# Lecture 26

Op- Amp

# Noninverting Amplifier

(1) Kirchhoff node equation at $V_+$ yields, $V_+ = V_i$
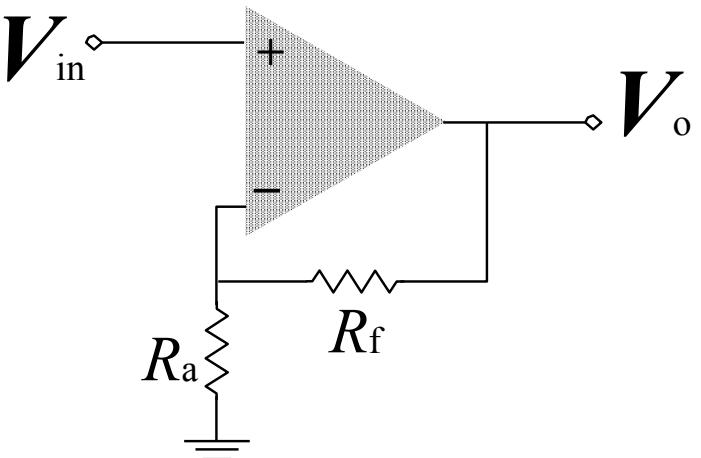
(2) Kirchhoff node equation at $V_-$ yields, $\dfrac{V_- - 0}{R_a} + \dfrac{V_- - V_o}{R_f} = 0$

(3) Setting $V_+ = V_-$ yields

$$\frac{V_i}{R_a} + \frac{V_i - V_o}{R_f} = 0 \quad \text{or} \quad \frac{V_o}{V_i} = 1 + \frac{R_f}{R_a}$$
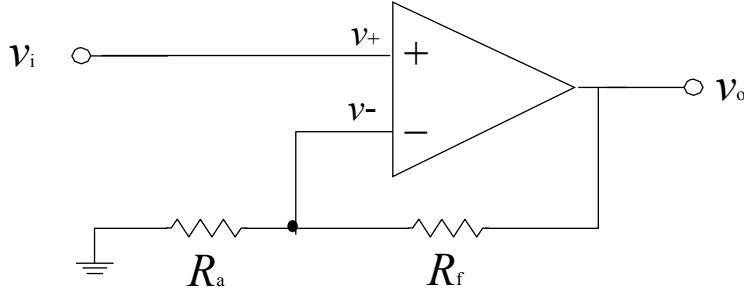
Noninverting amplifier

$$v_o = (1 + \frac{R_f}{R_a})v_i$$
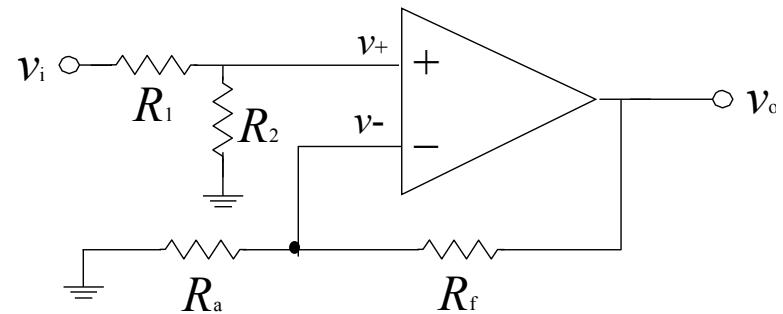


Noninverting input with voltage divider

$$v_o = (1 + \frac{R_f}{R_a})(\frac{R_2}{R_1 + R_2})v_i$$
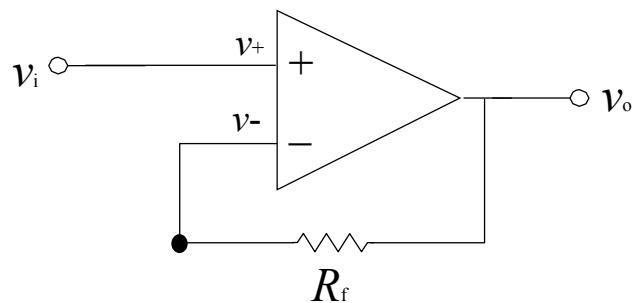


Voltage follower

$$v_o = v_i$$
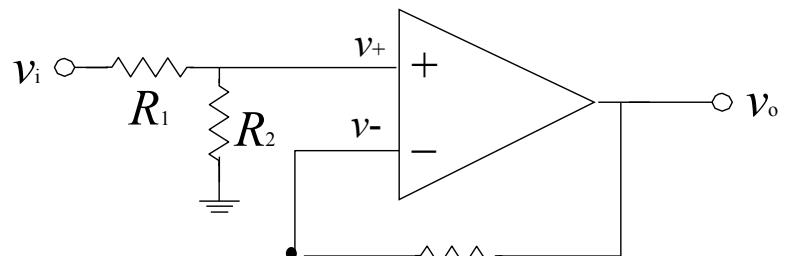


Less than unity gain

$$v_o = \frac{R_2}{R_1 + R_2}v_i$$

# Inverting Amplifier

(1)    Kirchhoff node equation at $V_+$ yields,    $V_+ = 0$

(2)    Kirchhoff node equation at $V_-$ yields,    $\dfrac{V_{in} - V_-}{R_a} + \dfrac{V_o - V_-}{R_f} = 0$

(3)    Setting $V_+ = V_-$ yields

$$\frac{V_o}{V_{in}} = \frac{-R_f}{R_a}$$
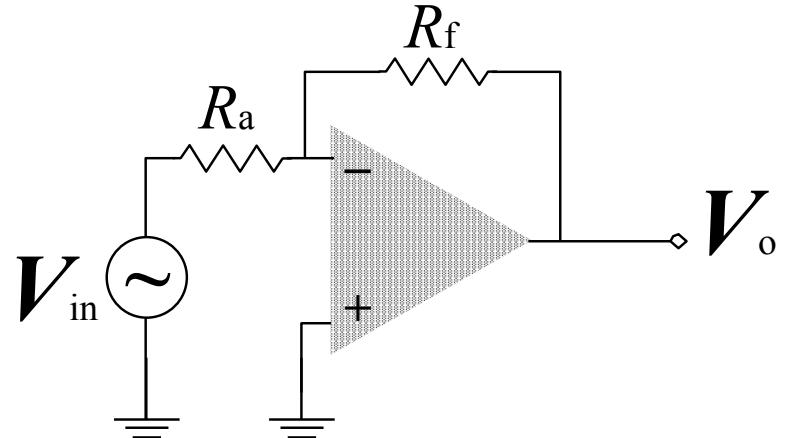
Notice: The closed-loop gain $V_o/V_{in}$ is dependent upon the ratio of two resistors, and is independent of the open-loop gain. This is caused by the use of feedback output voltage to subtract from the input voltage.

$R_f$

$R_a$

$V_{in}$

$-$

$+$

$V_o$

# Multiple Inputs

(1)  Kirchhoff node equation at $V_+$ yields, $$V_+ = 0$$

(2)  Kirchhoff node equation at $V_-$ yields,

$$\frac{V_- - V_o}{R_f} + \frac{V_- - V_a}{R_a} + \frac{V_- - V_b}{R_b} + \frac{V_- - V_c}{R_c} = 0$$
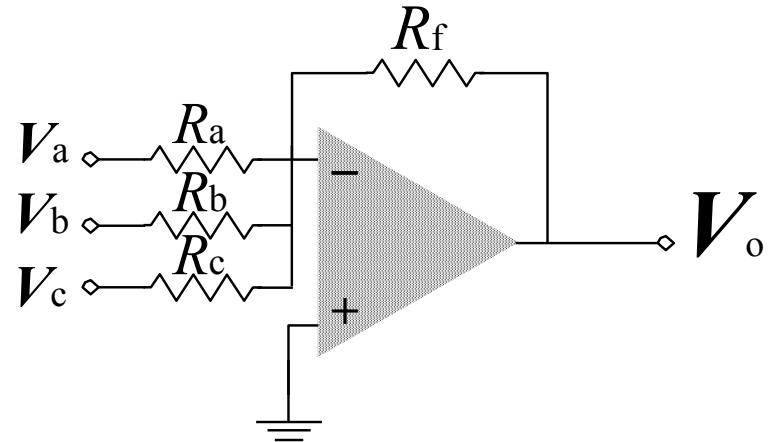
(3)  Setting $V_+ = V_-$ yields

$$V_o = -R_f \left( \frac{V_a}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c} \right) = -R_f \sum_{j=a}^{c} \frac{V_j}{R_j}$$

# Inverting Integrator

Now replace resistors $R_a$ and $R_f$ by complex components $Z_a$ and $Z_f$, respectively, therefore

$$V_o = \frac{-Z_f}{Z_a} V_{in}$$

Supposing

(i) The feedback component is a capacitor C, i.e.,

$$Z_f = \frac{1}{j\omega C}$$

(ii) The input component is a resistor R, $Z_a = R$

Therefore, the closed-loop gain ($V_o/V_{in}$) become:

$$v_o(t) = \frac{1}{RC} \int v_i(t) dt$$

$$v_i(t) = V_i e^{j\omega t}$$
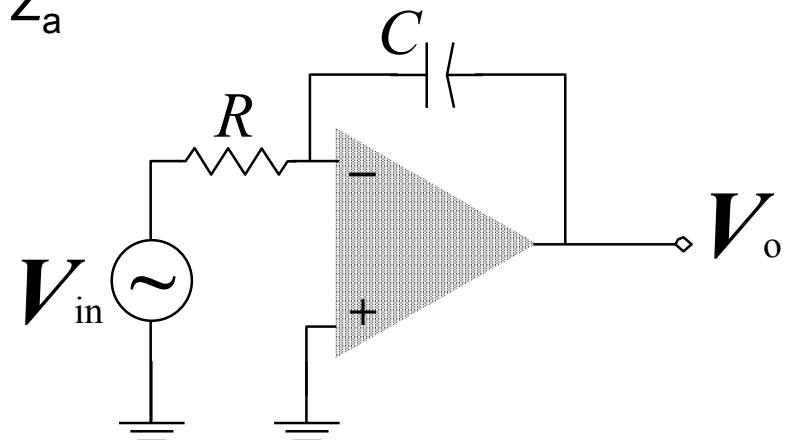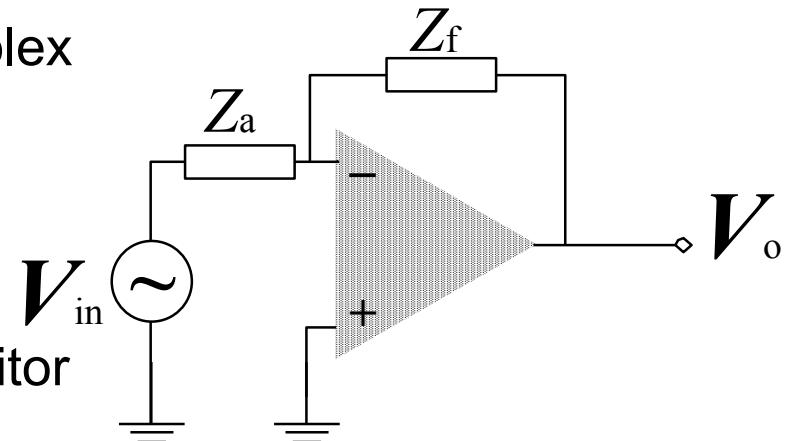
where

What happens if $Z_a = 1/j\omega C$ whereas, $Z_f = R$?
*Inverting differentiator*

# Op-Amp Integrator

Example:

(a) Determine the rate of change of the output voltage.

(b) Draw the output waveform.

$+5V$ ⎍⎍ $\cdots$

$0$

$\vdash$ 100µs $\dashv$

$V_i$

$C$ | | $0.01\mu F$

$R$

$10\ k\Omega$

$-$

$+$

$V_{o(max)}=10\ \mathbf{V}$

$V_o$
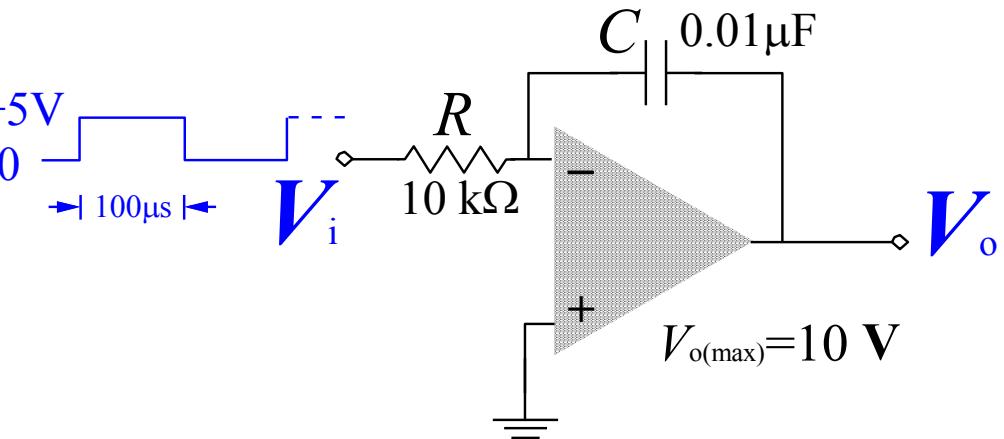
Solution:

(a) Rate of change of the output voltage

$$\frac{\Delta V_o}{\Delta t} = -\frac{V_i}{RC} = \frac{5\ \mathbf{V}}{(10\ \mathbf{k\Omega})(0.01\ \mu\mathbf{F})}$$

$$= -50\ \mathbf{mV}/\mu\mathbf{s}$$

(b) In 100 µs, the voltage decrease

$$\Delta V_o = (-50\ \mathbf{mV}/\mu\mathbf{s})(100\mathbf{\mu s}) = -5\mathbf{V}$$

$+5V$

$0$

$V_i$

$0$

$-5V$

$-10V$

$V_o$
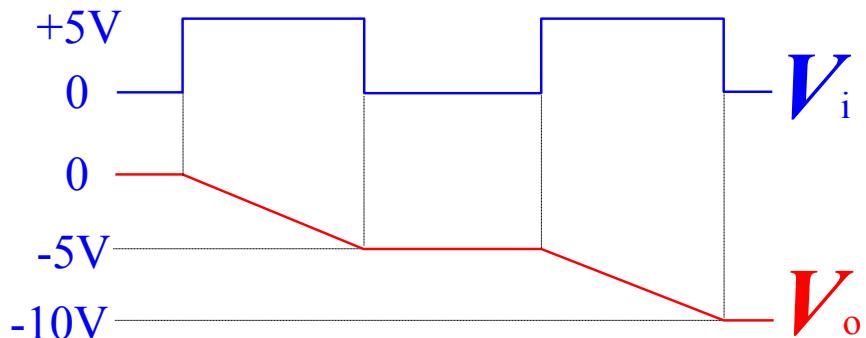
# Op-Amp Differentiator
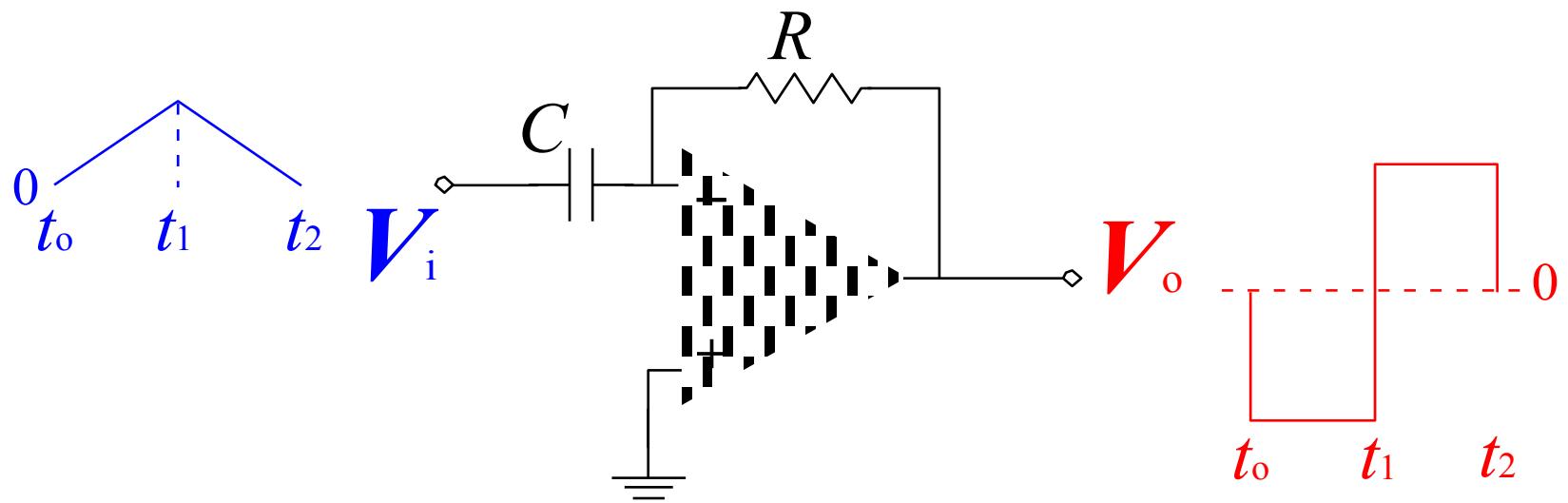
$$v_o = -\left(\frac{dV_i}{dt}\right)RC$$

# Non-ideal case (Inverting Amplifier)

$R_f$

$R_a$

$V_{in}$ $\sim$

$-$

$+$

$V_o$

*Practical op-amp*

$+$

$V_{in}$ $Z_{in}$ $Z_{out}$ $V_{out}$

$\sim$ $AV_{in}$

$-$
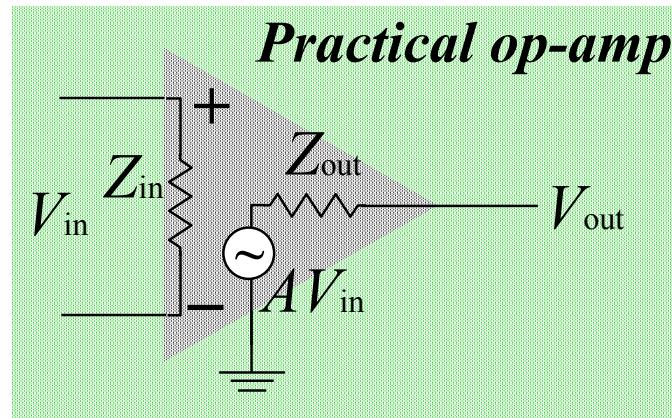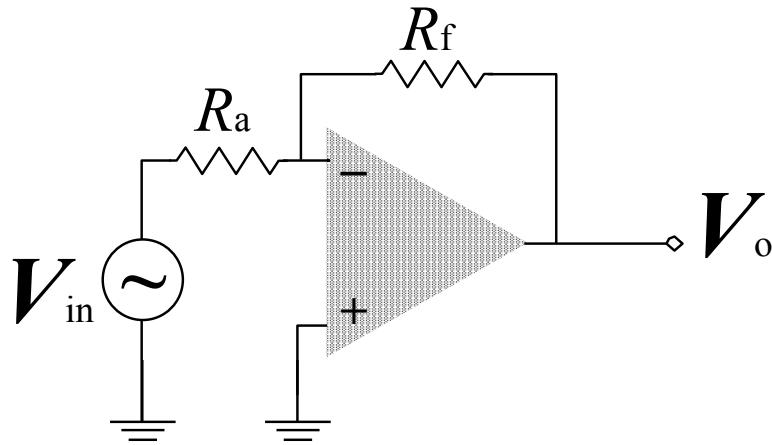
$\Downarrow$ Equivalent Circuit

$R_f$

$V_{in}$ $R_a$

$-$

$V_\pi$ $R_\pi$ $R_o$ $V_o$

$+$ $+$

$-AV_\pi$
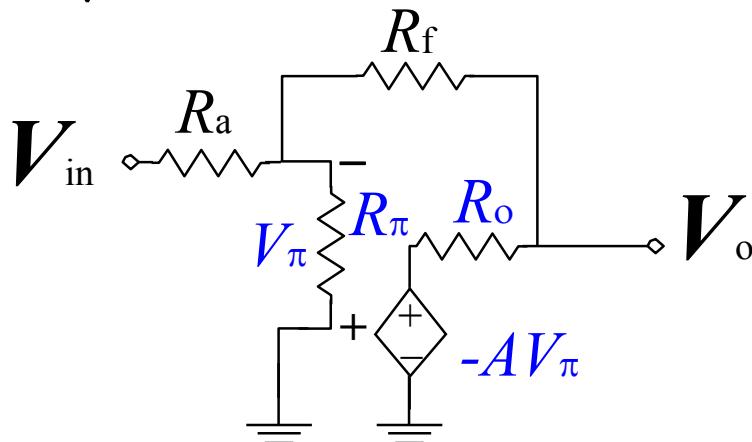
3 categories are considering

❑ Close-Loop Voltage Gain
❑ Input impedance
❑ Output impedance

# Close-Loop Gain

Applied KCL at V– terminal,

$$\frac{V_{in} - V_\pi}{R_a} + \frac{-V_\pi}{R_\pi} + \frac{V_o - V_\pi}{R_f} = 0$$

By using the open loop gain,

$$V_o = -AV_\pi$$

$$\Rightarrow \quad \frac{V_{in}}{R_a} + \frac{V_o}{AR_a} + \frac{V_o}{AR_\pi} + \frac{V_o}{R_f} + \frac{V_o}{AR_f} = 0$$

$$\Rightarrow \quad \frac{V_{in}}{R_a} = -V_o \frac{R_\pi R_f + R_a R_f + R_a R_\pi + AR_a R_\pi}{AR_a R_\pi R_f}$$
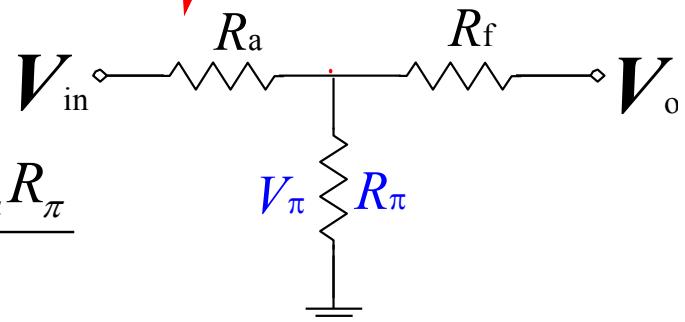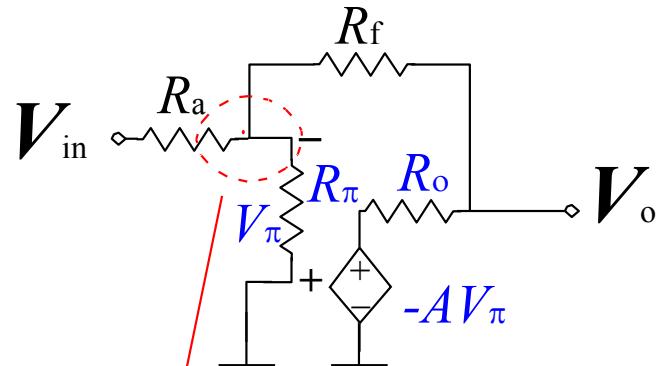
The Close-Loop Gain, $A_v$

$$A_v = \frac{V_o}{V_{in}} = \frac{-AR_\pi R_f}{R_\pi R_f + R_a R_f + R_a R_\pi + AR_a R_\pi}$$

# Close-Loop Gain

When the open loop gain is very large, the above equation become,

$$A_v \sim \frac{-R_f}{R_a}$$

Note : The close-loop gain now reduce to the same form
as an ideal case

# Input Impedance

Input Impedance can be regarded as,

$$R_{in} = R_a + R_\pi \,/\!/\, R'$$

where $R'$ is the equivalent impedance of the red box circuit, that is

$$R' = \frac{V_\pi}{i_f}$$

However, with the below circuit,

$$V_\pi - (-AV_\pi) = i_f (R_f + R_o)$$

$$\Rightarrow R' = \frac{V_\pi}{i_f} = \frac{R_f + R_o}{1 + A}$$
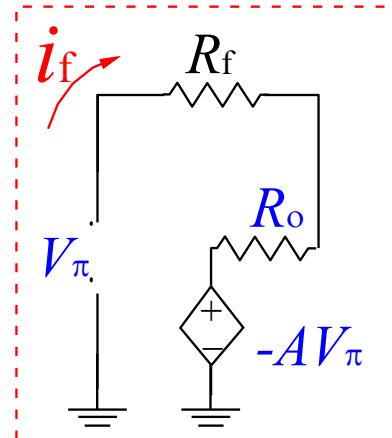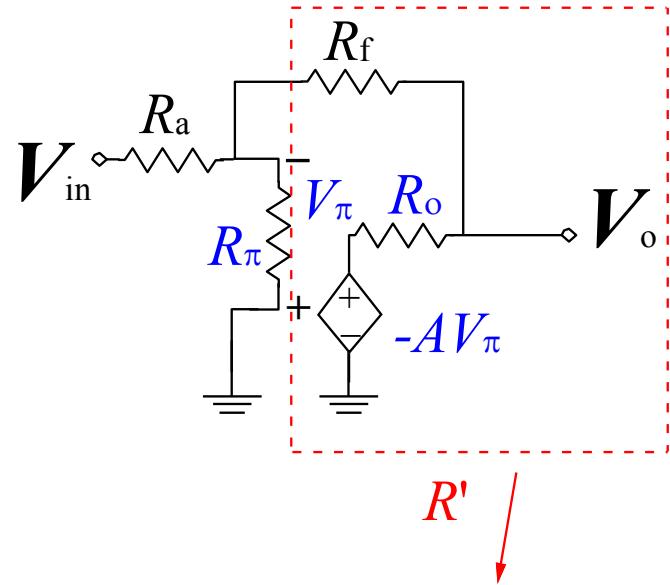
# Input Impedance

Finally, we find the input impedance as,

$$R_{in} = R_a + \left[ \frac{1}{R_\pi} + \frac{1+A}{R_f + R_o} \right]^{-1} \quad \Rightarrow \quad R_{in} = R_a + \frac{R_\pi (R_f + R_o)}{R_f + R_o + (1+A)R_\pi}$$

Since, $R_f + R_o << (1+A)R_\pi$, $R_{in}$ become,

$$R_{in} \sim R_a + \frac{(R_f + R_o)}{(1+A)}$$

Again with $R_f + R_o << (1+A)$

$$R_{in} \sim R_a$$

Note: The op-amp can provide an impedance isolated from input to output

# Output Impedance

Only source-free output impedance would be considered, i.e. $V_i$ is assumed to be 0

Firstly, with figure (a),

$$V_\pi = \frac{R_a // R_\pi}{R_f + R_a // R_\pi} V_o \Rightarrow V_\pi = \frac{R_a R_\pi}{R_a R_f + R_a R_\pi + R_f R_\pi} V_o$$

By using KCL, $i_o = i_1 + i_2$

$$i_o = \frac{V_o}{R_f + R_a // R_f} + \frac{V_o - (-AV_\pi)}{R_o}$$

By substitute the equation from Fig. (a),

The output impedance, $R_{out}$ is

$$\frac{V_o}{i_o} = \frac{R_o(R_a R_f + R_a R_\pi + R_f R_\pi)}{(1 + R_o)(R_a R_f + R_a R_\pi + R_f R_\pi) + (1 + A)R_a R_\pi}$$

$\therefore R_\pi$ and $A$ comparably large,

$$R_{out} \sim \frac{R_o(R_a + R_f)}{A R_a}$$



(a)   (b)