SYNTHESIS OF CERTAIN *R-L-C* FUNCTIONS

Under certain conditions, R-L-C driving-point functions may be synthesized with the use of either partial fractions or continued fractions. For example, the function

$$Z(s) = \frac{s^2 + 2s + 2}{s^2 + s + 1} \tag{11.58}$$

is neither L-C, R-C, nor R-L. Nevertheless, the function can be synthesized by continued fractions as shown.

$$\begin{array}{l}
s^2 + s + 1\overline{)s^2 + 2s + 2}(1 \leftarrow Z \\
\qquad\qquad\quad \underline{s^2 + \ \ s + 1} \\
\qquad\qquad\qquad\quad s + 1)s^2 + s + 1(s \leftarrow Y \\
\qquad\qquad\qquad\qquad\qquad \underline{s^2 + s} \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad 1)s + 1(s + 1 \leftarrow Z \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\ \ \underline{\underline{s + 1}}
\end{array}$$

The network derived from this expansion is given in Fig. 11.24.

In another case, the poles and zeros of the following admittance are all on the negative real axis, but they do not alternate.

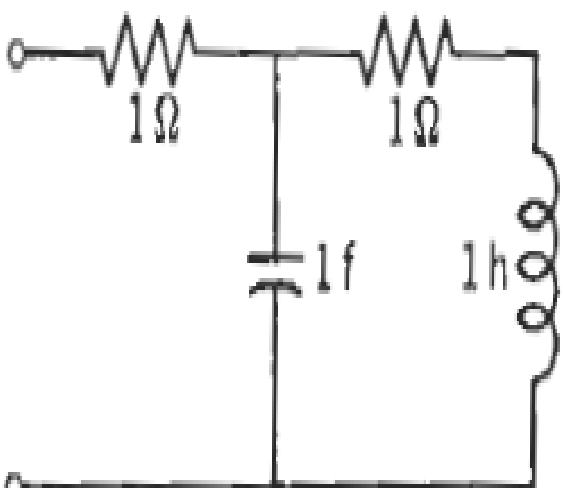

FIG. 11.24

$$Y(s) = \frac{(s+2)(s+3)}{(s+1)(s+4)} \qquad (11.59)$$

The partial fraction expansion for $Y(s)$ is

$$Y(s) = 1 + \frac{\frac{2}{3}}{s+1} + \frac{-\frac{2}{3}}{s+4} \qquad (11.60)$$

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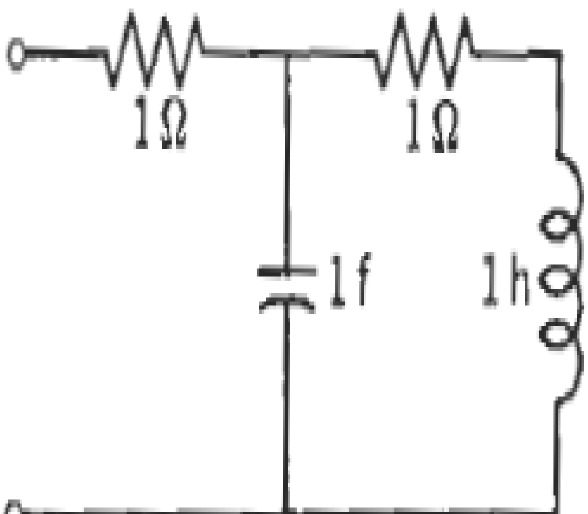

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Cont..

Since one of the residues is negative, we cannot use this expansion for synthesis. An alternate method would be to expand $Y(s)/s$ and then multiply the whole expansion by s.

$$\frac{Y(s)}{s} = \frac{\frac{3}{2}}{s} - \frac{\frac{2}{3}}{s+1} + \frac{\frac{1}{6}}{s+4} \qquad (11.61)$$

When we multiply by s, we obtain,

$$Y(s) = \frac{3}{2} - \frac{\frac{2}{3}s}{s+1} + \frac{\frac{1}{6}s}{s+4} \qquad (11.62)$$

Cont..

Note that $Y(s)$ also has a negative term. If we divide the denominator of this negative term into the numerator, we can rid ourselves of any terms with negative signs.

$$\begin{aligned} Y(s) &= \frac{3}{2} - \left(\frac{2}{3} - \frac{\frac{2}{3}}{s+1}\right) + \frac{\frac{1}{6}s}{s+4} \\ &= \frac{5}{6} + \frac{\frac{2}{3}}{s+1} + \frac{\frac{1}{6}s}{s+4} \end{aligned} \quad (11.63)$$

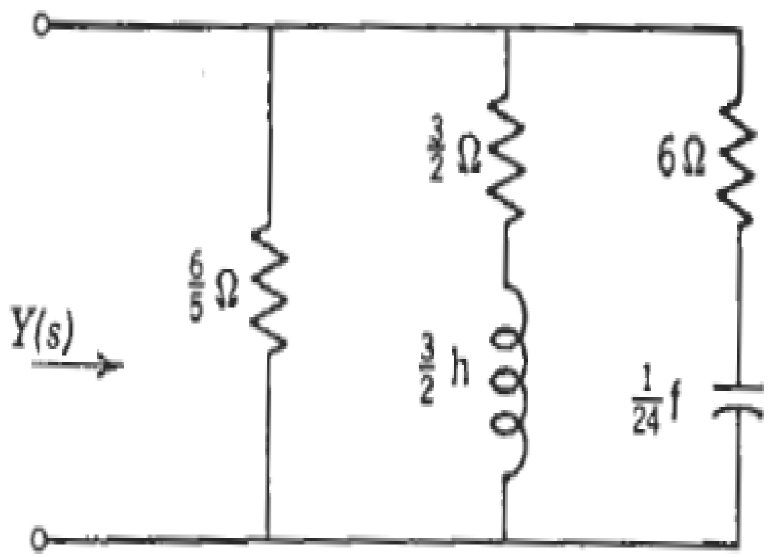

FIG. 11.25

The network that is realized from the expanded function is given in Fig. 11.25.