Digital Transmission Via Carrier Modulation

Carrier Phase Modulation

BPSK(Binary PSK) : M=2

QPSK(Quadrature PSK): M=4

MPSK(M-ary PSK): M > 2

Carrier Phase Modulation

- Carrier phase is used to transmit digital information via digital phase modulation
  - M-ary phase modulation ($M=2^k$)
    - $\theta_m = \frac{2\pi m}{M}, \quad m = 0,1,\ldots,M-1$
  - Binary phase modulation (M=2)
    - $\theta_0 = 0, \theta_0 = \pi$
- M carrier phase modulated signal waveform
  - $$u_m(t) = A g_T(t) \cos(2\pi f_c t + \frac{2\pi m}{M}), \quad m = 0,1,\ldots,M-1$$

Transmitting filter pulse shape

Amplitude

PSK(Phase Shift Keying)

- Digital phase modulation is called PSK
- PSK signals have equal energy
  - See Figure 7.7 page 289
  - $$\Im_m = \int_{-\infty}^{\infty} u_m^2(t)dt = \frac{A^2}{2}\int_{-\infty}^{\infty} g_T^2(t)dt \equiv \Im_s, \quad for\ all\ m$$
    - Normalized power case (Rectangular $g_T(t)$)
      - $$g_T(t) = \sqrt{2/T},\ 0 \le t \le T \qquad \frac{1}{2}\int_{-\infty}^{\infty} g_T^2(t)dt = 1 \qquad A = \sqrt{\Im_s}$$
      - $$u_m(t) = \sqrt{\frac{2\Im_s}{T}}\cos(2® f_c t + \frac{2® m}{M}), \quad m = 0,1,\ldots,M-1$$

Constant envelope

PSK with 2 orthogonal basis

- Changing representation of waveform

$$u_m(t) = A g_T(t)\cos(2\pi f_c t + \frac{2\pi m}{M})$$

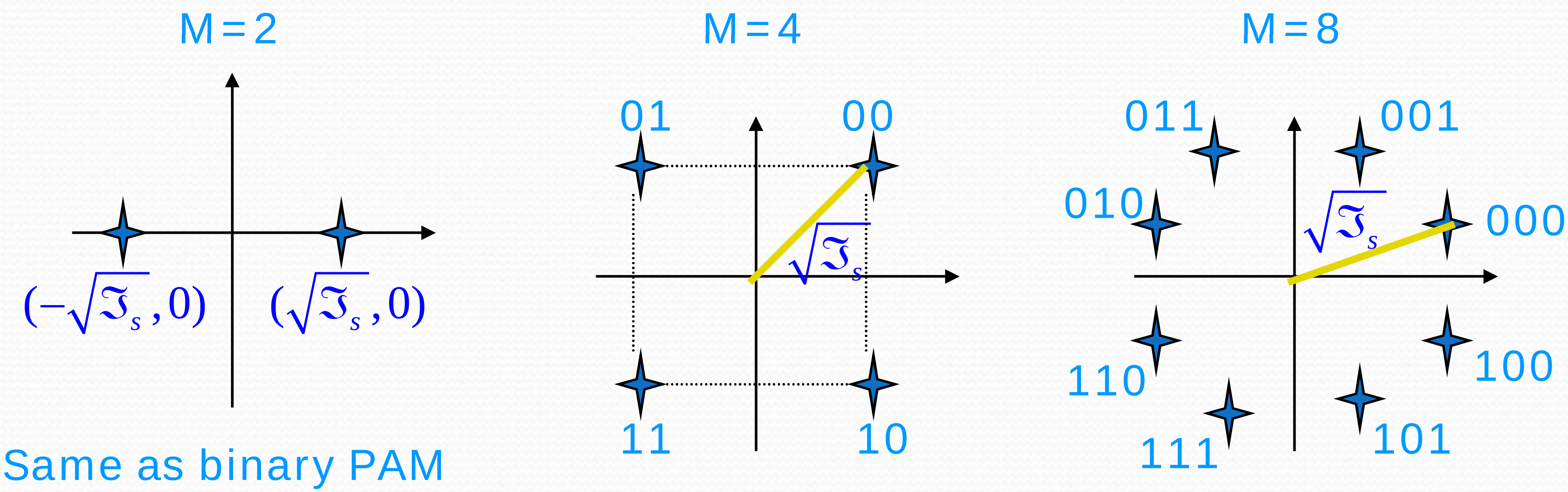
$$= \sqrt{\mathcal{E}_s}\, g_T(t)\{\cos(2\pi f_c t)\cos(\frac{2\pi m}{M}) - \sin(2\pi f_c t)\sin(\frac{2\pi m}{M})\}$$

$$= \underbrace{\sqrt{\mathcal{E}_s}\cos(\frac{2\pi m}{M})}_{s_{mc}}\underbrace{\{g_T(t)\cos(2\pi f_c t)\}}_{\psi_1(t)} + \underbrace{\sqrt{\mathcal{E}_s}\sin(\frac{2\pi m}{M})}_{s_{ms}}\underbrace{\{-g_T(t)\sin(2\pi f_c t)\}}_{\psi_2(t)}$$

  - Two orthogonal basis: $\psi_1(t), \psi_2(t)$
  - PSK can be represented geometrically as two-dimensional with components s_{mc}, s_{ms}

Constellation of PSK

- Two dimensional
  - $$s_m = (s_{mc}, s_{ms}) = (\sqrt{\Im_s}\cos\frac{2\text{®} m}{M}, \sqrt{\Im_s}\sin\frac{2\text{®} m}{M})$$



Same as binary PAM
with antipodal signals

Why Gray code is used ?

Phase demodulation and detection

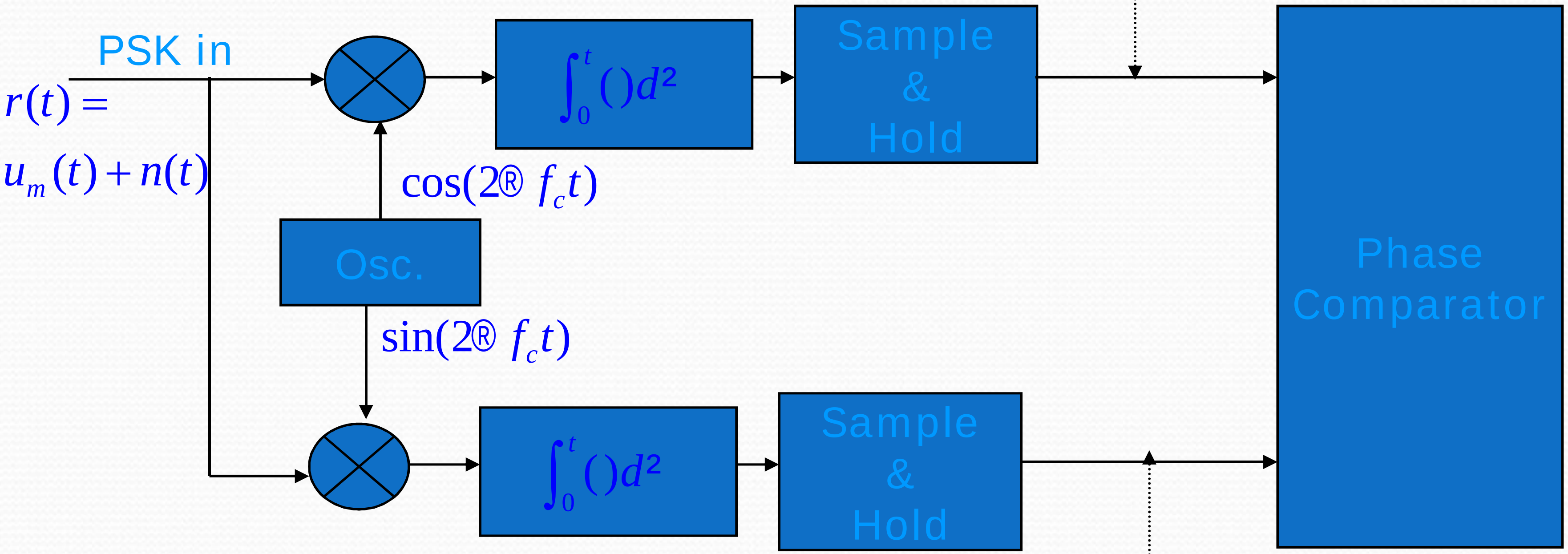
- Received signal through AWGN channel
  - $$\begin{aligned} r(t) &= u_m(t) + n(t) \\ &= s_{mc} g_T(t) \cos(2® f_c t) - s_{ms} g_T(t) \sin(2® f_c t) \\ &+ n_c(t) \cos(2® f_c t) - n_s(t) \sin(2® f_c t) \end{aligned}$$

- Optimal Receiver
  - Two correlator (or Matched Filter)
    - Two dimensional Signal
  - Optimum Detector

Optimum Receiver for PSK

- Two Correlator

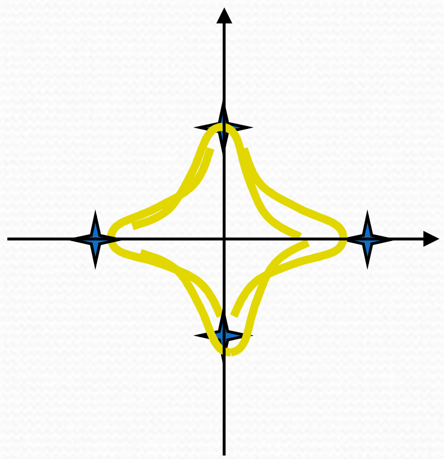
$$r_c = \sqrt{\Im_s}\cos\frac{2\pi m}{M} + \frac{1}{2}\int_{-\infty}^{\infty} g_T(t)n_c(t)dt = \sqrt{\Im_s}\cos\frac{2\pi m}{M} + n_c$$



$$r_s = \sqrt{\Im_s}\sin\frac{2\pi m}{M} + \frac{1}{2}\int_{-\infty}^{\infty} g_T(t)n_s(t)dt = \sqrt{\Im_s}\sin\frac{2\pi m}{M} + n_s$$

Noise component

- Because $n_c(t)$ and $n_s(t)$ are uncorrelated zero mean Gaussian process
  - Zero mean: $E[n_c] = E[n_s] = E[n_c n_s] = 0$
  - With variance: $E[n_c^2] = E[n_s^2] = \frac{N_0}{2}$
- Example of M=4



$$r = (r_c, r_s) = (\sqrt{\Im_s}\cos\frac{2\pi m}{M} + n_c, \sqrt{\Im_s}\sin\frac{2\pi m}{M} + n_c)$$

$$r_0 = (\sqrt{\Im_s} + n_c, n_c)$$

$$r_1 = (n_c, \sqrt{\Im_s} + n_c)$$

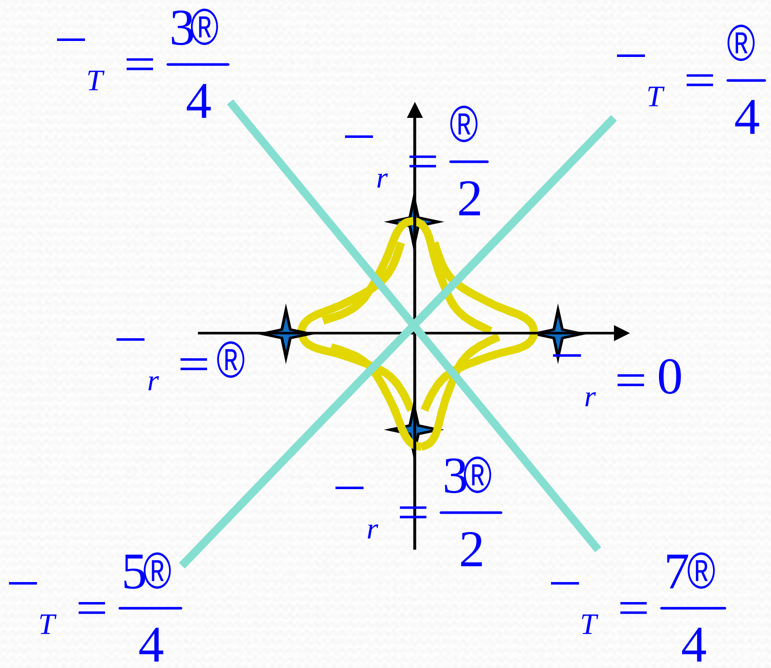
$$r_2 = (-\sqrt{\Im_s} + n_c, n_c)$$

$$r_3 = (n_c, -\sqrt{\Im_s} + n_c)$$

What do you think is optimal detector ??

Optimal Detector

- Example of M=4
  - Phase of received signal: $\theta_r = \tan^{-1}(\frac{r_s}{r_c}) = \frac{2\pi m}{M}$



Probability of Error

- Hard to get closed-form expression
  - M=2 case
    - Same as binary PAM : $P_2 = Q(\sqrt{\frac{2\Im_b}{N_0}})$
    - Where $\Im_b$ is energy per bit
  - M>4 case
    - Approximation: $P_M \approx 2Q(\sqrt{\frac{2\Im_s}{N_0}} \sin\frac{®}{M}) \approx 2Q(\sqrt{\frac{2k\Im_b}{N_0}} \sin\frac{®}{M})$
  - Equivalent bit-error probability for M-ary PSK
    - Approximation: $P_b \approx P_M / k$
- See figure 7.10 page 293, for Probability of Symbol error for M-ary PSK

DPSK(Differential PSK)

- Differential coding + PSK
- Easy to implement receiver
  - No estimation of carrier phase is required
    - No needs of PLL or Costas loop
- See Chapter 7.0 for details
- See figure 7.14 page 300, for Probability of error
  - Binary DPSK is 3dB poorer than Binary PSK
    - Performance Vs. Easy to implement