# Lecture 4
# Binary Arithmetic

# Binary Arithmetic

- Addition
- Subtraction
- Complements – 1’s and 2’s

# Binary Addition

(a)

$$\begin{array}{r} 0 \\ +\,0 \\ \hline 0 \end{array}$$

(b)

$$\begin{array}{r} 0 \\ +\,1 \\ \hline 1 \end{array}$$

(c)

$$\begin{array}{r} 1 \\ +\,0 \\ \hline 1 \end{array}$$

(d)

$$\begin{array}{r} 1 \\ +\,1 \\ \hline 1\,0 \end{array}$$

**Carry Bit**

# Binary Addition Examples

(a)
```
   1011
 + 1100
 ------
  10111
```

(b)
```
   1010
 +  100
 ------
   1110
```

(c)
```
   1011
 +  101
 ------
  10000
```

(d)
```
    101
 + 1001
 ------
   1110
```

(e)
```
   10011001
 +   101100
 ----------
   11000101
```

# Binary Complement

# (1s Complement) Operation

1 → 0

0 → 1

Example

1 1 0 0 1 0 1 1 0

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

0 0 1 1 0 1 0 0 1

# Two's Complement

The Two's complement of a binary number is obtained by first complementing the number and then adding 1 to the result.

```
1001110

0110001   <-- One's Complement
+     1
-------
0110010   <-- Two's Complement
```

# Binary Subtraction

Binary subtraction is implemented by adding the Two's complement of the number to be subtracted.

Example

```
  1101          1101
 -1001         +0111
               -----
               10100
```

Two's complement of 1001

If there is a carry then it is ignored. Thus, the answer is 0100.

# Basic Digital Arithmetic

- Signed Binary Number: A binary number of fixed length whose sign (+/-) is represented by one bit (usually MSB) and its magnitude by the remaining bits
- Unsigned Binary Number: A binary number of fixed length whose sign is not specified by a bit. All bits are magnitude and the sign is assumed +.

# Signed Binary Numbers

- Sign Bit: A bit (usually the MSB) that indicates whether a number is positive(=0) or negative (=1).
- Magnitude Bits: The bits of a signed binary number that tell how large it is in value.
- True Magnitude Form: A form of signed binary whose magnitude bits are the TRUE binary form (not complements).

# Signed Binary Numbers

- 1s Complement: A form of signed binary in which negative numbers are created by complementing all bits.
- 2s Complement: A form of signed binary in which the negative numbers are created by complementing all the bits and adding a 1 (1s Complement +1).

# Unsigned Binary Arithmetic

- Sum: Result of an Addition Operation of two (or more) binary numbers (operands).
- Carry: A digit (or bit) that is carried over to the next most significant bit during an N Bit addition operation.
- The carry bit is a 1 if the result was too large to be expressed in N bits.

# Basic Rules (Unsigned)

- One Bit Unsigned Addition

| | |
|---|---|
| 0 + 0 = | 0 0 |
| 1 + 0 = | 0 1 |
| 1 + 1 = | 1 0 |

1 + 1 + 1 = 1 1

CIN A B COUT SUM

# True Magnitude Form

- 5 Bit Numbers Negative = S=1

+25 = 011001

-25 = 111001

-12 = 101100

+12 = 001100

Note sign bit = MSB = S = 0

Same as +25 with S=1

True magnitude

# 2’s complement of a binary number:

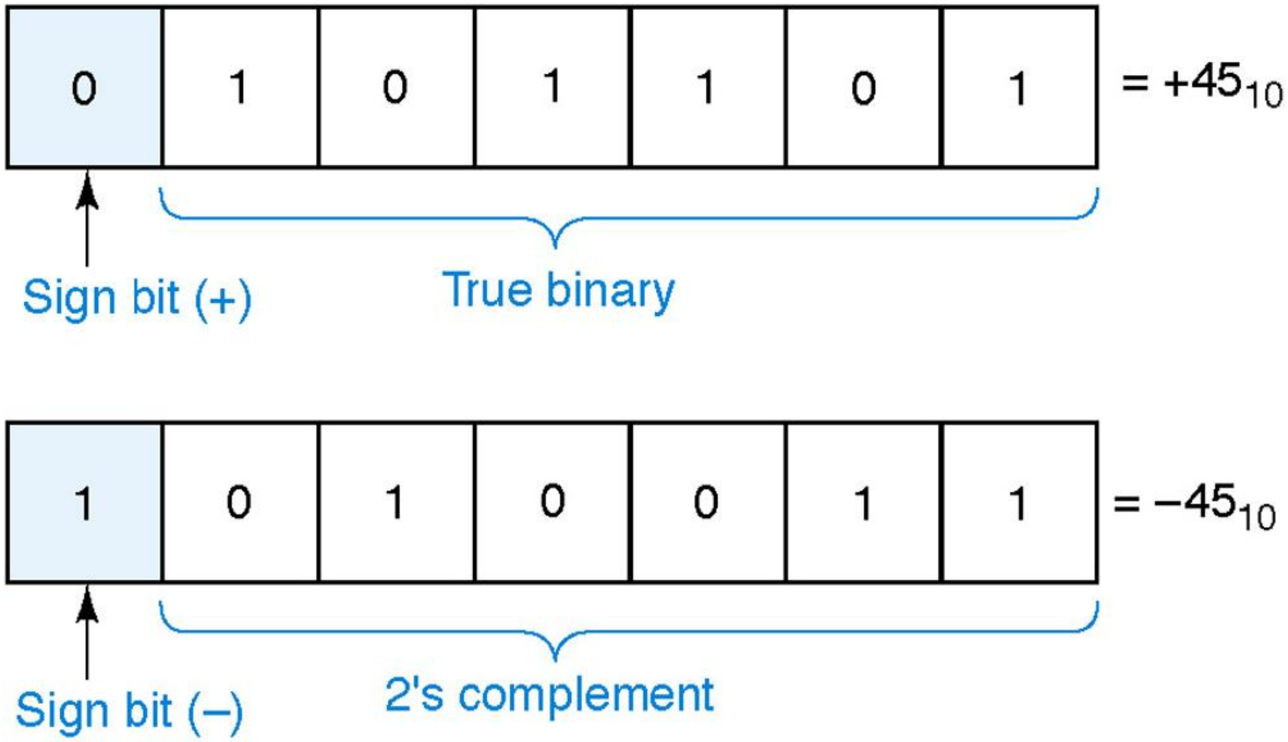
– Take the 1’s complement of the number
– Add 1 to the least-significant-bit position

| | |
|---|---|
| 101101 | binary equivalent of 45 |
| 010010 | complement each bit to form 1's complement |
| + 1 | add 1 to form 2's complement |
| 010011 | 2's complement of original binary number |

# Representing signed numbers using 2’s complement form

- If the number is positive, the magnitude is represented in its positional-weighted binary form, and a sign bit of 0 is placed in front of the MSB.
- If the number is negative, the magnitude is represented in its 2’s complement form, and a sign bit of 1 is placed in front of the MSB.

# example


0 1 0 1 1 0 1
= +45$_{10}$
Sign bit (+)
True binary
1 0 1 0 0 1 1
= −45$_{10}$
Sign bit (–)
2's complement

# Example

- Represent each of the following signed decimal numbers as a signed binary number in the 2’s-complement system. Use a total of five bits including the sign bit.

  (a) +13  (b) –9  (c) +3  (d) –2 (e) -8

# Addition in the 2’s-complement system

- Case I: Two Postive Numbers.

| | | | | |
|---|---|---|---|---|
| +9 | $\rightarrow$ | 0 | 1001 | (augend) |
| +4 | $\rightarrow$ | 0 | 0100 | (addend) |
| | | 0 | 1101 | (sum = +13) |

*Sign bits*

# Addition, cont.

- Case II: Positive Number and Smaller Negative Number

```
     +9 → 0 1001 (augend)
              Sign bits
     -4 →  1 1100 (addend)
_______________________________
         1  0 0101
```

*This carry is disregarded; the result is 0101(sum=+5)*

# Addition, cont.

- Case III: Positive Number and Larger Negative Number

*Negative sign bit*

$-9 \rightarrow 10111$

$\underline{+4 \rightarrow 00100}$

$11011$ (sum = -5)

# Addition, cont.

- Case IV: two negative Numbers

-9 $\rightarrow$ 10111

-4 $\rightarrow$ 11100

1 10011

*Sign bit*

*This carry is disregarded; the result is 10011(sum =-13)*

# Negative Result Example

- 2s Complement Negative Result (65-80)

| | | |
|---|---|---|
| +65 = | 0 100 0001 | 100 0001 |
| -80 = | 1 101 0000 (2s C.) | + 011 0000 |
| | | 111 0001 |
| | Invert | 000 1111 |
| | Add 1 | + 1 |
| | Final Result = -15 | 0000 1111 = 15(Neg.) |

# Addition, cont.

- Case V: Equal and Opposite Numbers

$$
\begin{array}{lll}
-9 \rightarrow & 1 & 0111 \\
\underline{+9 \rightarrow} & \underline{0} & \underline{1001} \\
0 \quad 1 & & 0000
\end{array}
$$

↑ *Disregard; the result is 0000(sum = +0)*

# Subtraction in the 2's-complement System

- The procedure for subtracting one binary number(the subtrahend) from another binary number(the minuend)
  - Negate the subtrahend. This will change the subtrahend to its equivalent value of opposite sign.
  - Add this to the minuend. The result of this addition will represent the difference between the subtrahend and the minuend.

# Addition and Subtraction of BCD and Excess-3 Code

# Unsigned Numbers BCD Addition

Use binary arithmetic to add the BCD digits:

| | | |
|---|---|---|
| 8 | 1000 | Eight |
| +5 | +0101 | Plus 5 |
| 13 | 1101 | is 13 (> 9) |

If result is > 9, it must generate a carry and be corrected!

To correct the digit, add 0110 in the result.

| |
|---|
| 3 |
| +5 |
| 8 OK (< 9) |

| | | |
|---|---|---|
| 8 | 1000 | Eight |
| +5 | +0101 | Plus 5 |
| 13 | 1101 | 13 ( is > 9) |
| | +0110 | so add 6 (always, for results > 9) |
| | carry = 1 0011 | giving 3 + carry |
| | 0001 \| 0011 | Final answer (two digits) |

**We try to avoid subtraction!**
**Replacing it with addition!**

The adder circuit utilizes the resulting carry bit by sending it as carry-in to the next digit

## Add $2905_{BCD}$ to $1857_{BCD}$ showing carries and digit corrections.

| | | | | | |
|---|---|---|---|---|---|
| 1010 | | 1 | 0 | 1 | 0 |
| 1857 | | 0001 | 1000 | 0101 | 0111 |
| +2905 | + | 0010 | 1001 | 0000 | 0101 |
| | | 0100 (4<9 → +0) | 10001 (17>9 → +6) | 0110 (6<9 → +0) | 1100 (12>9 → +6) |
| **Add 6 or 0** | + | 0000 | 0110 | 0000 | 0110 |
| 4762 | | 0100 | 1 0111 | 0110 | 1 0010 |

# Excess-3 Code

A BCD Code formed by adding 3 (0011) to its true 4-bit binary value.

Excess-3 is a self-complementing code:

A negative code equivalent can be found by inverting the binary bits of the positive code

Inverting the bits of the Excess -3 digit yields 9's Complement of the decimal equivalent.

Example : Excess -3 code of decimal 4 is 0111. (0100 + 0011 = 0111)

(4) = 0111

(-4) = 1000 (inverting the bits) which is Excess -3 code of decimal 5.

It is 9's complement of the decimal equivalent. (9 – 4 = 5)

Excess-3 Examples

3 = 0011 + 0011 = 0110 = 6 in E3.

1 = 0001 + 0011 = 0100 = 4 in E3.

If we complement 1 = 1011 in E3, this

is the code for an 8.

9’s Complement of 1 = (9 – 1) = 8 (SelfComplement)