Transfer Functions , Zeros of Transmission and Network Synthesis with 1 ohm Terminations

TRANSFER FUNCTIONS

- **Properties of Transfer Functions [T(s)]**
- (I) T(s) is real for real s. This property is satisfied when T(s) is a rational function with real coefficients.

(II)(a) T(s) has no poles in the right half of s-plane

(b) There are no multiple poles on the jw axis.

(c) If T(s) = P(s) / Q(s) , then the degree of numerator polynomial P(s) cannot exceed the degree of denominator Q(s) by more than unity.

(d) Q(s) must be Hurwitz polynomial.

(III) Suppose P(s) and Q(s) are given in terms of even and odd parts as

$$P(s) = M_1(s) + N_1(s) = \text{even} + \text{odd and}$$

$$Q(s) = M_2(s) + N_2(s) = \text{even} + \text{odd}$$

Properties of Transfer Functions (Contd)

$$\text{Then } T(s) = \frac{P(s)}{Q(s)} = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)} \quad \text{or}$$

$$T(jw) = \frac{P(jw)}{Q(jw)} = \frac{M_1(jw) + N_1(jw)}{M_2(jw) + N_2(jw)}$$ which shows that Amplitude Response is an even function in w and the phase response is an odd function in w.

(Already discussed while doing Amplitude and phase responses)

$V_1 = z_{11}I_1 + z_{12}I_2$ $\quad\quad$ $I_1 = y_{11}V_1 + y_{12}V_2$

$V_2 = z_{21}I_1 + z_{22}I_2$ $\quad\quad$ $I_2 = y_{21} V_1 + y_{22} V_2$

- $V_2 / V_1 = z_{21} / z_{11}$ $\quad\quad$ $V_2 / V_1 = - y_{21} / y_{11}$

Generally there are source and load impedances to be accounted for.

Properties of Transfer Functions (Contd)

- For two port networks ,

$z_{12} = V_1 / I_2 \quad I_1=0$ and $z_{21} = V_2 / I_1$ when $I_2 = 0$

$V2 / V1 = z_{21} / z_{11}$ (in terms of open circuit impedances)

$V2 / V1 = - y_{21} / y_{22}$ (in terms of short circuit admittances)

When the network is terminated at output port by a resistance R then the transfer impedance of the overall network is

$$Z_{21} = \frac{V_2}{I_1} = \frac{z_{21}R}{z_{22} + R} \qquad (V_2 = - I_2 R)$$

$$Y_{21} = \frac{I_2}{V_1} = \frac{y_{21}G}{y_{22} + G} \qquad (V_2 = - I_2 R)$$

Properties of Transfer Functions (Contd)

- When both the ports are terminated in resistors yhe voltage ratio transfer function V2 / Vg is given by

$$\frac{V_2}{V_g} = \frac{z_{21}R_2}{(z_{11} + R_1)(z_{22}+R_2) - z_{21}z_{12}} \quad \text{where}$$

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$V_1 = V_g - I_1R_1$$

$$V_2 = - I_2R_2$$

similarly we can describe other transfer functions such as current ratio transfer functions in terms of open and short circuit parameters.

Specific Properties of Open-Circuit and Short-circuit Parameters

- I) The poles of $z_{21}(s)$ are also the poles of $z_{11}(s)$ and $z_{22}(s)$. However , not all the poles of $z_{11}(s)$ and $z_{22}(s)$ are the poles of $z_{21}(s)$.

(II) The poles of $y_{12}(s)$ are also the poles of $y_{11}(s)$ and $y_{22}(s)$. However , not all the poles of $y_{11}(s)$ and $y_{22}(s)$ are the poles of $y_{12}(s)$

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Example Π- network . Series branch 5F capacitor .Parallel branches 1/2 H and 1/3 H

The y-parameters for this Π- network are

$y_{11}(s) = 2/s + 5s$, $y_{22}(s) = 3/s + 5s$, $y_{12}(s) = - 5s$

It can be (seen that $y_{11}(s)$ and $y_{22}(s)$ have poles at $s = 0$ and at $s = \infty$, whereas $y_{12}(s)$ has pole only at $s = \infty$. Thus the property is verified.

(III) Suppose $y_{11}(s)$, $y_{22}(s)$ & $y_{12}(s)$ all have poles at $s=s_1$

Let residue of the poles at s_1 of the function $y_{11}(s)$ be k_{11} .

Let residue of the poles at s_1 of the function $y_{22}(s)$ be k_{22} .

Similarly, let the residue of the poles at s_1 of the function $y_{12}(s)$ be k_{12} .

General property of L-C , R-C ,or R-L two port networks is that

$k_{11} k_{22} – k_{12}^2 \geq 0$ This equation is known as RESIDUE CONDITION

Specific Properties of Open-Circuit and Short-circuit Parameters

General property of L-C , R-C ,or R-L two port networks is that

$k_{11} k_{22} – k_{12}^2 \geq 0$. This equation is known as RESIDUE CONDITION

Consider the same above example of L-C Π- network whose y-parameters are

$y_{11}(s) = 2/s + 5s$, $y_{22}(s) = 3/s + 5s$, $y_{12}(s) = - 5s$

(i) Corresponding to pole at $s=\infty$,

$k_{11} = 5$, $k_{22} = 5$ and $k_{12} = - 5$

$k_{11} k_{22} – k_{12}^2 = (5)(5) – (- 5)^2 = 0$ residue condition is satisfied.

(ii) Corresponding to pole at $s=0$,

$k_{11} = 2$, $k_{22} = 3$ and $k_{12} = - 0$

$k_{11} k_{22} – k_{12}^2 = (2)(3) – (0)^2 = 6 > 0$ residue condition is satisfied

Specific Properties of Open-Circuit and Short-circuit Parameters

- Ex : **The open –circuit impedance functions of a typical network are**

$$Z_{11} = \frac{4s^4 + 7s^2 + 1}{s(4s^2 + 3)}, \quad Z_{22} = \frac{s(2s^2 + 3)}{4s^2 + 3}, \quad Z_{21} = \frac{2s}{4s^2 + 3}$$

Show that the residue condition is satisfied at the poles at $s = \pm \sqrt{3}/2$ s=o, and at s=∞

SOLUTION By partial fraction method we have

$Z_{11} = s + 1/3s + (8/3) / 4(s^2 + 3/4)$

$Z_{22} = 0 + s/2 + (3s/2) / 4(s^2 + 3/4)$

$Z_{21} = 0 + 0 + 2s / 4(s^2 + 3/4)$

(i) **For the pole at $s = \pm \sqrt{3}/2$**

the residues of Z_{11},,, Z_{22} and Z_{21} are

$k_{11} = (1/4)(8/3) = 2/3$, $k_{22} = (1/4)(3/20 = 3/8$ and $k_{12} = (1/4)(2) = 1/2$

$k_{11} k_{22} - k_{12}^2 = (2/3)(3/8) - (1/2)^2 = (1/4) - (1/4) = 0 \quad (\geq 0)$

Specific Properties of Open-Circuit and Short-circuit Parameters

(ii) For the pole at s=0 the residues of $Z_{11,,,}$ Z_{22} and Z_{21} are

$k_{11} = 1/3$, $k_{22} = 0$ and $k_{12} = 0$

$k_{11} k_{22} - k_{12}^2 = (1/3)(0) - (0)^2 = 0$ (≥ 0)

(iii) For the pole at s=∞ the residues of $Z_{11,,,}$ Z_{22} and Z_{21} are

$k_{11} = 1$, $k_{22} = 1/2$ and $k_{12} = 0$

$k_{11} k_{22} - k_{12}^2 = (1)(1/2) - (0)^2 = 1/2$ (≥ 0)

Zeroes of Transmission

- A zero of Transmission is a zero of transfer Function. At a zero of transmission , there is a zero output for an input of the same frequency .
- The zeroes of transmission of a 2-port network are defined as the frequencies at which the network results in zero output for a finite input.
- s = 0 means w= 0 Capacitor in Series behaves open circuit.

  1 / cs or 1 / wc = infinity

  s = 0 means w= 0 Inductance in parallel branch acts short cct.

  Ls or Lw = 0 at s/w =0

At s = 0 Single L in SHUNT or Single C in SERIES give zero output

Similarly at S = ∞ , C in SHUNT arm or L in SERIES arm give Zero output

Zeroes of Transmission (Contd)

- **L-C parallel combination in series arm**

$z(s) = (1/cs) / (s^2 + 1/LC)$ gives zero output when $z(s) = \infty$ means open circuit.

$s^2 = - 1/LC$

$s = \pm j /\sqrt{LC}$

Zeroes at $\pm j /\sqrt{LC}$

L-C Series combination in shunt arm

$z(s) = Ls + 1/cs = (s^2 + 1/LC) / (1/L) s$

$= 0$ at $s^2 = - 1/LC$ or $s = \pm j /\sqrt{LC}$ zeros

Zeroes of Transmission (Contd)

- **R-C parallel combination in series arm**

  $z(s) = (1/C)/(s + 1/RC)$

  $= \infty$ at $s = - (1 / RC)$ OC , $z = \infty$

R-C Series combination in shunt arm

$z(s) =(s + 1/RC) / (1/R)s$

$= 0$ at $s= - (1/RC)$ SC , $z(s) = 0$

R-L Series combination in parallel arm

$z(s) = Ls + R = L\{ s+ R/L \}$

$= 0$ at $s = - (R/L)$

R-L parallel combination in series arm

$z(s) = (Rs) / (s + R/L)$

$= \infty$ at $s = - R/L$ OC , $z = \infty$

Synthesis with 1 ohm termination

- 3. Identify N(s) of $Y_{21}(s)$ as Even or odd. $Y_{21}(s) = 1/(S^3 + 2s^2 + 2s + 1)$

  In this case Even .

  4. D(s) of $Y_{21}(s)$

  Even part $2s^2 + 1$

  Odd part $S^3 + 2s$

  5. $Y_{21}(s) = \dfrac{1/ (S^3 + 2s)}{1 + (2s^2 + 1) /(S^3 + 2s)} = \dfrac{y_{21}(s)}{1 + y_{22}(s)}$

  $y_{21}(s) = 1/ (S^3 + 3s)$ and $y_{22}(s) = (2s^2 + 1) /(S^3 + 2s)$

  6. Synthesize $y_{22}(s)$ in Ladder n /w to have three zeros at $s = \infty$

  USE CAUER - I NETWORK

  (Degree of Denominator higher , so invert .First element will be series element L and y_{22}. from 1 ohm side)