Network topology, cut-set and

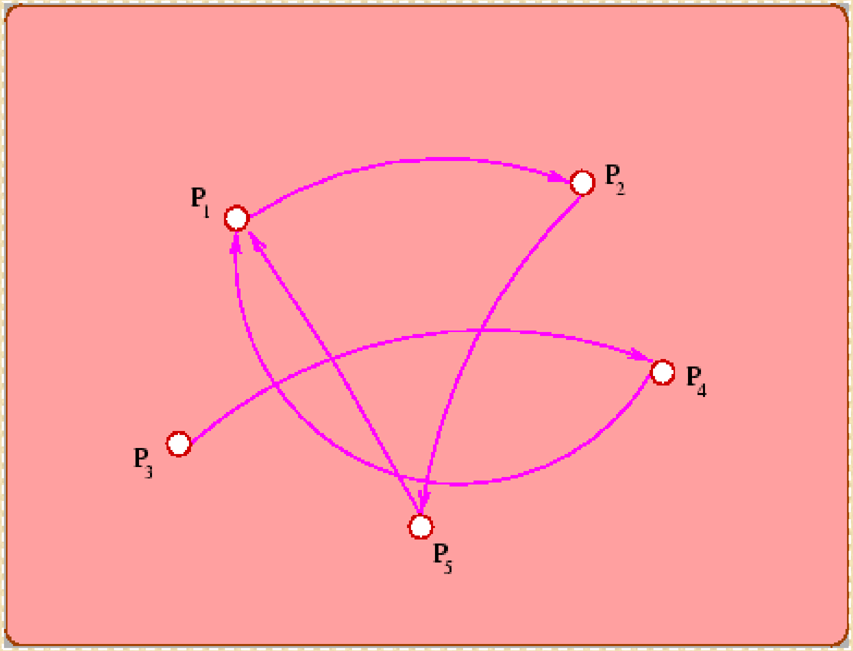
loop equation

Definitions

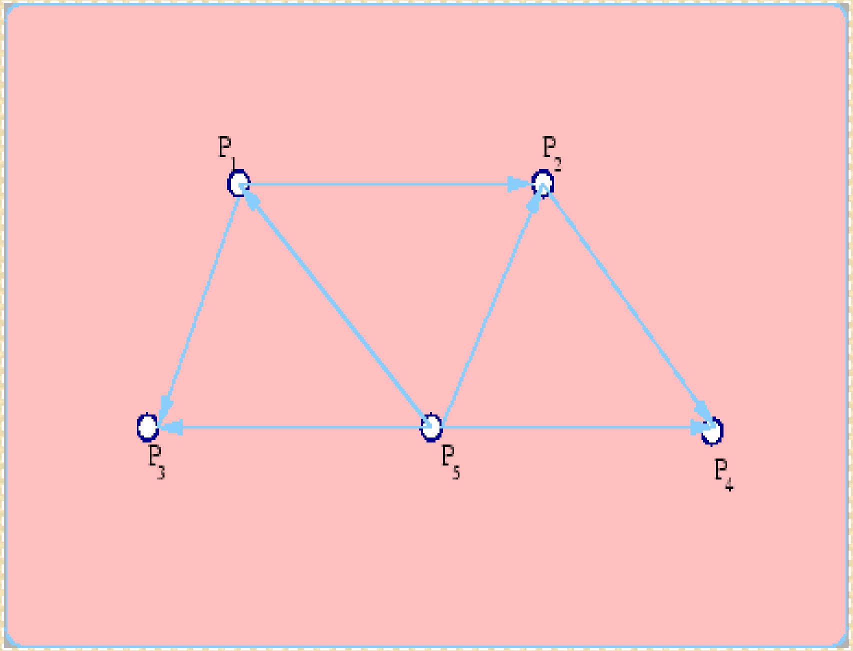
- Connected Graph : A lumped network graph is said to be connected if there exists at least one path among the branches (disregarding their orientation) between any pair of nodes.

- Sub Graph : A sub graph is a subset of the original set of graph branches along with their corresponding nodes.

(A) Connected Graph



(B) Disconnected Graph



Cut – Set

- Given a connected lumped network graph, a set of its branches is said to constitute a cut-set if its removal separates the remaining portion of the network into two parts.

Tree

- Given a lumped network graph, an associated tree is any connected subgraph which is comprised of all of the nodes of the original connected graph, but has no loops.

Loop

- Given a lumped network graph, a loop is any closed connected path among the graph branches for which each branch included is traversed only once and each node encountered connects exactly two included branches.

Theorems

- (a) A graph is a tree if and only if there exists exactly one path between an pair of its nodes.
- (b) Every connected graph contains a tree.
- (c) If a tree has n nodes, it must have n-1 branches.

Fundamental cut-sets

- Given an n - node connected network graph and an associated tree, each of the n -1 fundamental cut-sets with respect to that tree is formed of one tree branch together with the minimal set of links such that the removal of this entire cut-set of branches would separate the remaining portion of the graph into two parts.

Fundamental cutset matrix

$$q_{ij} \equiv \begin{cases} +1: \text{if branch } j \text{ is in cutset } i \text{ and has the same orientation with} \\ \quad \text{regard to the closed surface defining cutset } i \text{ as the tree} \\ \quad \text{branch associated with cut-set } i. \\ \\ 0: \text{if branch } j \text{ is not in cutset } i. \\ \\ +1: \text{if branch } j \text{ is in cutset } i \text{ and has the opposite oriientation} \\ \quad \text{with regard to the closed surface defining} \\ \quad \text{cutset } i \text{ as the tree branch associated with} \\ \quad \text{cutset } i. \end{cases}$$

Nodal incidence matrix

The fundamental cutset equations may be obtained as the appropriately signed sum of the Kirchhoff `s current law node equations for the nodes in the tree on either side of the corresponding tree branch, we may always write

$$Q = WA_a$$

(A is nodal incidence matrix)

Loop incidence matrix

Loop incidence matrix defined by

$$b_{ij} \equiv \begin{cases} +1: \text{if branch } j \text{ is in loop } i \text{ and is oriented} \\ \quad\;\; \text{in the same direction as the loop.} \\ \\ 0: \text{if branch } j \text{ is not in loop } i. \\ \\ -1: \text{if branch } j \text{ is in loop } i \text{ and is oriented} \\ \quad\;\; \text{in the opposite direction as the loop.} \end{cases}$$