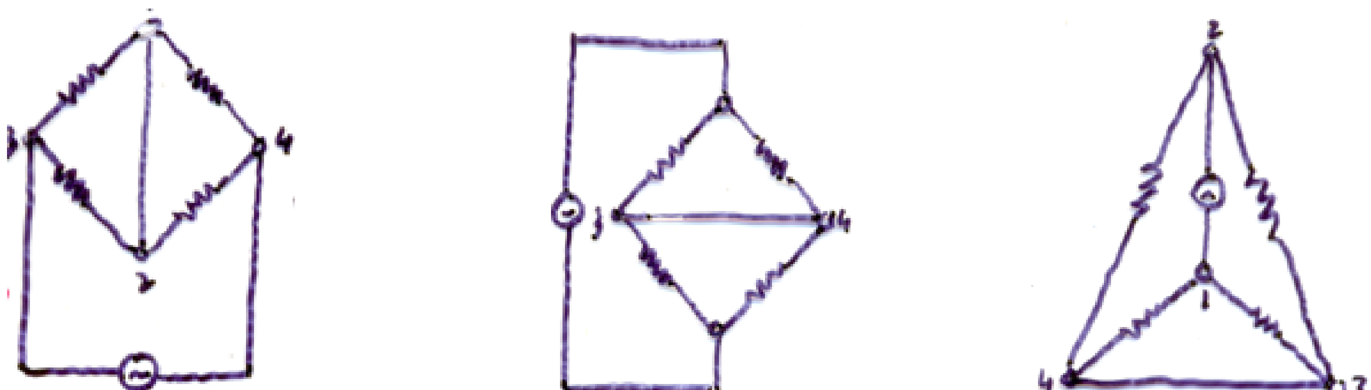
GRAPH THEORY

NETWORK TOPOLOGY

- A network is an interconnection of passive elements(R,L,C) and active elements (voltage source, current source).
- Network topology deals with those properties of the networks which are unaffected when we stretch, twist or otherwise distort the size and shape of the network.
- We use this when we have a more elaborate circuit with more nodes and more loops.

- Example:

These three configurations are of same network. These are topologically equal in the sense voltage and current through any branch will be same in all three cases.



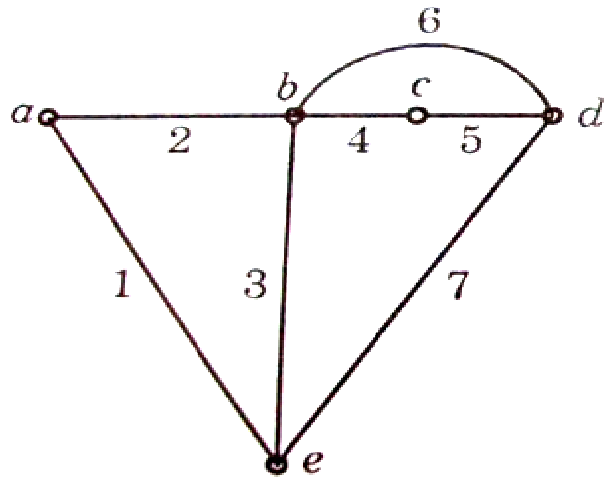
(a) (b) (c)

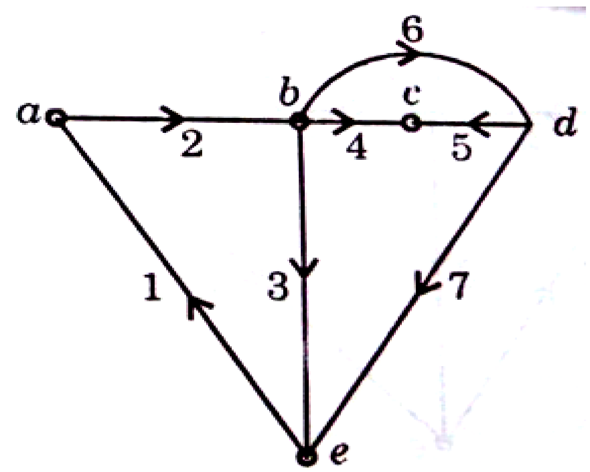
<u>Note</u>: The ideal voltage and current sources are replaced by short circuit and open circuit respectively.

DEFINITIONS

- Node: Terminal common to two or more elements is called a node.
- Branch: Line replacing the network element in a graph.

  Each branch joins two distinct nodes.
- Graph: A collection of nodes and branches. It shows the geometrical interconnection of the elements of a network.
  - Directed: A graph whose branches are oriented.
  - Undirected: A graph whose branches are not oriented.

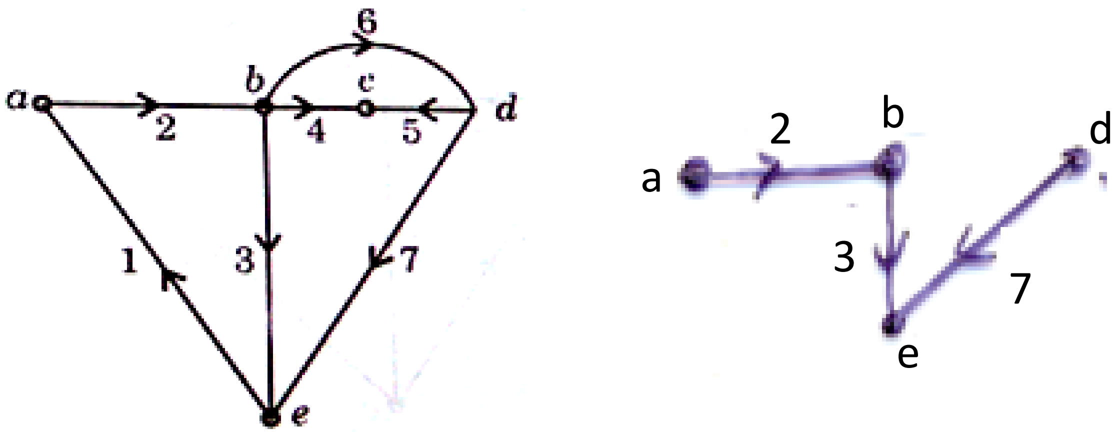




Undirected and Directed graphs

- Rank of a Graph: It is (*n-1*), where *n* is the number of nodes or vertices of the graph.
- Subgraph: It is a subset of the nodes and the branches of the graph. The subset is proper if it does not contain all the branches and nodes of the graph.
- Path: It is a particular (improper) subgraph consisting of an ordered sequence of branches having the following properties:-

- At the terminating nodes, only one branch is incident
- At the remaining nodes, called the internal nodes, two branches are incident.



Branches (2), (3) and (7) constitute a path

- <u>Tree</u>: A tree is a connected subgraph of a connected graph containing all the nodes of the graph but containing no loops, i.e., there is a unique path between every pair of nodes.

- A tree is also defined as any set of branches in the original graph that is just sufficient to connect all the nodes.
- There branches are (n-1).

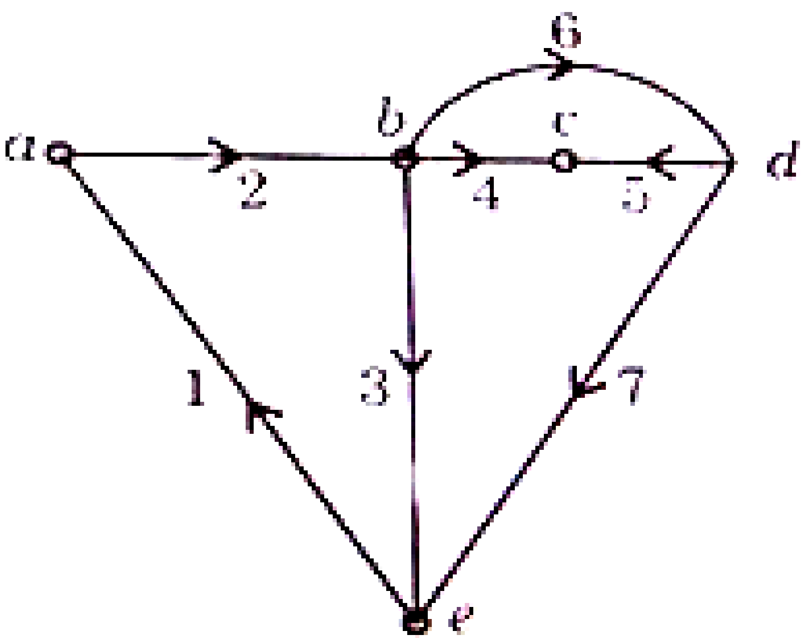
- Twig: The branches of the tree are called twigs.
- Link or Chord: Those branches of the graph which are not in the tree.
- Co-tree: All the links of a tree together constitute complement of the tree and is called co-tree, in which the number of branches are: $b-(n-1)$ where b is the number of branches of the graph.

Number of twigs: $nt = n-1$;

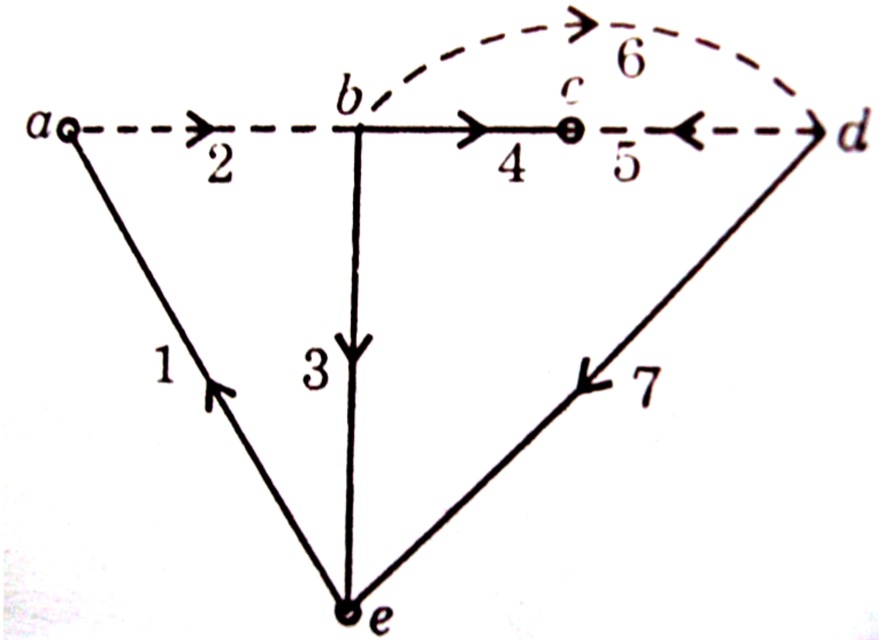
Number of links: $nl = b-nt = b-n+1$

Example

<u>A Graph</u>



<u>Corresponding tree and co-tree</u>



——— twigs ; branches of tree [1, 3, 4, 7]

- - - - - - links ; branches of co-tree [2, 5, 6]

branches (b) = 7

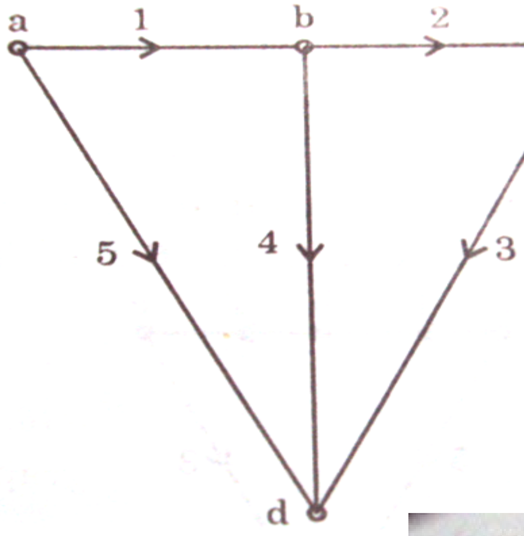
nodes (n) =5

twigs (t) = 4

links (l) = 3

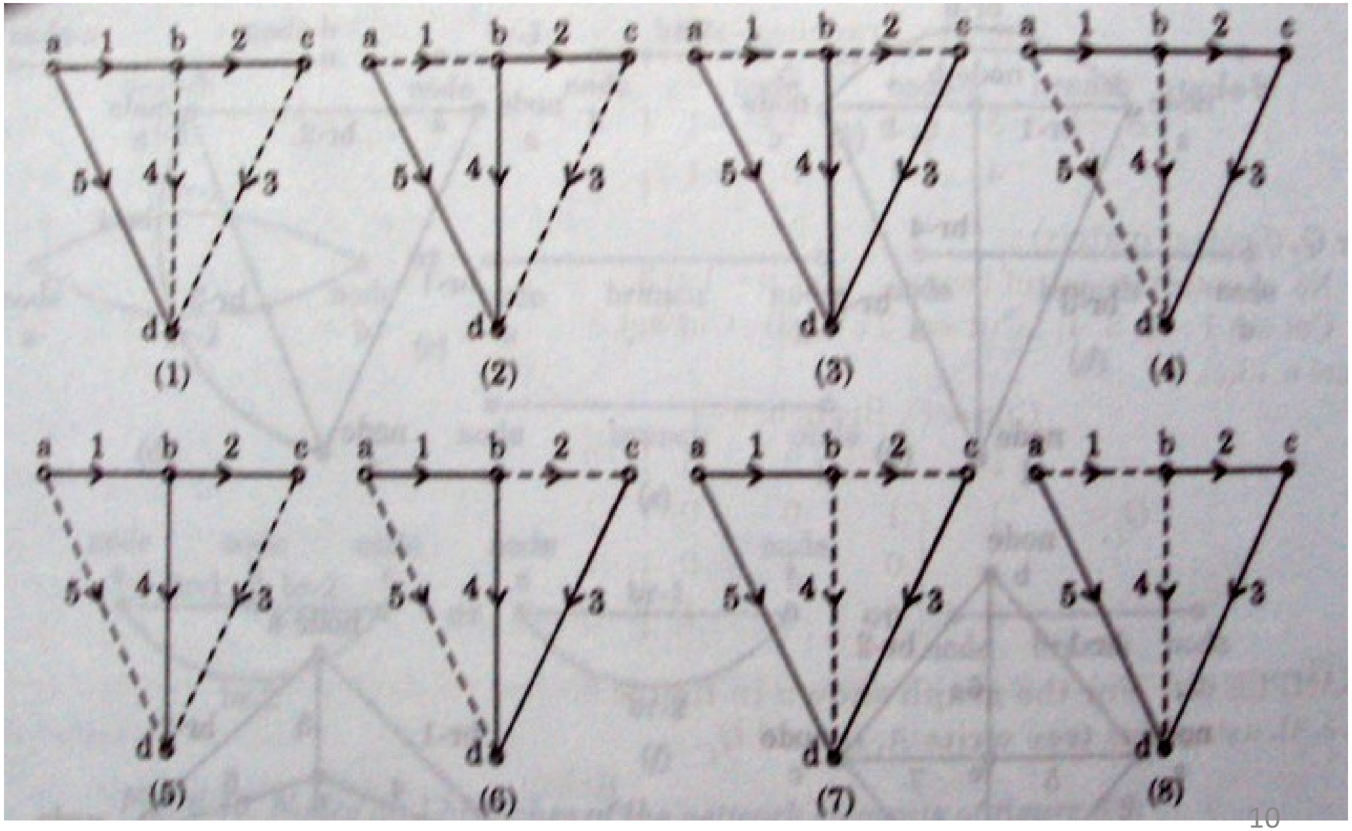
Properties of Trees

- Each tree has *(n-1)* branches.
- The rank of a tree is *(n-1)*. This is also the rank of the graph to which the tree belongs.
- Tree has all the nodes. It has no closed loops.
- The number of terminal nodes or end vertices of every tree are two.
- Every connected graph has atleast one tree.
- A connected graph's subgraph is a tree if there exists only one path between any pair of nodes in it.



a) A Graph

b) All the trees and the corresponding co-trees



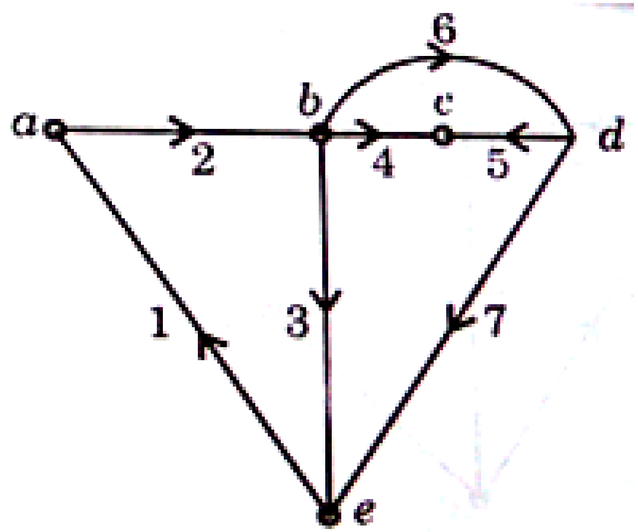
Incidence Matrix [A]

- The incident matrix translates the graphical data of a network into algebraic form.
- For a graph with *n* nodes and *b* branches, the complete incidence matrix A_C is an $n*b$ matrix whose elements are defined by:

a_{ij} = 1; if branch j leaves node i

-1; if branch j enters node i

0; if branch j is not incident with node i.



Nodes\Branches →

↓

$A_C =$

Node	1	2	3	4	5	6	7
a	−1	+1	0	0	0	0	0
b	0	−1	+1	+1	0	+1	0
c	0	0	0	−1	−1	0	0
d	0	0	0	0	+1	−1	+1
e	+1	0	−1	0	0	0	−1

Properties of Complete Incidence Matrix

- The rank of a complete incidence matrix is *(n-1)*.
- Determinant of the complete incidence matrix is always zero.
- The sum of the entries in any column is zero.
- If one complete row is removed from a complete incidence matrix, it results into a reduced incidence matrix $[A_R]$.
- The total number of possible trees of any graph= determinant of $[A\ A^T]$.

Drawing graph from Complete Incidence Matrix

a) Given Matrix

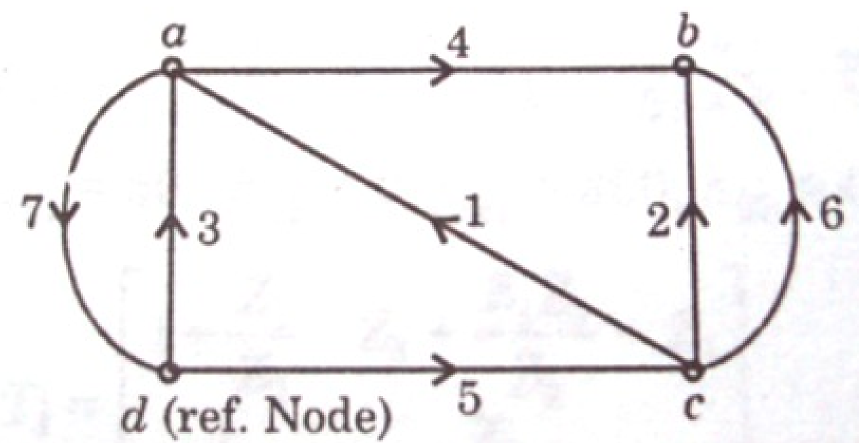
$$A_C = \begin{bmatrix} -1 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

b) Numbering of rows and branches

Nodes\Branches →

$$A = \begin{array}{c} \downarrow \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} -1 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \end{array}$$

c) Mark 3 nodes n draw the graph



Drawing graph from Reduced Incidence Matrix

a) Given reduced incidence matrix

Nodes\Branches

$$A = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

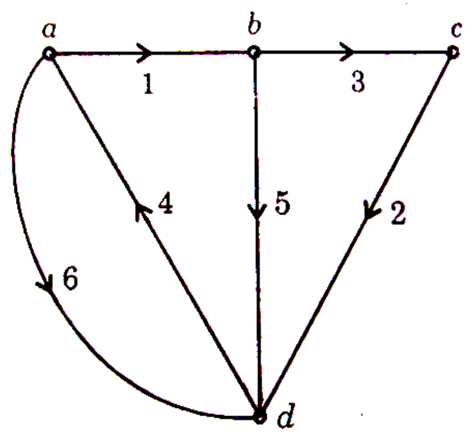
b) Make complete incidence matrix

Nodes\Branches

$$A = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$d \; [\, 0 \quad -1 \quad 0 \quad 1 \quad -1 \quad -1 \,]$$

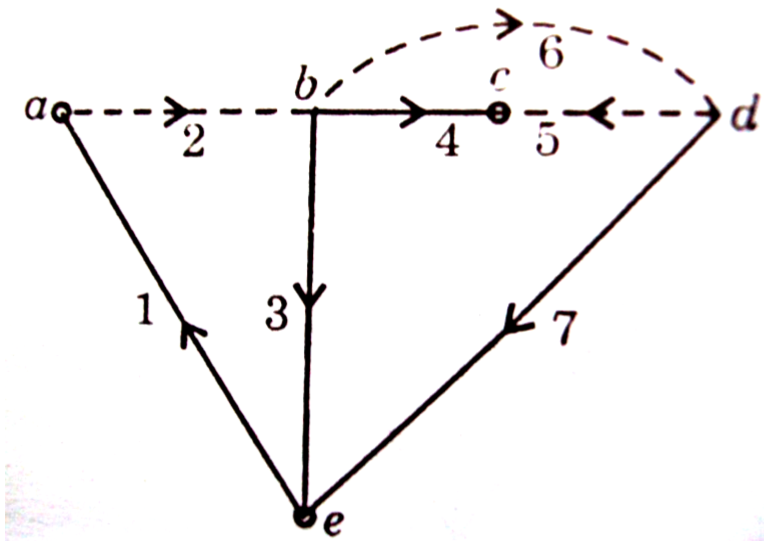
c) Make the graph with 4 nodes

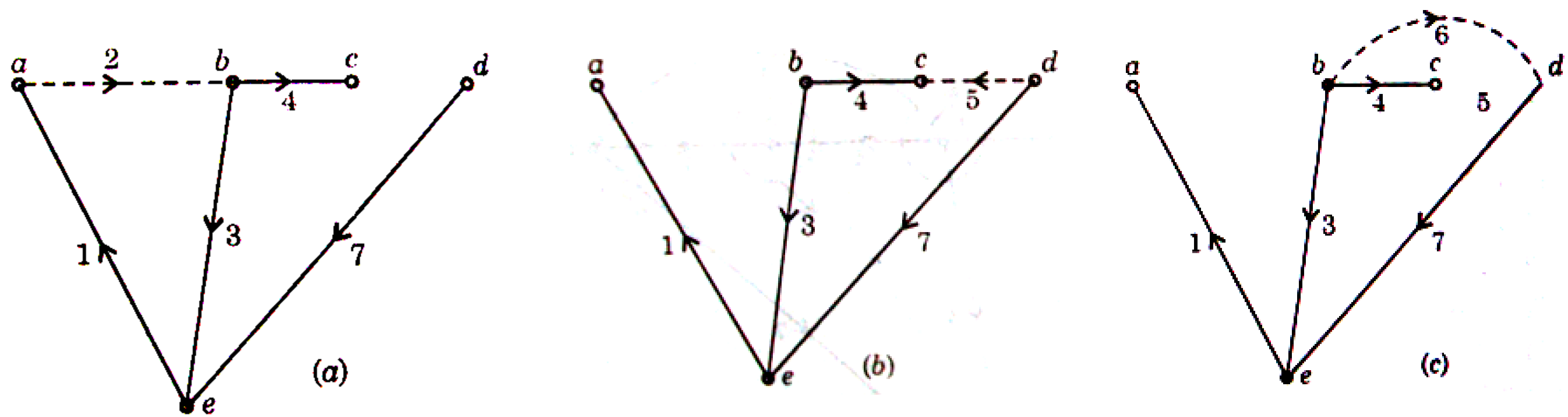


Tie-Set Matrix

- Given a graph, select a tree. Then each link gives rise to a loop or a circuit.
- Loops formed in this way are the minimum number of loops of a graph.
- They are the *fundamental loops (f loops)* or the *Tiesets*. The number of tiesets is equal to *nl*, the number of links.
- The orientation of the loop is defined by the orientation of the corresponding link.
- Writing the tieset, with link as underlined and serial of tieset as the link.

Given graph





Tiesets of the tree

a) Tieset 2: [1,2,3]
b) Tieset 5: [3,4,5,7]
c) Tieset 6: [3,6,7]

The tieset matrix is

$$
\begin{array}{c|cccc|ccc}
\text{Tiesets}\backslash\text{Branches} \rightarrow & & \text{twigs} & & & & \text{links} & \\
\downarrow & 1 & 3 & 4 & 7 & 2 & 5 & 6 \\
\hline
2 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
5 & 0 & 1 & -1 & -1 & 0 & 1 & 0 \\
6 & 0 & -1 & 0 & 1 & 0 & 0 & 1
\end{array}
= B_f
$$

This is a non singular matrix of rank $nl = b - nt = b - n + 1 = 3$.

$$[B_f] = [B_{ft} : B_{fl}] = [B_{ft} : U]$$

B_{fl} : for links

U: identity matrix

B_{ft} : for twigs