## Lecture 27

> Op- Amp

## Op-Amp Differentiator



$$
v_{o}=-\left(\frac{d V_{i}}{d t}\right) R C
$$

## Non-ideal case (Inverting Amplifier)



## Close-Loop Gain

Applied KCL at V- terminal,

$$
\frac{V_{i n}-V_{\pi}}{R_{a}}+\frac{-V_{\pi}}{R_{\pi}}+\frac{V_{o}-V_{\pi}}{R_{f}}=0
$$

By using the open loop gain,

$$
\begin{aligned}
& V_{o}=-A V_{\pi} \\
\Rightarrow & \frac{V_{i n}}{R_{a}}+\frac{V_{o}}{A R_{a}}+\frac{V_{o}}{A R_{\pi}}+\frac{V_{o}}{R_{f}}+\frac{V_{o}}{A R_{f}}=0
\end{aligned}
$$



$$
\Rightarrow \frac{V_{i n}}{R_{a}}=-V_{o} \frac{R_{\pi} R_{f}+R_{a} R_{f}+R_{a} R_{\pi}+A R_{a} R_{\pi}}{A R_{a} R_{\pi} R_{f}}
$$

The Close-Loop Gain, $A_{v}$

$$
A_{v}=\frac{V_{o}}{V_{i n}}=\frac{-A R_{\pi} R_{f}}{R_{\pi} R_{f}+R_{a} R_{f}+R_{a} R_{\pi}+A R_{a} R_{\pi}}
$$

## Close-Loop Gain

When the open loop gain is very large, the above equation become,

$$
A_{v} \sim \frac{-R_{f}}{R_{a}}
$$

Note : The close-loop gain now reduce to the same form as an ideal case

## Input Impedance

Input Impedance can be regarded as,

$$
R_{i n}=R_{a}+R_{\pi} / / R^{\prime}
$$

where $R^{\prime}$ is the equivalent impedance of the red box circuit, that is

$$
R^{\prime}=\frac{V_{\pi}}{i_{f}}
$$

However, with the below circuit,

$$
\begin{aligned}
& V_{\pi}-\left(-A V_{\pi}\right)=i_{f}\left(R_{f}+R_{o}\right) \\
& \Rightarrow R^{\prime}=\frac{V_{\pi}}{i_{f}}=\frac{R_{f}+R_{o}}{1+A}
\end{aligned}
$$



## Input Impedance

Finally, we find the input impedance as,

$$
R_{i n}=R_{a}+\left[\frac{1}{R_{\pi}}+\frac{1+A}{R_{f}+R_{o}}\right]^{-1} \Rightarrow \quad R_{i n}=R_{a}+\frac{R_{\pi}\left(R_{f}+R_{o}\right)}{R_{f}+R_{o}+(1+A) R_{\pi}}
$$

Since, $\quad R_{f}+R_{o} \ll(1+A) R_{\pi}, R_{\text {in }}$ become,

$$
R_{i n} \sim R_{a}+\frac{\left(R_{f}+R_{o}\right)}{(1+A)}
$$

Again with $\quad R_{f}+R_{o} \ll(1+A)$

$$
R_{i n} \sim R_{a}
$$

Note: The op-amp can provide an impedance isolated from input to output

## Output Impedance

Only source-free output impedance would be considered, i.e. $V_{i}$ is assumed to be 0

Firstly, with figure (a),

$$
V_{\pi}=\frac{R_{a} / / R_{\pi}}{R_{f}+R_{a} / / R_{\pi}} V_{o} \Rightarrow V_{\pi}=\frac{R_{a} R_{\pi}}{R_{a} R_{f}+R_{a} R_{\pi}+R_{f} R_{\pi}} V_{o}
$$

By using KCL, $i_{0}=i_{1}+i_{2}$
$i_{o}=\frac{V_{o}}{R_{f}+R_{a} / / R_{f}}+\frac{V_{o}-\left(-A V_{\pi}\right)}{R_{o}}$


By substitute the equation from Fig. (a),
The outputimpedance, $R_{\text {out }}$ is
$\frac{V_{o}}{i_{o}}=\frac{R_{o}\left(R_{a} R_{f}+R_{a} R_{\pi}+R_{f} R_{\pi}\right)}{\left(1+R_{o}\right)\left(R_{a} R_{f}+R_{a} R_{\pi}+R_{f} R_{\pi}\right)+(1+A) R_{a} R_{\pi}{ }^{\prime}}$
$\therefore R_{\pi}$ and $A$ comparably large,

$$
R_{\text {out }} \sim \frac{R_{o}\left(R_{a}+R_{f}\right)}{A R_{a}}
$$

