Loop incidence matrix & KVL

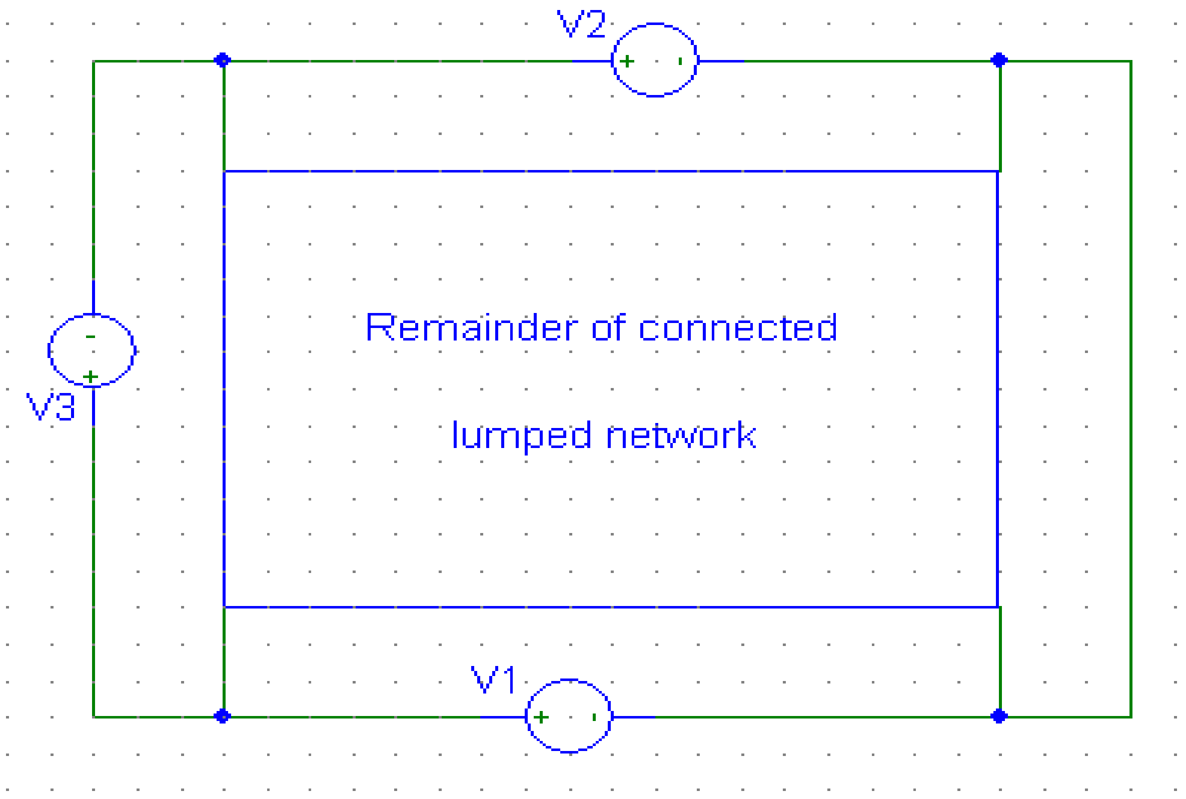
We define branch voltage vector

$$v_b(t) \equiv [v_1(t), v_2(t), \ldots, v_b(t)]`$$

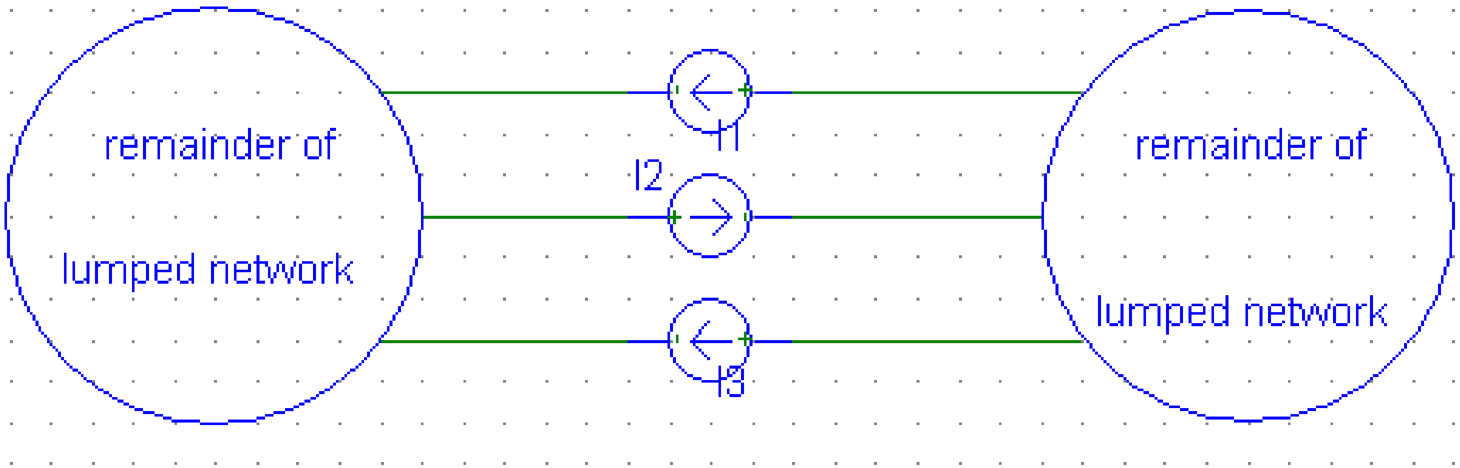
We may write the KVL loop equations conveniently in vector – matrix form as

$$B_a v_b(t) = 0 \quad \text{for all t}$$

General Case



$$v_1(t) - v_2(t) - v_3(t) = 0 \quad \text{(for all t)}$$



$$i_1(t) - i_2(t) + i_3(t) = 0 \text{ (for all t)}$$

To obtain the cut set equations for an n-node , b-branch connected lumped network, we first write Kirchhoff `s law

$$Qi_b(t) \equiv 0 \quad v_b(t) \equiv Q`v_t(t)$$

The close relation of these expressions with

$$Ai_b(t) \equiv 0 \quad v_b(t) \equiv A`v_n(t)$$

$$i_b(t) = y_b v_b(t) + \tau_b$$

$$y_b \equiv diag(y_k)$$

$$y_k \equiv \begin{cases} 0 : \text{if } k\text{th branch contains an indepedent voltage source.} \\ \mathrm{C_k} D : \text{if } k\text{th branch contains a capacitance of value } \mathrm{C_k} \\ \dfrac{1}{\mathrm{R_k}} : \text{if } k\text{th branch contains a resistance of value } \mathrm{R_k} \\ \dfrac{1}{\mathrm{L_k} D} : \text{if } k\text{th branch contains an inductance of value } \mathrm{L_k} \\ 0 : \text{if } k\text{th branch contains an independent current source} \end{cases}$$

And current vector τ_b is specified as follows

$$\tau_k \equiv \begin{cases} i_k(t): \text{ if } k\text{th branch contains an independent voltage source} \\ 0: \text{if } k\text{th branch contains a capacitance} \\ 0: \text{if } k\text{th branch contains a resistance} \\ i_{k0}: \text{if } k\text{th branch contains an inductance with the} \\ \quad \text{initial condition } i_k(\mathrm{t}_0) = i_{k0} \\ \hat{i}_k(t): \text{if } k\text{th branch contains an independent current} \\ \quad \text{source specified by the time function } \hat{i}_k \end{cases}$$

$$0 = Qi_b(t) = Qy_bQ`v_t(t) + Q\tau_b$$

Hence,

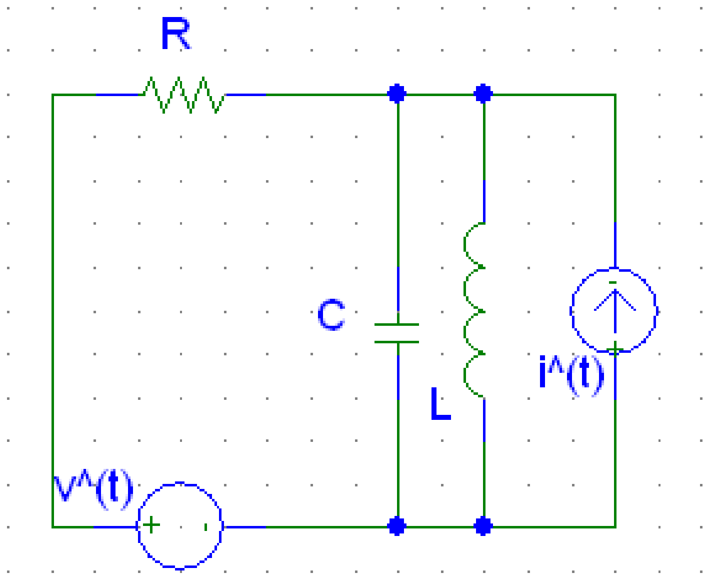
$$Qy_bQ`v_t(t) = -Q\tau_b$$

$$v_b(t) \equiv \hat{Q}`v_{\hat{i}}(t)$$

We obtain cutset equations

$$Qy_b\hat{Q}`v_{\hat{i}}(t) = -Q\tau_b$$

Example



$$i_b(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{LD} & 0 \\ 0 & 0 & 0 & 0 & CD \end{bmatrix} v_b(t) + \begin{bmatrix} i_1(t) \\ 0 \\ \hat{i}(t) \\ i_4(t_0) \\ 0 \end{bmatrix}$$

hence the fundamental cutset matrix

$$Q = \begin{bmatrix} +1 & 0 & -1 & -1 & -1 \\ 0 & +1 & -1 & -1 & -1 \end{bmatrix}$$

yields the cutset equations

$$\begin{bmatrix} \frac{1}{LD} + CD & \frac{1}{LD} + CD \\ \frac{1}{LD} + CD & \frac{1}{R} + \frac{1}{LD} + CD \end{bmatrix} \begin{bmatrix} \hat{v}(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} -i_1(t) + \hat{i}(t) + i_4(t_0) \\ \hat{i}(t) + i_4(t_0) \end{bmatrix}$$

In this case we need only solve

$$\frac{1}{R}v_2(t)+\frac{1}{L}\int_{t_0}^{t}v_2(\tau)d\tau+C\frac{dv_2(t)}{dt}=-\frac{1}{L}\int_{t_0}^{t}\hat{v}(\tau)d\tau-C\frac{d\hat{v}(t)}{dt}+\hat{i}(t)+i_4(t_0)$$

for the voltage function v_2 to obtain every branch variable.