Synthesis of R-C Impedance /R-L Admittance Networks

- **Properties of R-C Impedance or R-L Admittance Networks**
- **1. The poles and zeros lie on the negative real axis including origin of the complex s-plane.**
- **2. The poles and zeros interlace (or alternate) along the negative real axis.**
- **3. The residues of the poles must be real and positive.**
- **4. The singularity nearest to or at the origin must be a pole.**

  Function $Z_{R\text{-}C}(s)$ or $Y_{R\text{-}L}(s) \rightarrow \infty$ with $s \rightarrow 0$
- **5. The singularity nearest to or at the minus infinity must be a zero.**

  Function $Z_{R\text{-}L}(s)$ or $Y_{R\text{-}C}(s) \rightarrow 0$ with $s \rightarrow \infty$

Synthesis of R-C Impedance /R-L Admittance Networks

- If the following represent $Z_{R\text{-}C}(s)$ or $Y_{R\text{-}L}$ N/W

(a) F(s) =[(s+1)(s-3)] / [(s+4)(s+8)] No (1)

(b) F(s) =[(s+2)(s+5)] / [s(s+1)] No (2)

(c) **F(s) =[s (s+8)] / [(s+1)(s+9)] No (4)**

(d) F(s) =[(s+1)(s+4)] / [s(s+2)(s+6)] Yes

SYNTHESIS OF *R-C* IMPEDANCES OR *R-L* ADMITTANCES

Consider the following example.

$$F(s) = \frac{3(s+2)(s+4)}{s(s+3)} \tag{11.48}$$

The partial fraction expansion of the remainder function is obtained as

$$F(s) = \frac{8}{s} + \frac{1}{s+3} + 3 \tag{11.49}$$

where $F(\infty) = 3$. If $F(s)$ is an impedance $Z(s)$, it must be an *R-C* impedance and it is realized in the series Foster form in Fig. 11.16. On the other hand, if $F(s)$ represents an admittance, we realize $Y(s)$ as an *R-L* network in the parallel Foster form (Fig. 11.17).

Cont...

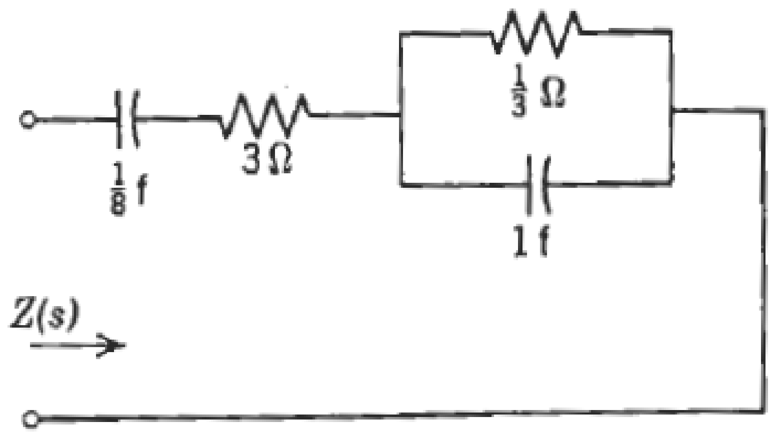

FIG. 11.16

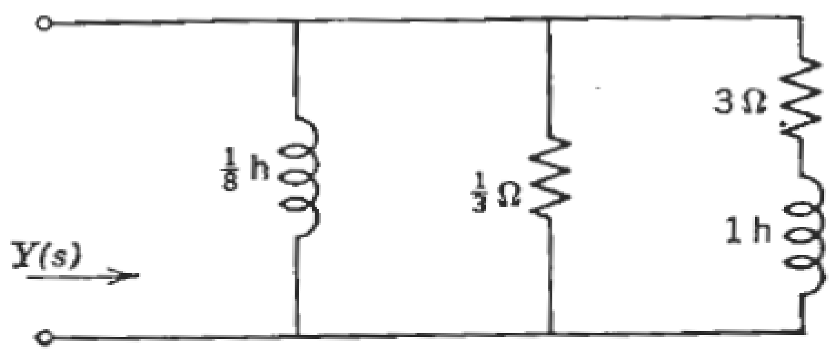

FIG. 11.17

Cont..

For example, the continued fraction expansion of $F(s)$ in Eq. 11.48 is

$$s^2 + 3s\overline{)3s^2 + 18s + 24}(3$$

$$3s^2 + 9s$$

$$\overline{\qquad 9s + 24)s^2 + 3s(\tfrac{1}{9}s}$$

$$s^2 + \tfrac{8}{3}s$$

$$\overline{\qquad \tfrac{1}{3}s)9s + 24(27}$$

$$9s$$

$$\overline{\qquad 24)\tfrac{1}{3}s(s/72}$$

$$\underline{\underline{\tfrac{1}{3}s}}$$

Cont..

If $F(s)$ is an impedance $Z(s)$, the resulting network is shown in Fig. 11.18. If $F(s)$ is an admittance $Y(s)$, we have the *R-L* network of Fig. 11.19.

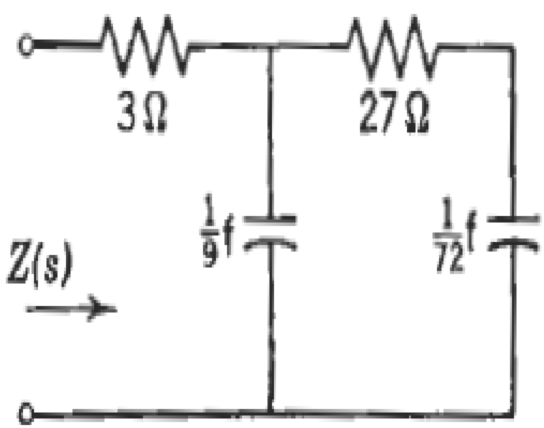

FIG. 11.18

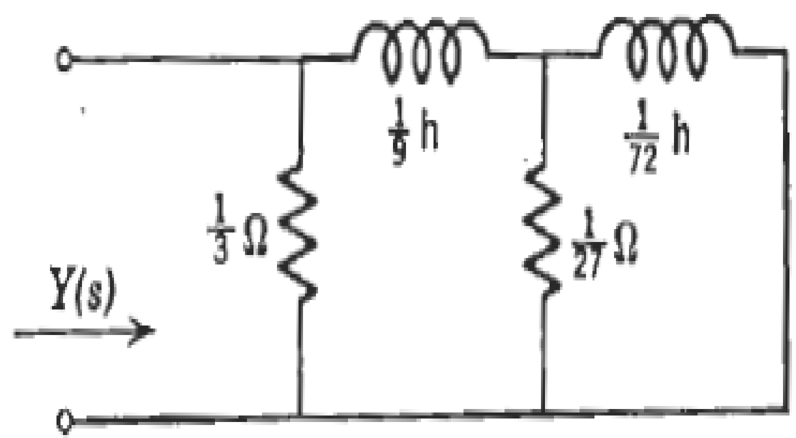

FIG. 11.19

PROPERTIES OF *R-L* IMPEDANCES AND *R-C* ADMITTANCES

The immittance that represents a series Foster *R-L* impedance or a parallel Foster *R-C* admittance is given as

$$F(s) = K_\infty s + K_0 + \frac{K_i s}{s + \sigma_i} + \cdots \qquad (11.50)$$

The significant difference between an *R-C* impedance and an *R-L* impedance is that the partial fraction expansion term for the *R-C* "tank" circuit is $K_i/(s + \sigma_i)$; whereas, for the *R-L* impedance, the corresponding term must be multiplied by an s in order to give an *R-L* tank circuit consisting of a resistor in parallel with an inductor.

The properties of *R-L* impedance or *R-C* admittance functions can be derived in much the same manner as the properties of *R-C* impedance functions. Without going into the derivation of the properties, the more significant ones are given in the following:

1. Poles and zeros of an *R-L* impedance or *R-C* admittance are located on the negative real axis, and they alternate.
2. The singularity nearest to (or at) the origin is a zero. The singularity nearest to (or at) $s = \infty$ must be a pole.
3. The residues of the poles must be real and *negative*.

Cont..

Because of the third property, a partial fraction expansion of an *R-L* impedance function would yield terms as

$$-\frac{K_i}{s+\sigma_i} \tag{11.51}$$

This does not present any trouble, however, because the term above does not represent an *R-L* impedance at all. To obtain the Foster form of an *R-L* impedance, we will resort to the following artifice. Let us first expand $Z(s)/s$ into partial fractions. If $Z(s)$ is an *R-L* impedance, we will state without proof here that the partial fraction expansion of $Z(s)/s$ yields positive residues.[5] Thus, we have

$$\frac{Z(s)}{s} = \frac{K_0}{s} + K_\infty + \frac{K_i}{s+\sigma_i} + \cdots \tag{11.52}$$