Synthesis of Transfer Functions of 2 –port n /w with 1 ohm terminated Load

- L –C Networks with zeros at $S = 0$ or $s = \infty$
- Steps to be followed
  1. Identify no of zeros of transfer function and where they are located. $S = 0$ or $s = \infty$.
  2. Identify the circuit (LP filter or HP filter) or how L & C located.
  3. Identify $N(s)$ of $Z_{21}(s) / Y_{21}(s)$ whether Even or Odd.
  4. Segregate $D(s)$ of $Z_{21}(s) / Y_{21}(s)$ into Even and Odd parts.
- 5. Express $Z_{21}(s) / Y_{21}(s)$ as $z_{21}(s)$

$$Z_{21} = \frac{z_{21}(s)}{y_{21}(s)}$$

or $Y_{21} = \frac{y_{21}(s)}{z_{21}(s)}$

(Divide $N(s)$ & $D(s)$ By even part if $N(s)$ is odd and by odd part if $N(s)$ is even)

6. Synthesise $z_{22}(s)$ or $y_{22}(s)$ as Ladder network starting from 1 ohm side.

7. NOTE: If $Z_{21}(s)$ has all ZEROS at $S = \infty$ only then CAUER – I NETWORK.

   If $Z_{21}(s)$ has all ZEROS AT $S = 0$ only then CAUER – II NETWORK.
Synthesis with 1 ohm termination

• $Z_{21}(s) = \frac{1}{S^3 + 3S^2 + 3S + 2}$

• 1. No of zeros of transmission = 3 and are at $s = \infty$

• 2. Identify the L-C circuit as LP filter.

  Circuit shown in next slide
Synthesis with 1 ohm termination

3. Identify N(s) of \( Z_{21}(s) \) as Even or odd. \( Z_{21}(s) = \frac{1}{S^3 + 3s^2 + 3s + 2} \)
   In this case Even.

4. D(s) of \( Z_{21}(s) \)
   Even part  \( 3s^2 + 2 \)
   Odd part \( S^3 + 3s \)

5. \( Z_{21}(s) = \frac{1}{S^3 + 3s} \)

\[ \frac{z_{21}(s)}{1 + \frac{(3s^2 + 2)}{(S^3 + 3s)}} = 1 + z_{22}(s) \]
\[ z_{21}(s) = \frac{1}{S^3 + 3s} \quad \text{and} \quad z_{22}(s) = \frac{(3s^2 + 2)}{(S^3 + 3s)} \]

6. Synthesize \( z_{22}(s) \) in Ladder n/w to have three zeros at \( s = \infty \)

   USE CAUER - I NETWORK

   (Degree of Denominator higher, so invert. First element will be shunt element C and \( y_{22} \) from 1 ohm side)
Synthesis with 1 ohm termination

- $Y_{21}(s) = \frac{1}{S^3 + 2s^2 + 2s + 1}$
- 1. No of zeros of transmission = 3 and are at $s = \infty$
- 2. Identify the L-C circuit as LP filter.
Synthesis with 1 ohm termination

- \( Z_{21}(s) = \frac{S^3}{S^3 + 3s^2 + 4s + 2} \)
- 1. No of zeros of transmission = 3 and are at \( s = 0 \)
- 2. Identify the L-C circuit as HP filter.

Circuit shown in next slide
Synthesis with 1 ohm termination

• 3. Identify N(s) of $Z_{21}(s)$ as Even or odd. $Z_{21}(s) = \frac{S^3}{S^3 + 3s^2 + 4s + 2}$
In this case Odd.

4. $D(s)$ of $Z_{21}(s)$
   Even part $3s^2 + 2$
   Odd part $S^3 + 4s$

5. $Z_{21}(s) = \frac{S^3}{(3s^2 + 2)}$
   $\frac{z_{21}(s)}{1 + \frac{(S^3 + 4s)}{(3s^2 + 2)}} = \frac{1 + z_{22}(s)}{1 + \frac{z_{22}(s)}{z_{21}(s)}}$
   $z_{21}(s) = \frac{S^3}{(3s^2 + 2)}$ and $z_{22}(s) = \frac{(S^3 + 4s)}{(3s^2 + 2)}$

6. Synthesize $z_{22}(s)$ in Ladder n /w to have three zeros at $s = 0$
   USE CAUER -II NETWORK $z_{22}(s) = \frac{(4s + S^3)}{(2 + 3s^2)}$
   (Degree of Denominator Lower, so invert. First element will be shunt element L and $y_{22}$ from 1 ohm side. $Y_2$, $z_3$ and $y_4$ respectively)
Synthesis with 1 ohm termination

- \( Y_{21}(s) = \frac{S^3}{S^3 + 3s^2 + 3s + 2} \)
- 1. No of zeros of transmission = 3 and are at \( s = 0 \)
- 2. Identify the L-C circuit.
3. Identify $N(s)$ of $Y_{21}(s)$ as Even or odd. $Y_{21}(s) = \frac{S^3}{S^3 + 3s^2 + 3s + 2}$
   In this case Odd.

4. $D(s)$ of $Z_{21}(s)$
   - Even part: $3s^2 + 2$
   - Odd part: $S^3 + 3s$

5. $Y_{21}(s) = \frac{S^3}{(3s^2 + 2)}$

   $$\frac{y_{21}(s)}{y_{21}(s)} = \frac{1 + y_{22}(s)}{1 + \frac{(S^3 + 3s)}{(3s^2 + 2)}}$$

   $y_{21}(s) = \frac{S^3}{(3s^2 + 2)}$ and $y_{22}(s) = \frac{(S^3 + 3s)}{(3s^2 + 2)}$

6. Synthesize $y_{22}(s)$ in Ladder n/w to have three zeros at $s = 0$

   **USE CAUER -II NETWORK** $z_{22}(s) = \frac{(4s + S^3)}{(2 + 3s^2)}$

   (Degree of Denominator Lower, so invert. First element will be series element C and $y_{22}$ from 1 ohm side. $Z_2$, $Y_3$ and $Z_4$ respectively)
Synthesis with 1 ohm termination

- \( Y_{21}(s) = \frac{S^2}{S^3 + 3s^2 + 4s + 2} \)

1. Two zeros of transmission are at \( s = 0 \) and one at \( s = \infty \)

2. Identify the L-C circuit. This is neither as HP filter and nor as LP filter type circuit.

3. Identify \( N(s) \) of \( Z_{21}(s) \) as Even or odd. \( Z_{21}(s) = \frac{S^2}{S^3 + 3s^2 + 4s + 2} \)
   
   In this case Even

4. \( D(s) \) of \( Y_{21}(s) \)
   
   Even part \( 3s^2 + 2 \)
   
   Odd part \( S^3 + 4s \)

   \( \frac{S^2}{S^3 + 4s} \)

5. \( Y_{21}(s) = \frac{\text{Even part}}{\text{Odd part}} = \frac{S^2}{(S^3 + 3s + 4s)} \)

   \( Y_{21}(s) = S^2 / (S^3 + 3s) \)

   and \( y_{22}(s) = \frac{(3s^2 + 2)}{(S^3 + 4s)} \)

6. \( y_{22}(s) = (3s^2 + 2) / s(s^2 + 4s) \).

7. A parallel inductor gives a zero of transmission at \( s = 0 \)
Synthesis with 1 ohm termination

- $y_{22}(s) = \frac{(3s^2 + 2)}{s(s^2 + 4s)}.$
  
  $$= \frac{1}{2s} + \frac{5s}{2(s^2 + 4)}$$
  
  $$= \frac{1}{2s} + 1/ [ (2/5)s + (8/5)s]$$
  
  $$= Y_A + Y_B$$

$Y_A = (1/2s)$ represents an inductance of 2H

$Y_B = 1/ [ (2/5)s + (8/5)s] = 1 / Z_B$

or $Z_B$ represents that it is a combination of two elements in series which give zeros at $s = 0$ and at $s = \infty$

These give an inductor of $2/5$ H and a capacitor of $5/8$ F

overall network as looked from 1 ohm side is shown on the next slide.
Synthesis with 1 ohm termination

- \( Y_{21}(s) = \frac{1}{s^3 + 2s^2 + 2s + 1} \)
  
  T- n/w series branches \( L = \frac{3}{2} \) H and \( \frac{1}{2} \) H. Shunt branch \( \frac{4}{3} \) F

- \( Y_{21}(s) = \frac{s^3}{s^3 + 3s^2 + 3s + 2} \)
  
  T- n/w series branches \( C = \frac{3}{7} \) F and \( \frac{3}{2} \) F. Shunt branch \( \frac{7}{9} \) H

- \( z_{21}(s) = \frac{s}{s^3 + 3s^2 + 3s + 2} \)
  
  (\( \frac{9}{7} \) H in series with \( \frac{7}{6} \) F with \( \frac{1}{3} \) F capacitor in parallel with 1 ohm resistor)