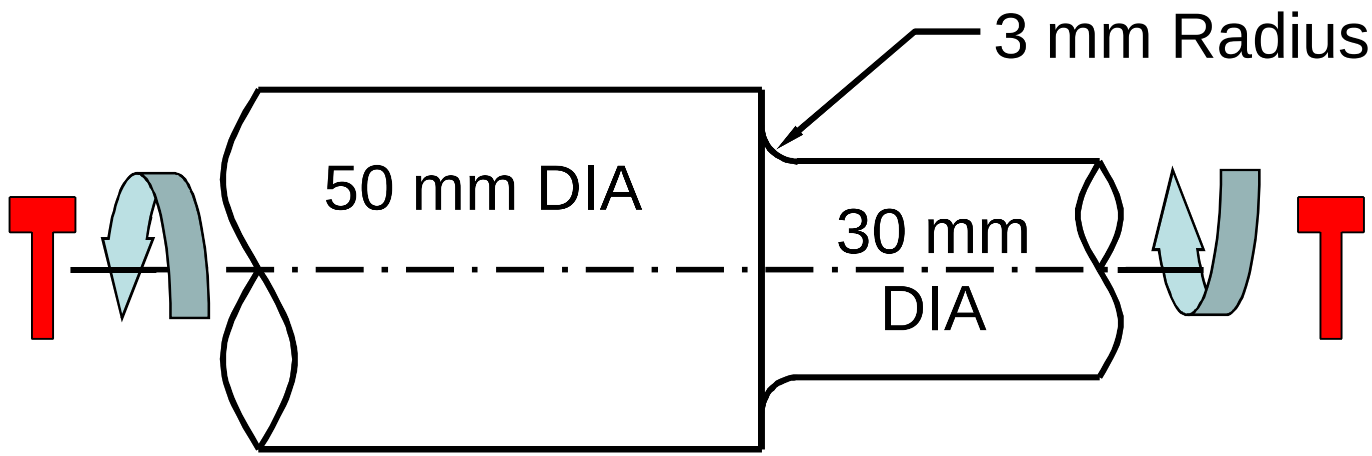
.8 — Tension; .86 — Size; Reliability 50%

$$N = \frac{1}{.297} = \mathbf{3.36}$$

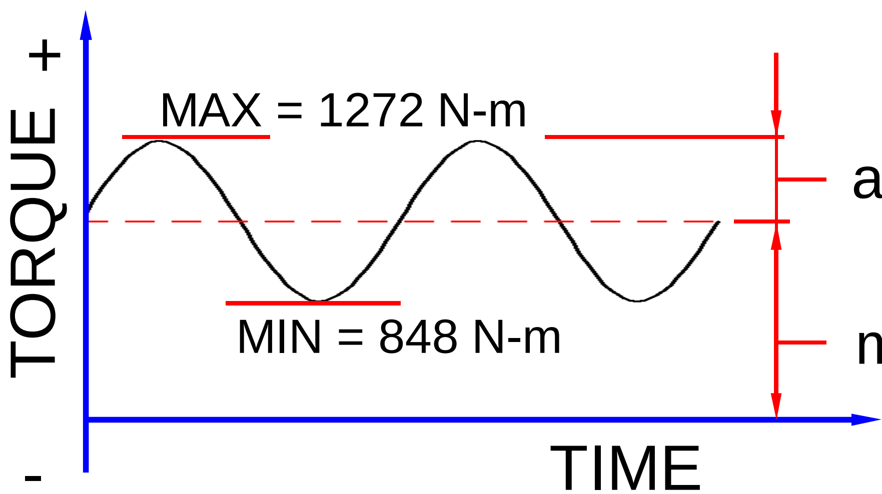
3.36 is good, need further information on Sn for titanium.

Example:

Find a suitable steel for N = 3 & 90% reliable.



T varies from 848 N-m to 1272 N-m



$$\text{alt} = \frac{1272 - 848}{2} = 212\ \text{N}\cdot\text{m}$$

$$\text{mean} = \frac{1272 + 848}{2} = 1060\ \text{N}\cdot\text{m}$$

T = 1060 ± 212 N-m

Example: continued.

- Stress concentration from App. A15-1:

$$\frac{D}{d} = \frac{50\text{ mm}}{30\text{ mm}} = 1.667; \quad \frac{r}{d} = \frac{3\text{ mm}}{30\text{ mm}} = .1 \quad \Rightarrow \mathbf{K_t} = \mathbf{1.38}$$

- Find the mean shear stress:

$$\tau_m = \frac{T_m}{Z_p} = \frac{1060\text{ N}\cdot\text{m}(1000\,\frac{\text{mm}}{\text{m}})}{\frac{\pi}{16}(30\text{ mm})^3} = \mathbf{200\ MPa}$$

- Find the alternating shear stress:

$$\tau_a = \frac{T_a}{Z_p} = \frac{212000\text{ N}\cdot\text{mm}}{5301\,\text{mm}^3} = \mathbf{40\ MPa}$$

Example: continued.

- So, $t = 200 \pm 40$ MPa. Guess a material.

TRY: AISI 1040 OQT 400°F

S_u = 779 MPa, S_y = 600 MPa, %E = 19%

Ductile

- Verify that $t_{max} \leq S_{ys}$:

$$t_{max} = 200 + 40 = 240 \text{ MPa} \leq S_{ys} \approx 600/2 = 300\text{MPa}$$

- Find the ultimate shear stress:

$$S_{us} = .75S_u = .75(779 \text{ MPa}) = 584 \text{ MPa}$$

Example: continued.

- Assume machined surface, $S_n \approx 295$ MPa

(Fig. 5-8)

- Find actual endurance strength:

$$S'_{sn} = S_n(C_m)(C_{st})(C_R)(C_S)$$

$$= \underline{295\text{ MPa}}(1.0)(.577)(.9)(.86) = \mathbf{132\ MPa}$$

S_n

Wrought steel

Shear Stress

90% Reliability

Size – 30 mm

Example: continued.

- Goodman: $$\frac{K_t \tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$$ (Eqn. 5-28)

$$\frac{1.38(40\,MPa)}{132\ MPa} + \frac{200\,MPa}{584\,MPa} = \frac{1}{N} = .7606$$

$$N = \frac{1}{.7606} = \mathbf{1.31}$$

No Good!!! We wanted $N \geq 3$

Need a material with Su about 3 times bigger than this guess or/and a better surface finish on the part.

Example: continued.

- Guess another material.

TRY: AISI 1340 OQT 700°F

S_u = 1520 MPa, S_y = 1360 MPa, %E = 10%

Ductile

- Find the ultimate shear stress:

S_{us} = .75S_u = .75(779 MPa) = 584 MPa

- Find actual endurance strength:

S'_{sn} = $S_n(C_m)(C_{st})(C_R)(C_S)$

= 610 MPa(1.0)(.577)(.9)(.86) = **272 MPa**

S_n shear size

wrought reliable

Example: continued.

- Goodman: $$\frac{K_t\,\tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$$ (Eqn. 5-28)

$$\frac{1.38(40\,\text{MPa})}{272\ \text{MPa}} + \frac{200\,\text{MPa}}{1140\,\text{MPa}} = \frac{1}{N} = .378$$

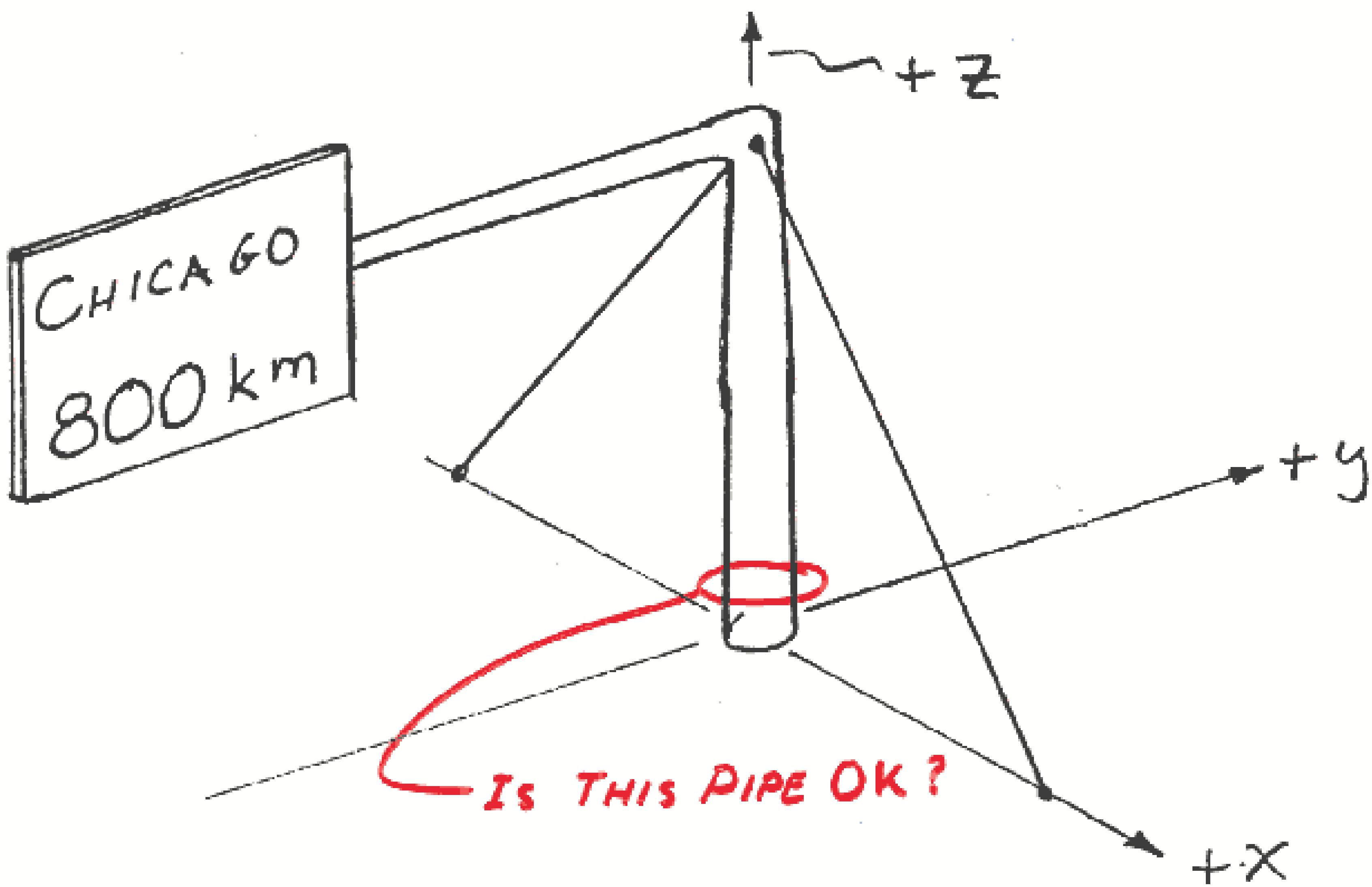
$$N = \frac{1}{.378} = \mathbf{2.64}$$

No Good!!! We wanted $N \geq 3$

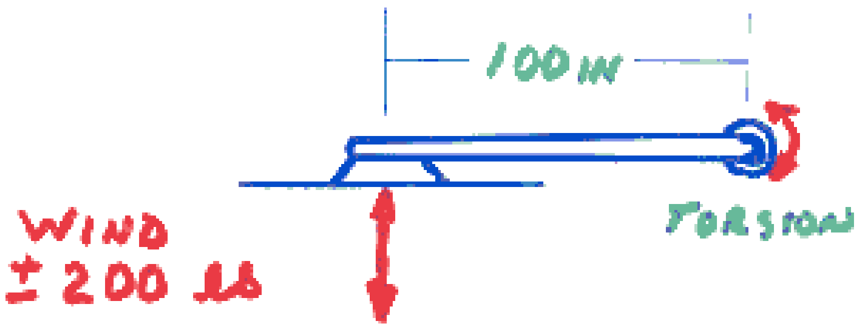
Decision Point:

- Accept 2.64 as close enough to 3.0?
- Go to polished surface?
- Change dimensions? Material? (Can't do much better in steel since S_n does not improve much for S_u > 1500 MPa

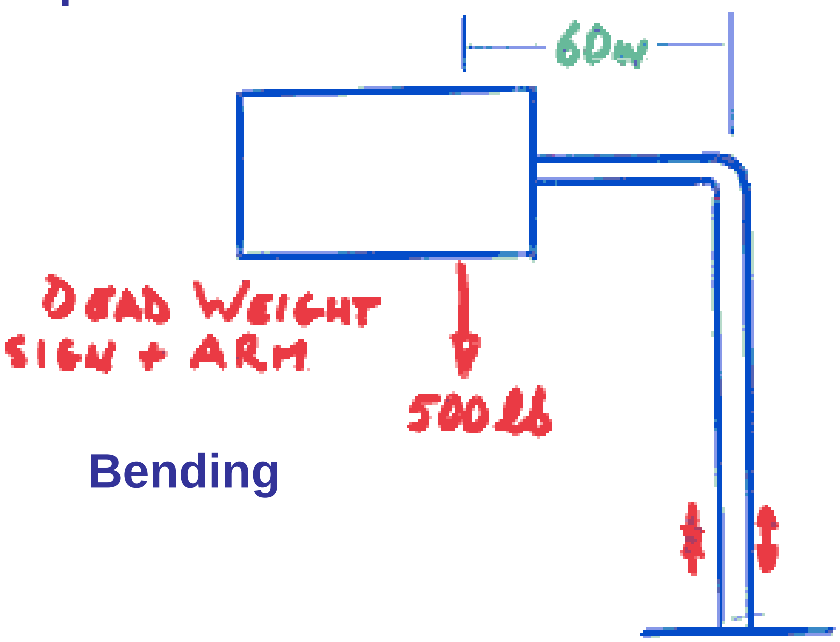
Example: Combined Stress Fatigue



Example: Combined Stress Fatigue Cont'd



Reversed, Repeated

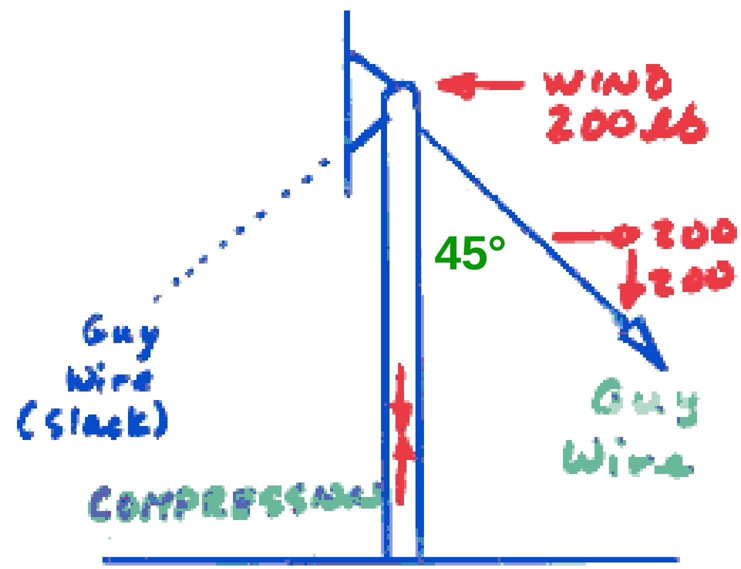


Bending

PIPE: TS4 x .237 WALL

MATERIAL: ASTM A242 Equivalent

DEAD WEIGHT:
SIGN + ARM + POST = **1000#**
(Compression)



Repeated one direction

Example: Combined Stress Fatigue Cont'd

Stress Analysis:

Dead Weight:

$$\sigma = \frac{P}{A} = \frac{1000\#}{3.17\ in^2} = 315.5\ psi \qquad \text{(Static)}$$

Vertical from Wind:

$$\sigma = \frac{P}{A} = \frac{200\#}{3.17\ in^2} = 63.09\ psi \qquad \text{(Cyclic)}$$

Bending:

$$\sigma = \frac{M}{Z} = \frac{500\#(60\ in)}{3.21\ in^3} = 9345.8\ psi \qquad \text{(Static)}$$

Example: Combined Stress Fatigue Cont'd

Stress Analysis:

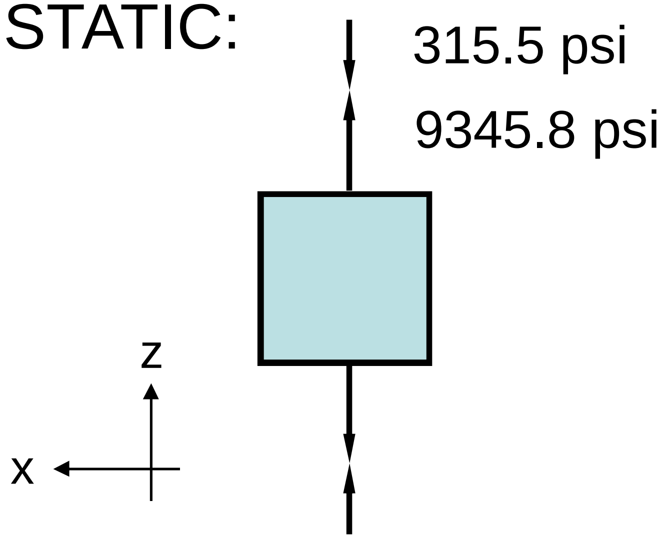
Torsion:

$$\tau = \frac{T}{Z_P} = \frac{200\#(100\,in)}{2(3.21\,in^3)} = 3115.3\ psi \quad \text{(Cyclic)}$$

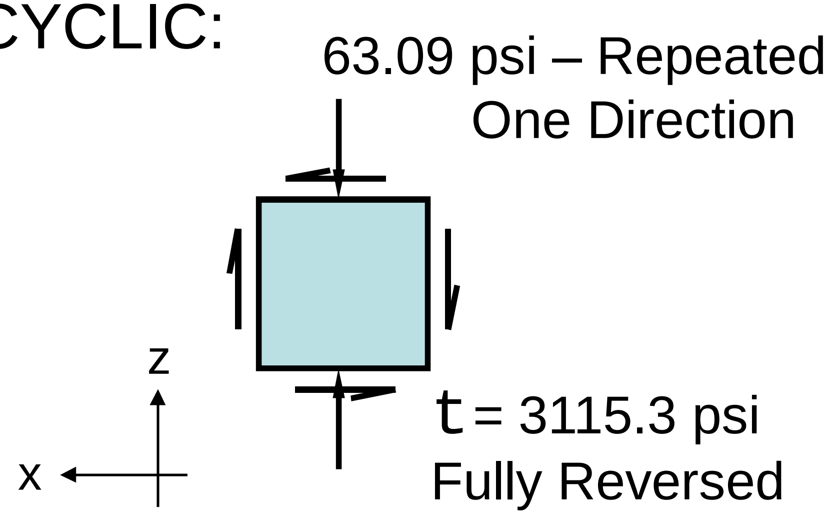
Stress Elements:

(Viewed from +y)

STATIC:



CYCLIC:



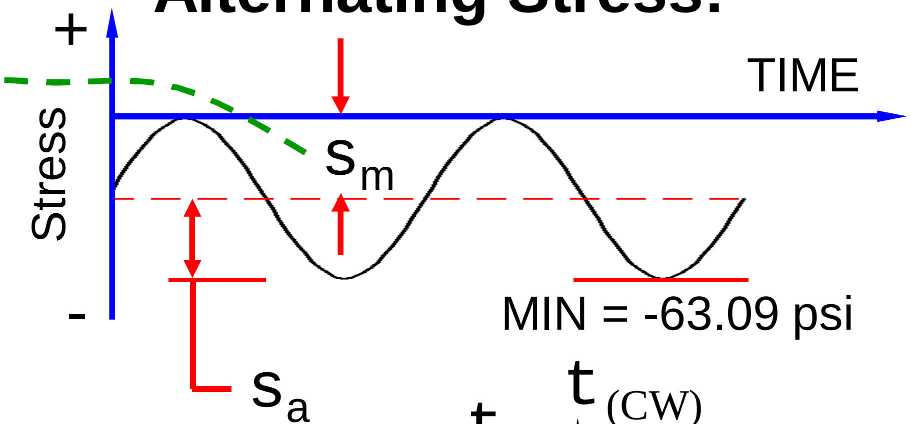
Example: Combined Stress Fatigue Cont'd

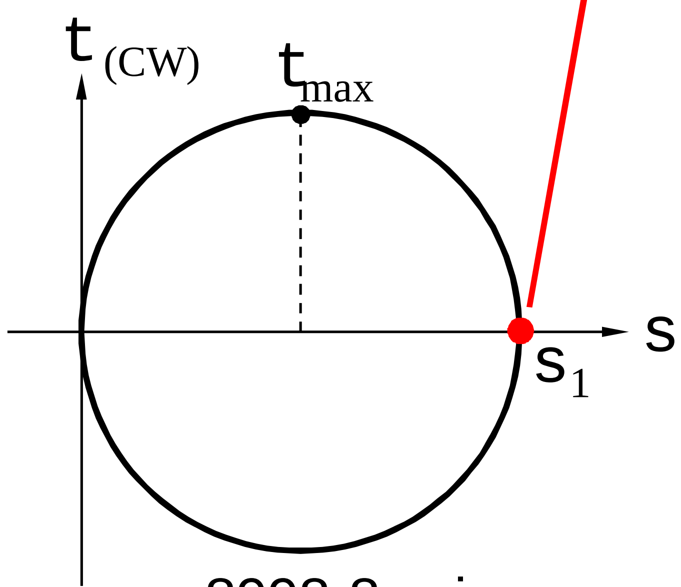
Mean Stress:

9345.8 } Static
-315.5
-31.5 } Repeated / 2

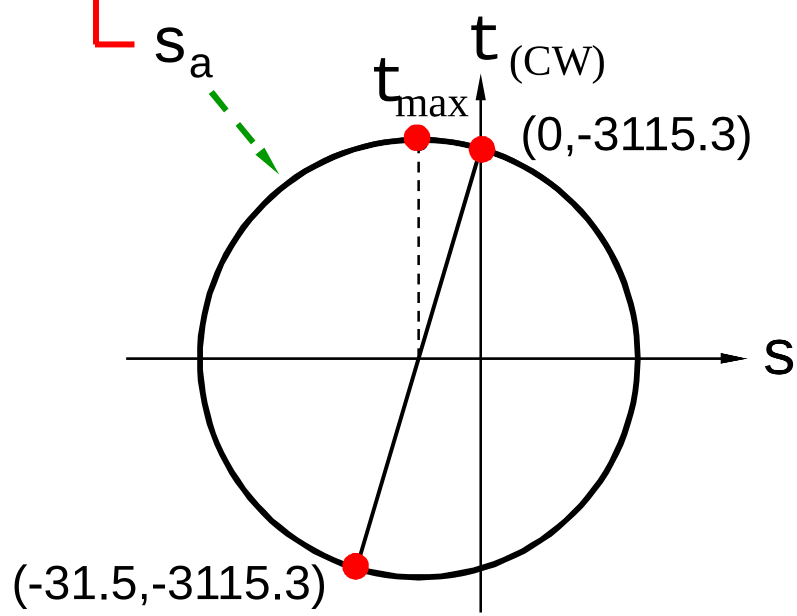
8998.8 psi

Alternating Stress:





$$\tau_{max} = \frac{8998.8 \text{ psi}}{2} = 4499.4 \text{ psi}$$



$$\tau_{max} = 3115.34 \text{ psi}$$

Example: Combined Stress Fatigue Cont'd

Determine Strength:

Try for N = 3 → some uncertainty

Size Factor? OD = 4.50 in, Wall thickness = .237 in
ID = 4.50" – 2(.237") = 4.026 in

Max. stress at OD. The stress declines to 95% at 95% of the OD = .95(4.50") = 4.275 in. Therefore, amount of steel at or above 95% stress is the same as in 4.50" solid.

ASTM A242: S_u = 70 ksi, S_y = 50 ksi, %E = 21%

$t \leq 3/4"$ Ductile

Example: Combined Stress Fatigue Cont'd

We must use S_{su} and S'_{sn} since this is a combined stress situation. (Case I1, page 197)

S_{us} = .75S_u = .75(70 ksi) = **52.5 ksi**

S'_{sn} = $S_n(C_m)(C_{st})(C_R)(C_S)$
= 23 ksi(1.0)(.577)(.9)(.745) = **8.9 ksi**

Hot Rolled Surface

Wrought steel

Combined or Shear Stress

90% Reliability

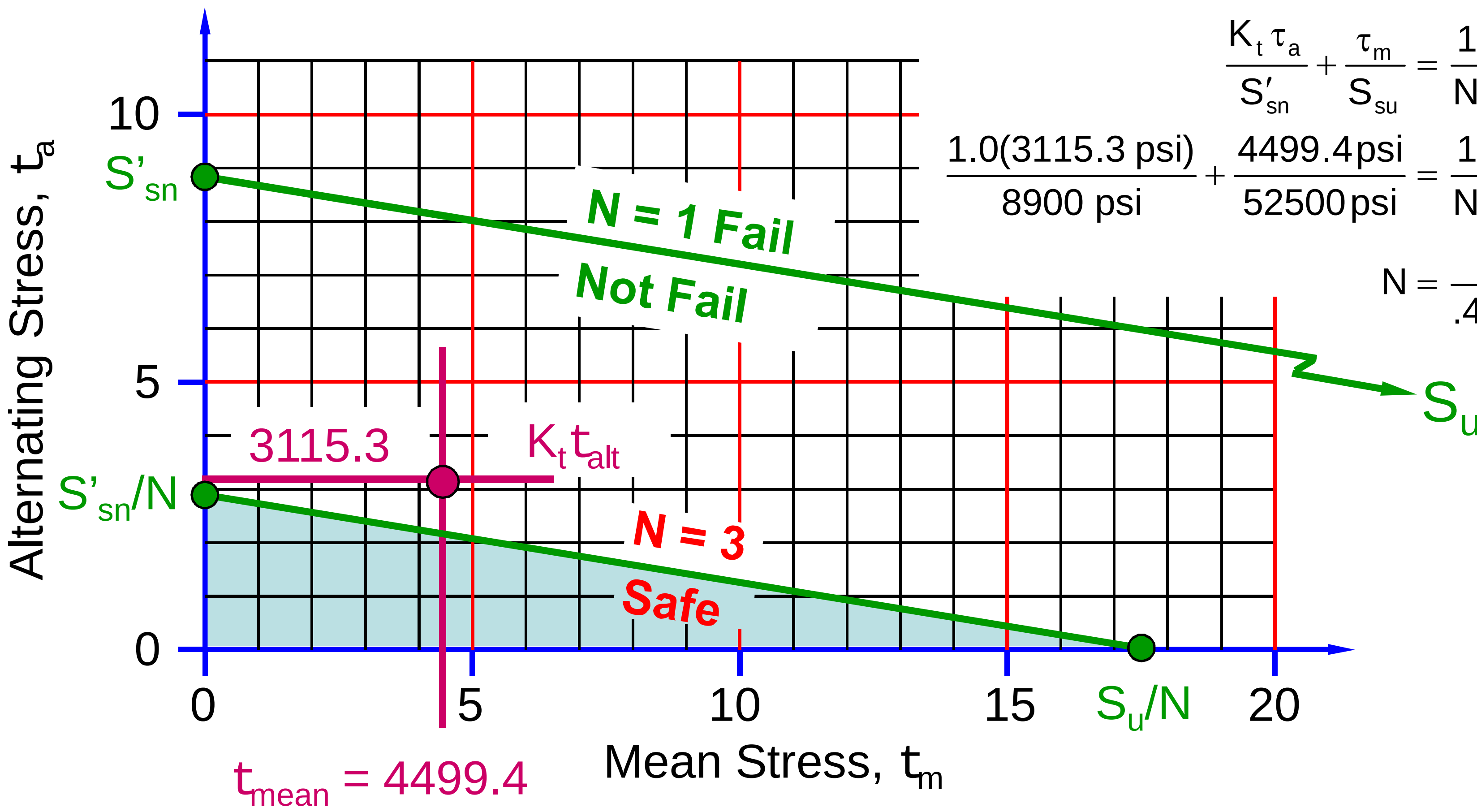
Size – 4.50" dia

Example: Combined Stress Fatigue Cont'd

"Safe" Line for Goodman Diagram:

t_a = S'_{sn} / N = 8.9 ksi / 3 = **2.97 ksi**

t_m = S_{su} / N = 52.5 ksi / 3 = **17.5 ksi**



$$\frac{K_t \tau_a}{S'_{sn}} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$$

$$\frac{1.0(3115.3\text{ psi})}{8900\text{ psi}} + \frac{4499.4\text{ psi}}{52500\text{ psi}} = \frac{1}{N} = .426$$

$$N = \frac{1}{.426} = \mathbf{2.29}$$

SUMMARY: FAILURE THEORIES!!

Design Factors, N
(a.k.a. Factor of Safety)

FOR DUCTILE MATERIALS:

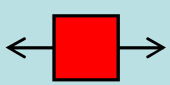
•N = 1.25 to 2.0	Static loading, high level of confidence in all design data
•N = 2.0 to 2.5	Dynamic loading, average confidence in all design data
•N = 2.5 to 4.0	Static or dynamic with uncertainty about loads, material properties, complex stress state, etc…

•N = 4.0 or higher Above + desire to provide extra safety

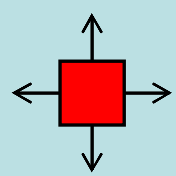
Failure Theory:	When Use?	Failure When:	Design Stress:
1. Maximum Normal Stress	Brittle Material/ Uniaxial Static Stress	$\sigma_{\max} = Kt * \sigma \geq Sut$ (for tension) $\sigma_{\max} = Kt * \sigma \geq Suc$ (for compression)	$\sigma_d = Sut / N$ (for tension) $\sigma_d = Suc / N$ (for compression)
2. Yield Strength (Basis for MCH T 213)	Ductile Material/ Uniaxial Static Normal Stress	$\sigma_{\max} \geq Syt$ (for tension) $\sigma_{\max} \geq Syc$ (for compression) Note: Syt $\approx$ Syc for ductile/wrought material	$\sigma_{\mathrm{d}} = Syt / N$ (for tension) $\sigma_{\mathrm{d}} = Syc / N$ (for compression)
3. Maximum Shear Stress (Basis for MCH T 213)	Ductile Material/ Bi-axial Static Stress	$\tau_{\max} \geq Sys$ where Sys $\approx$ Sy/2	$\tau_d = Sys / N$ where Sys $\approx$ Sy/2
4. Distortion Energy (von Mises)	Ductile Material/ Bi-axial Static Stress	$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} \geq Sy$ where σ' = von Mises stress	$\sigma'_d = Sy / N$ *see* Figure 5-13

Failure Theories for STATIC Loading

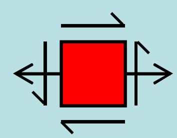
Uniaxial:



Bi-axial:



or



Failure Theory:	When Use?	Failure When:	Design Stress:
Failure Theories for FATIGUE Loading			
5. Goodman Method	a. Ductile Material/ Fluctuating Normal Stress (Fatigue Loading)	$\frac{K_t \sigma_a}{S_n'} + \frac{\sigma_m}{S_u} \geq 1$	$\frac{K_t \sigma_a}{S_n'} + \frac{\sigma_m}{S_u} = \frac{1}{N}$ *see* Figure 5.15
	b. Ductile Material/ Fluctuating Shear Stress (Fatigue Loading)	$\frac{K_t \tau_a}{S_{sn}'} + \frac{\tau_m}{S_{su}} \geq 1$ where $S_{sn}' = 0.577 S_n'$ and $S_{su} = 0.75 S_u$	$\frac{K_t \tau_a}{S_{sn}'} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$ where $S_{sn}' = 0.577 S_n'$ and $S_{su} = 0.75 S_u$
	c. Ductile Material/ Fluctuating Combined Stress (Fatigue Loading)	$\frac{K_t (\tau_a)_{\max}}{S_{sn}'} + \frac{(\tau_m)_{\max}}{S_{su}} \geq 1$ where $S_{sn}' = 0.577 S_n'$ and $S_{su} = 0.75 S_u$	$\frac{K_t (\tau_a)_{\max}}{S_{sn}'} + \frac{(\tau_m)_{\max}}{S_{su}} \geq 1$ where $S_{sn}' = 0.577 S_n'$ and $S_{su} = 0.75 S_u$

Failure Theory:	When Use?	Failure When:	Design Stress:
1. Maximum Normal Stress	Brittle Material/ Uniaxial Static Stress	$\sigma_{\max} = Kt * \sigma \geq Sut$ (for tension) $\sigma_{\max} = Kt * \sigma \geq Suc$ (for compression)	$\sigma_d = Sut / N$ (for tension) $\sigma_d = Suc / N$ (for compression)
2. Yield Strength (Basis for MCH T 213)	Ductile Material/ Uniaxial Static Normal Stress	$\sigma_{\max} \geq Syt$ (for tension) $\sigma_{\max} \geq Syc$ (for compression) Note: $\mathrm{Syt} \approx \mathrm{Syc}$ for ductile/wrought material	$\sigma_{\mathrm{d}} = Syt / N$ (for tension) $\sigma_{\mathrm{d}} = Syc / N$ (for compression)
3. Maximum Shear Stress (Basis for MCH T 213)	Ductile Material/ Bi-axial Static Stress	$\tau_{\max} \geq Sys$ where $\mathrm{Sys} \approx \mathrm{Sy}/2$	$\tau_d = Sys / N$ where $\mathrm{Sys} \approx \mathrm{Sy}/2$
4. Distortion Energy (von Mises)	Ductile Material/ Bi-axial Static Stress	$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \geq Sy$ where σ' = von Mises stress	$\sigma'_d = Sy / N$ *see* Figure 5-13
5. Goodman Method	a. Ductile Material/ Fluctuating Normal Stress (Fatigue Loading)	$\frac{K_t \sigma_a}{S_n'} + \frac{\sigma_m}{S_u} \geq 1$	$\frac{K_t \sigma_a}{S_n'} + \frac{\sigma_m}{S_u} = \frac{1}{N}$ *see* Figure 5.15
	b. Ductile Material/ Fluctuating Shear Stress (Fatigue Loading)	$\frac{K_t \tau_a}{S_{sn}'} + \frac{\tau_m}{S_{su}} \geq 1$ where $S_{sn}' = 0.577 S_n'$ and $S_{su} = 0.75 S_u$	$\frac{K_t \tau_a}{S_{sn}'} + \frac{\tau_m}{S_{su}} = \frac{1}{N}$ where $S_{sn}' = 0.577 S_n'$ and $S_{su} = 0.75 S_u$
	c. Ductile Material/ Fluctuating Combined Stress (Fatigue Loading)	$\frac{K_t (\tau_a)_{\max}}{S_{sn}'} + \frac{(\tau_m)_{\max}}{S_{su}} \geq 1$ where $S_{sn}' = 0.577 S_n'$ and $S_{su} = 0.75 S_u$	$\frac{K_t (\tau_a)_{\max}}{S_{sn}'} + \frac{(\tau_m)_{\max}}{S_{su}} \geq 1$ where $S_{sn}' = 0.577 S_n'$ and $S_{su} = 0.75 S_u$

General Comments:

1. Failure theory to use depends on material (ductile vs. brittle) and type of loading (static or dynamic). Note, ductile if elongation > 5%.
2. Ductile material static loads – ok to neglect Kt (stress concentrations)
3. Brittle material static loads – must use Kt
4. Terminology:
    - Su (or Sut) = ultimate strength in tension
    - Suc = ultimate strength in compression
    - Sy = yield strength in tension
    - Sys = 0.5*Sy = yield strength in shear
    - Sus = 0.75*Su = ultimate strength in shear
    - Sn = endurance strength = 0.5*Su or get from Fig 5-8 or S-N curve
    - S'n = estimated actual endurance strength = Sn(C_m) (C_{st}) (C_R) (C_s)
    - S'sn = 0.577* S'n = estimated actual endurance strength in shear

5.9 What Failure Theory to Use:

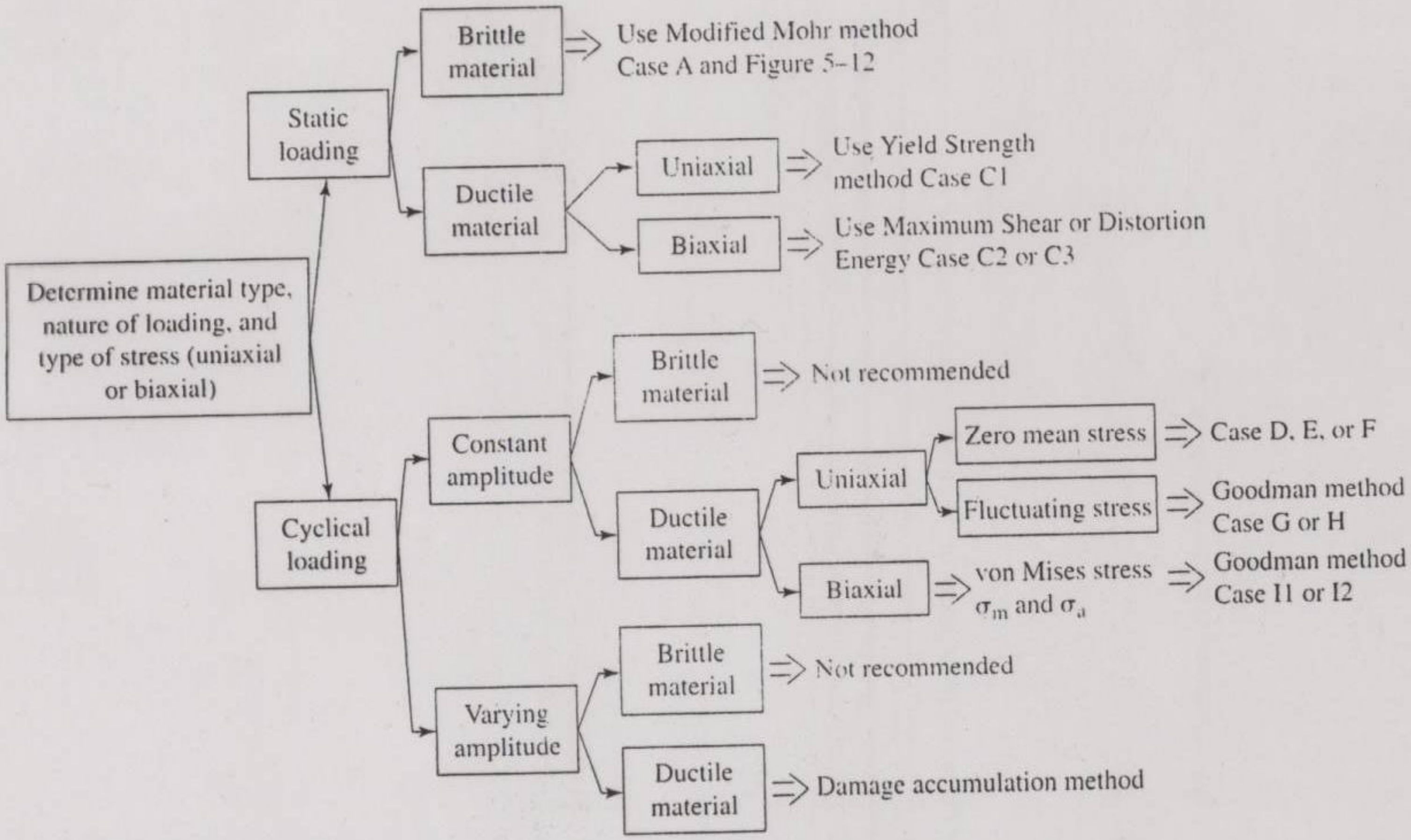


FIGURE 5–17 Logic diagram for visualizing methods of design analysis