Lecture 9
Quine-McCluskey (Tabular) MinimizationMethod

Quine-McCluskey (Tabular) Minimization

- Two step process utilizing tabular listings to:
  - Identify prime implicants (implicant tables)
  - Identify minimal PI set (cover tables)
- All work is done in tabular form
  - Number of variables is not a limitation
  - Basis for many computer implementations
  - Don’t cares are easily handled
- Proper organization and term identification are key factors for correct results

Quine-McCluskey Minimization (cont.)

- Terms are initially listed one per line in groups
  - Each group contains terms with the same number of true and complemented variables
  - Terms are listed in numerical order within group
- Terms and implicants are identified using one of three common notations
  - full variable form
  - cellular form
  - 1,0,- form

Notation Forms

- Full variable form - variables and complements in algebraic form
  - hard to identify when adjacency applies
  - very easy to make mistakes
- Cellular form - terms are identified by their decimal index value
  - Easy to tell when adjacency applies; indexes must differ by power of two (one bit)
  - Implicants identified by term nos. separated by comma; differing bit pos. in () following terms

Notation Forms (cont.)

- 1,0,- form - terms are identified by their binary index value
  - Easier to translate to/from full variable form
  - Easy to identify when adjacency applies, one bit is different
  - - shows variable(s) dropped when adjacency is used
- Different forms may be mixed during the minimization

Example of Different Notations

$F(A, B, C, D) = \Sigma\, m(4,5,6,8,10,13)$

	Full variable	Cellular	1,0,-
1	$A\bar{B}CD$	4	0100
	$\bar{A}BCD$	8	1000
2	$A\bar{B}C\bar{D}$ –	5	0101
	$A\bar{B}CD$ –	6	0110
	$A\bar{B}\bar{C}D$	10	1010
3	$A\bar{B}C\bar{D}$ –	13	1101
	– –		
	–		

Implication Table (1,0,-)

- Quine-McCluskey Method
  - Tabular method to systematically find all prime implicants
  - $f(A,B,C,D) = \Sigma\, m(4,5,6,8,9,10,13) + \Sigma\, d(0,7,15)$
  - Part 1: Find all prime implicants
  - Step 1: Fill Column 1 with active-set and DC-set minterm indices. Group by number of true variables (# of 1's).

NOTE: DCs <u>are</u> included in this step!

Implication Table		
Column I		
0000		
0100		
1000		
0101		
0110		
1001		
1010		
0111		
1101		
1111		

Implication Table (cellular)

- Quine-McCluskey Method
  - Tabular method to systematically find all prime implicants
  - $f(A,B,C,D) = \Sigma\, m(4,5,6,8,9,10,13) + \Sigma\, d(0,7,15)$
  - Part 1: Find all prime implicants
  - Step 1: Fill Column 1 with active-set and DC-set minterm indices. Group by number of true variables (# of 1's).

NOTE: DCs <u>are</u> included in this step!

Implication Table		
Column I		
0		
4		
8		
5		
6		
9		
10		
7		
13		
15		

Minimization - First Pass (1,0,-)

- Quine-McCluskey Method
  - Tabular method to systematically find all prime implicants
  - $f(A,B,C,D) = \Sigma\, m(4,5,6,8,9,10,13) + \Sigma\, d(0,7,15)$
  - Part 1: Find all prime implicants
  - Step 2: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. One bit difference implies adjacent. Eliminate variable and place in next column.

    E.g., 0000 vs. 0100 yields 0-00

    0000 vs. 1000 yields -000

    When used in a combination, mark with a check. If cannot be combined, mark with a star. These are the prime implicants.

    Repeat until nothing left.

Implication Table

Column I	Column II	
0000 ✓	0-00	
	-000	
0100 ✓		
1000 ✓	010-	
	01-0	
0101 ✓	100-	
0110 ✓	10-0	
1001 ✓		
1010 ✓	01-1	
	-101	
0111 ✓	011-	
1101 ✓	1-01	
1111 ✓	-111	
	11-1	

Minimization - First Pass (cellular)

- Quine-McCluskey Method
  - Tabular method to systematically find all prime implicants
  - $f(A,B,C,D) = \Sigma\, m(4,5,6,8,9,10,13) + \Sigma\, d(0,7,15)$
  - Part 1: Find all prime implicants
  - Step 2: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. 2^n difference implies adjacent. Next col is numbers with diff in parentheses.

    E.g., 0 vs. 4 yields 0,4(4)

    5 vs. 7 yields 5,7(2)

    When used in a combination, mark with a check. If cannot be combined, mark with a star. These are the prime implicants.

    Repeat until nothing left.

Implication Table

Column I	Column II
0 ✓	0,4(4)
	0,8(8)
4 ✓	
8 ✓	4,5(1)
	4,6(2)
5 ✓	8,9(1)
6 ✓	8,10(2)
9 ✓	
10 ✓	5,7(2)
	5,13(8)
7 ✓	6,7(1)
13 ✓	9,13(4)
15 ✓	7,15(8)
	13,15(2)

Minimization - Second Pass (1,0,-)

- Quine-McCluskey Method
  - Step 2 cont.: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. One bit difference implies adjacent. Eliminate variable and place in next column.

    E.g., 0000 vs. 0100 yields 0-00

    0000 vs. 1000 yields -000

    When used in a combination, mark with a check. If cannot be combined, mark with a star. These are the prime implicants.

    Repeat until nothing left.

Implication Table

Column I	Column II	Column III
0000 ✓	0-00 *	01-- *
	-000 *	
0100 ✓		-1-1 *
1000 ✓	010- ✓	
	01-0 ✓	
0101 ✓	100- *	
0110 ✓	10-0 *	
1001 ✓		
1010 ✓	01-1 ✓	
	-101 ✓	
0111 ✓	011- ✓	
1101 ✓	1-01 *	
1111 ✓	-111 ✓	
	11-1 ✓	

Minimization - Second Pass (cellular)

- Quine-McCluskey Method
  - Step 2 cont.: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. 2^n difference implies adjacent. Next col is numbers with differences in parentheses.

    E.g., 4,5(1) and 6,7(1) yields 4,5,6,7(3)

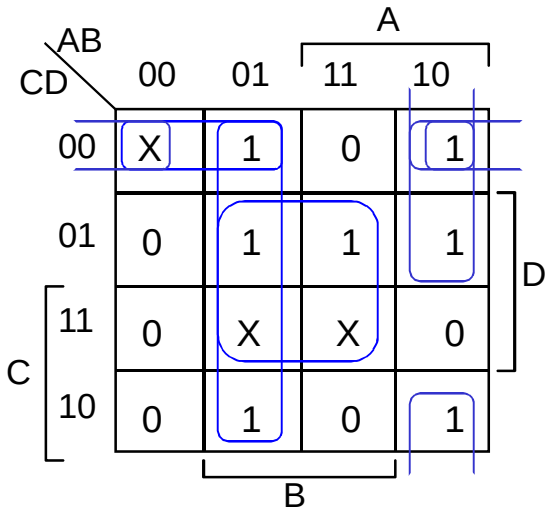
    When used in a combination, mark with a check. If cannot be combined, mark with a star. These are the prime implicants.

    Repeat until nothing left.

Implication Table

Column I	Column II	Column III
0 ✓	0,4(4) *	4,5,6,7(3) *
	0,8(8) *	
4 ✓		5,7,13,15
8 ✓	4,5(1) ✓	(10) *
	4,6(2) ✓	
5 ✓	8,9(1) *	
6 ✓	8,10(2) *	
9 ✓		
10 ✓	5,7(2) ✓	
	5,13(8) ✓	
7 ✓	6,7(1) ✓	
13 ✓	9,13(4) *	
15 ✓	7,15(8) ✓	
	13,15(2) ✓	

Prime Implicants



Prime Implicants:

$0\text{-}00 = \overline{A}\,\overline{C}\,\overline{D}$

$\text{-}000 = \overline{B}\,\overline{C}\,\overline{D}$

$100\text{-} = A\,\overline{B}\,\overline{C}$

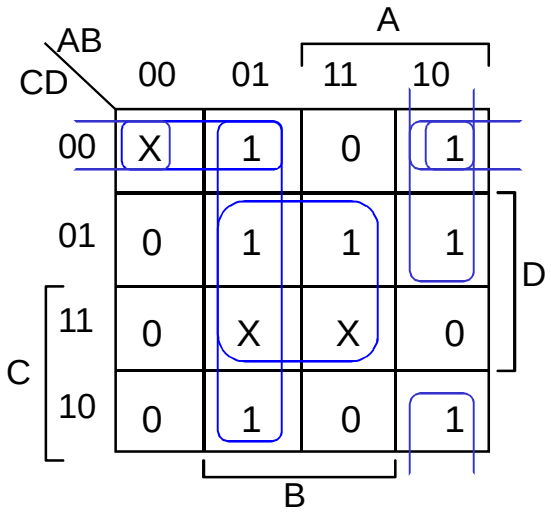
$10\text{-}0 = A\,\overline{B}\,\overline{D}$

$1\text{-}01 = A\,\overline{C}\,D$

$\text{-}1\text{-}1 = B\,D$

$01\text{--} = \overline{A}\,B$

Prime Implicants (cont.)



Prime Implicants:

$$0\text{-}00 = \overline{A}\,\overline{C}\,\overline{D} \qquad \text{-}000 = \overline{B}\,\overline{C}\,\overline{D}$$

$$100\text{-} = A\,\overline{B}\,\overline{C} \qquad 10\text{-}0 = A\,\overline{B}\,\overline{D}$$

$$1\text{-}01 = A\,\overline{C}\,D \qquad \text{-}1\text{-}1 = B\,D$$

$$01\text{--} = \overline{A}\,B$$

Stage 2: find smallest set of prime implicants that cover the active-set

recall that essential prime implicants must be in final expression

Coverage Table

Coverage Chart

	4	5	6	8	9	10	13
0,4(0-00)	X						
0,8(-000)				X			
8,9(100-)				X	X		
8,10(10-0)				X		X	
9,13(1-01)					X		X
4,5,6,7(01--)	X	X	X				
5,7,13,15(-1-1)		X					X

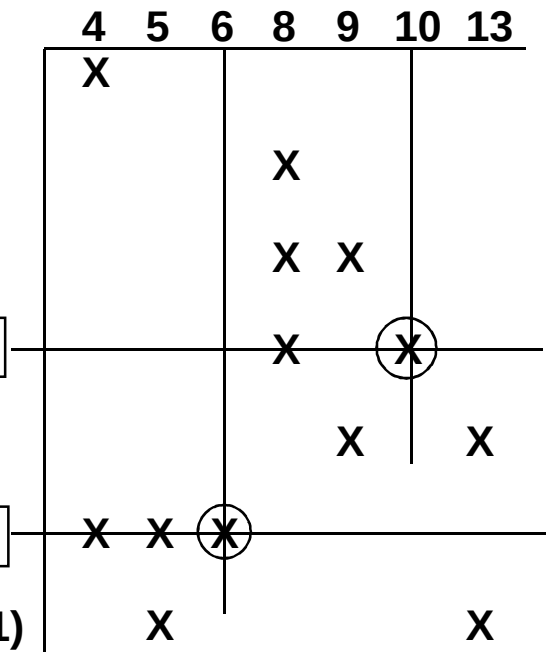
rows = prime implicants
columns = ON-set elements
place an "X" if ON-set element is
covered by the prime implicant

Note: <u>Don't</u> include DCs in coverage table; they don't have covered by the final logic expression!

Coverage Table (cont.)

Coverage Chart

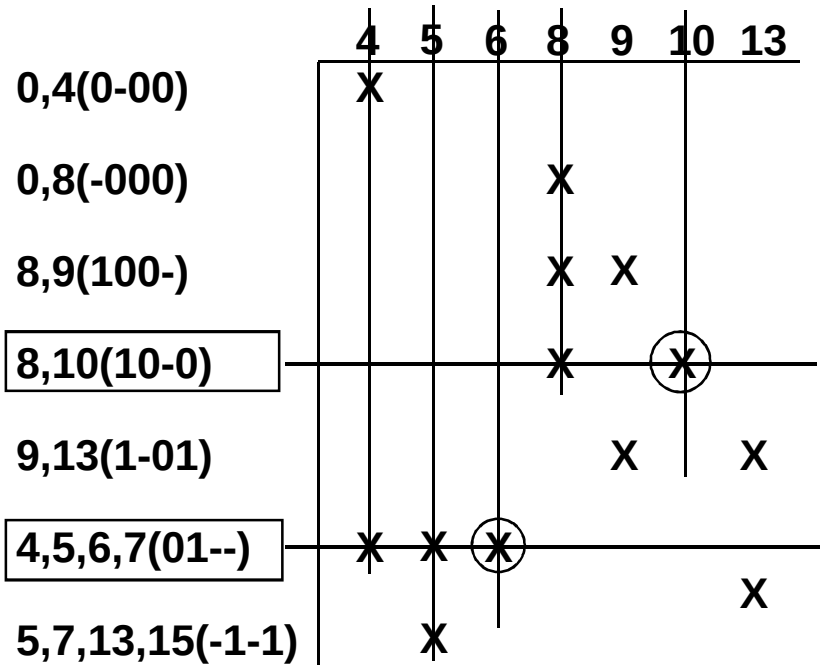
	4	5	6	8	9	10	13
0,4(0-00)	X						
0,8(-000)				X			
8,9(100-)				X	X		
8,10(10-0)				X		X	
9,13(1-01)					X		X
4,5,6,7(01--)	X	X	X				
5,7,13,15(-1-1)		X					X



rows = prime implicants
columns = ON-set elements
place an "X" if ON-set element is covered by the prime implicant

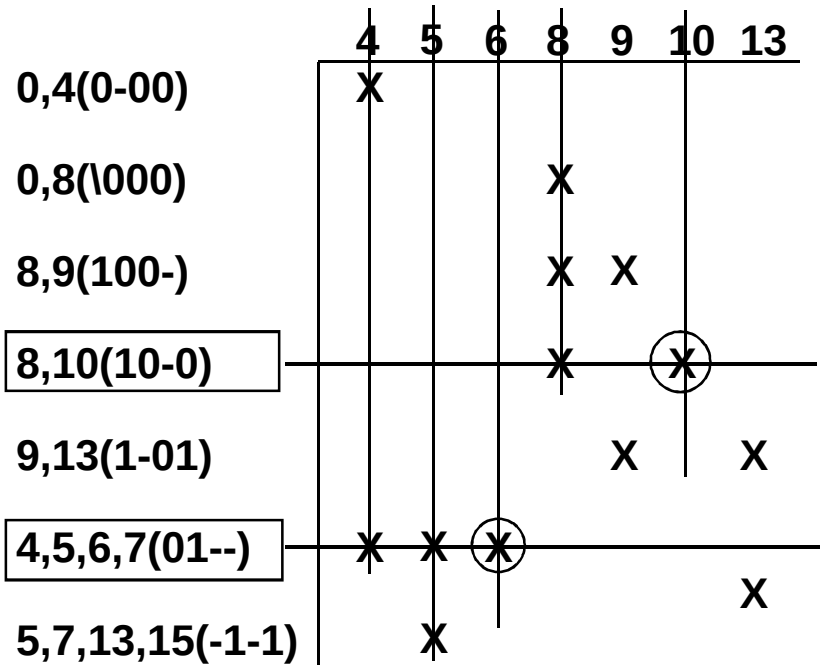
If column has a single X, than the implicant associated with the row is essential. It must appear in minimum cover

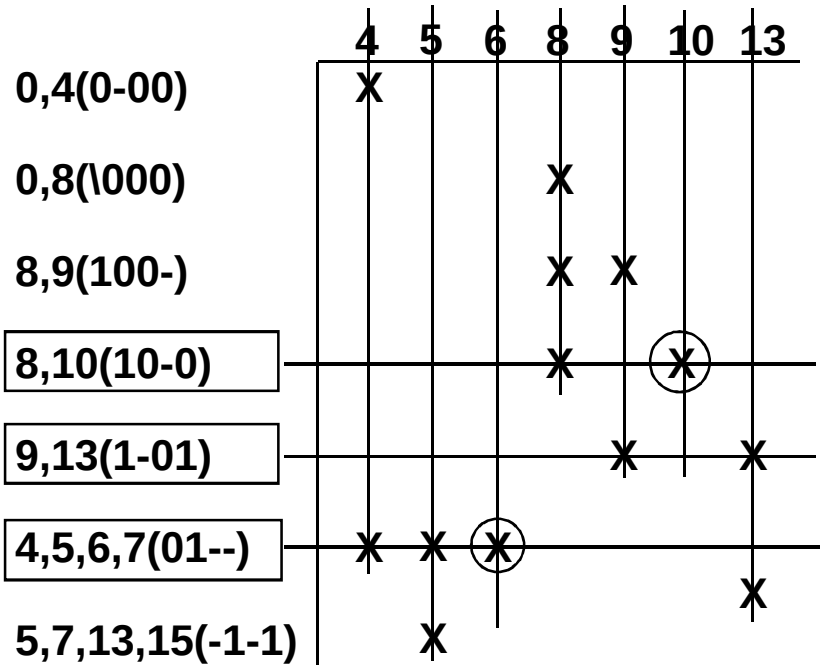
Coverage Table (cont.)



Eliminate all columns covered by essential primes

Coverage Table (cont.)





Eliminate all columns covered by essential primes

Find minimum set of rows that cover the remaining columns

$$F = A\overline{B}\,\overline{D} + A\overline{C}D + \overline{A}B$$