Synthesis of R-L Impedance / R-C Admittance Networks

- **Properties of R-L Impedance or R-C Admittance Networks**
- **1. The poles and zeros lie on the negative real axis including origin of the complex s-plane.**
- **2. The poles and zeros interlace (or alternate) along the negative real axis.**
- **3. The residues of the poles must be real and negative.**
- **4. The residues of the poles of [$Z_{R-L}(s)$ /s] or [$Y_{R-L}(s)$]**
- **must be real and positive.**
- **5. The singularity nearest to or at the origin must be a zero**

  Function $Z_{R-L}(s)$ or $Y_{R-C}(s) \rightarrow 0$ with $s \rightarrow 0$
- **6. The singularity nearest to or at the minus infinity must be a pole.**

  Function $Z_{R-L}(s)$ or $Y_{R-C}(s) \rightarrow \infty$ with $s \rightarrow \infty$

Synthesis of R-L Impedance /R-C Admittance Networks

- If the following represent $Z_{R-L}(s)$ or Y_{R-C} N/W

(a) $F(s) = [(s+4)(s+8)] / [(s+2)(s-5)]$ No (1)

(b) $F(s) = [s(s+1)] / [(s+2)(s+5)]$ No (2)

(c) **$F(s) = [(s+1)(s+8)(s+12)] / [s(s+2)(s+10)]$ No (5)**

(d) $F(s) = [s(s+2)(s+6)] / [(s+1)(s+4)]$ Yes

SYNTHESIS OF L-C DRIVING-POINT IMMITTANCES

an L-C immittance is a positive real function

with poles and zeros on the $j\omega$ axis only. The partial fraction expansion of an L-C function is expressed in general terms as

$$F(s) = \frac{K_0}{s} + \frac{2K_2 s}{s^2 + \omega_2^2} + \cdots + K_\infty s \qquad (11.22)$$

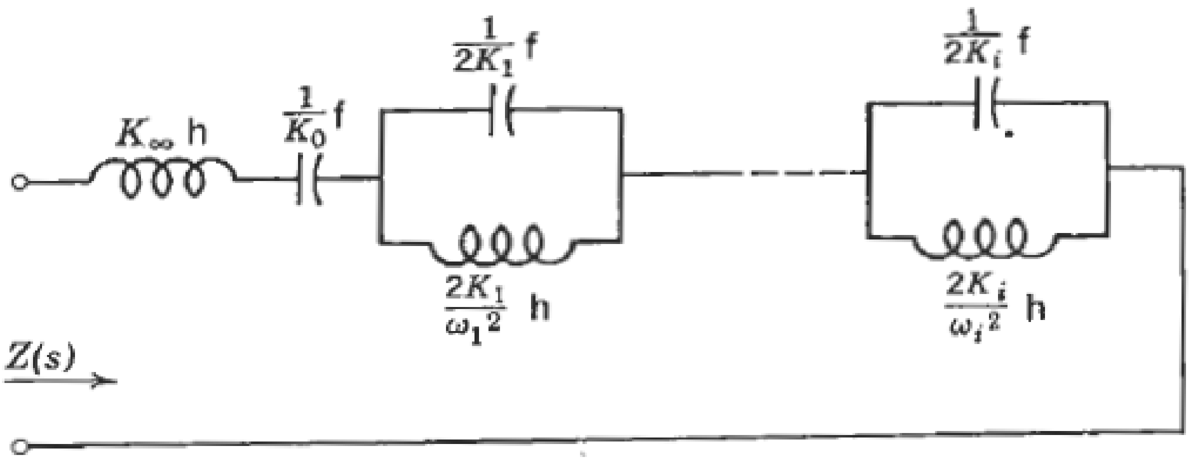


FIG. 11.3

The synthesis is accomplished directly from the partial fraction expansion by associating the individual terms in the expansion with network elements. If $F(s)$ is an impedance $Z(s)$, then the term K_0/s represents a capacitor of $1/K_0$ farads; the term $K_\infty s$ is an inductance of K_∞ henrys, and the term

$2K_i s/(s^2 + \omega_i^2)$ is a parallel tank circuit that consists of a capacitor of $1/2K_i$ farads in parallel with an inductance of $2K_i/\omega_i^2$. Thus a partial fraction expansion of a general L-C impedance would yield the network shown in Fig. 11.3. For example, consider the following L-C function.

$$Z(s) = \frac{2(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)} \tag{11.23}$$

A partial fraction expansion of $Z(s)$ gives

$$Z(s) = 2s + \frac{\frac{9}{2}}{s} + \frac{\frac{15}{2}s}{s^2 + 4} \tag{11.24}$$

Cont....

We then obtain the synthesized network in Fig. 11.4.

The partial fraction expansion method is based upon the elementary synthesis procedure of removing poles on the $j\omega$ axis. The advantage with *L-C* functions is that *all* the poles of the function lie on the $j\omega$ axis so that we can remove all the poles simultaneously. Suppose $F(s)$ in Eq. 11.22 is an admittance $Y(s)$. Then the partial fraction expansion of $Y(s)$ gives us a circuit consisting of parallel branches shown in Fig. 11.5. For example,

$$Y(s) = \frac{s(s^2 + 2)(s^2 + 4)}{(s^2 + 1)(s^2 + 3)} \tag{11.25}$$

Cont..

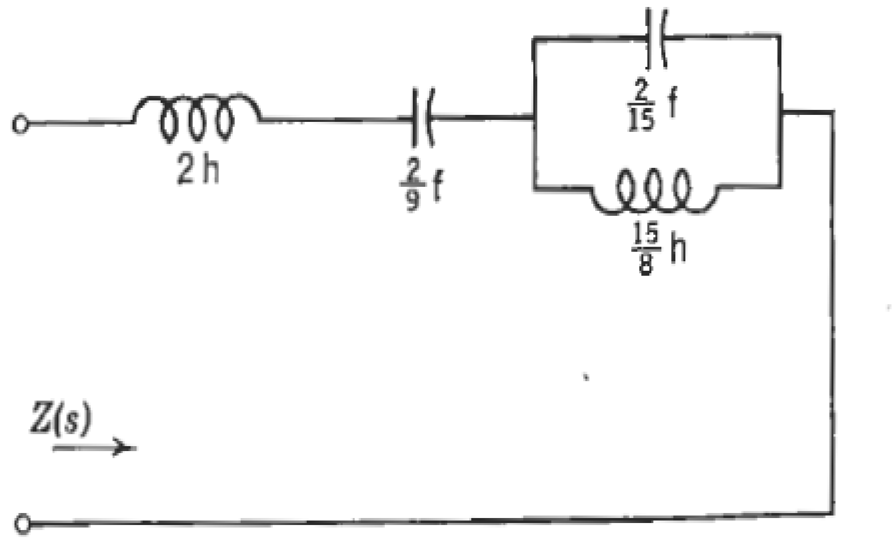


FIG. 11.4

Cont..

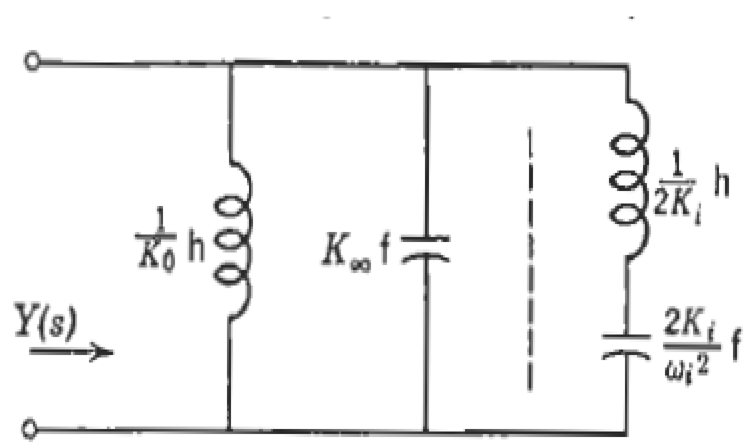
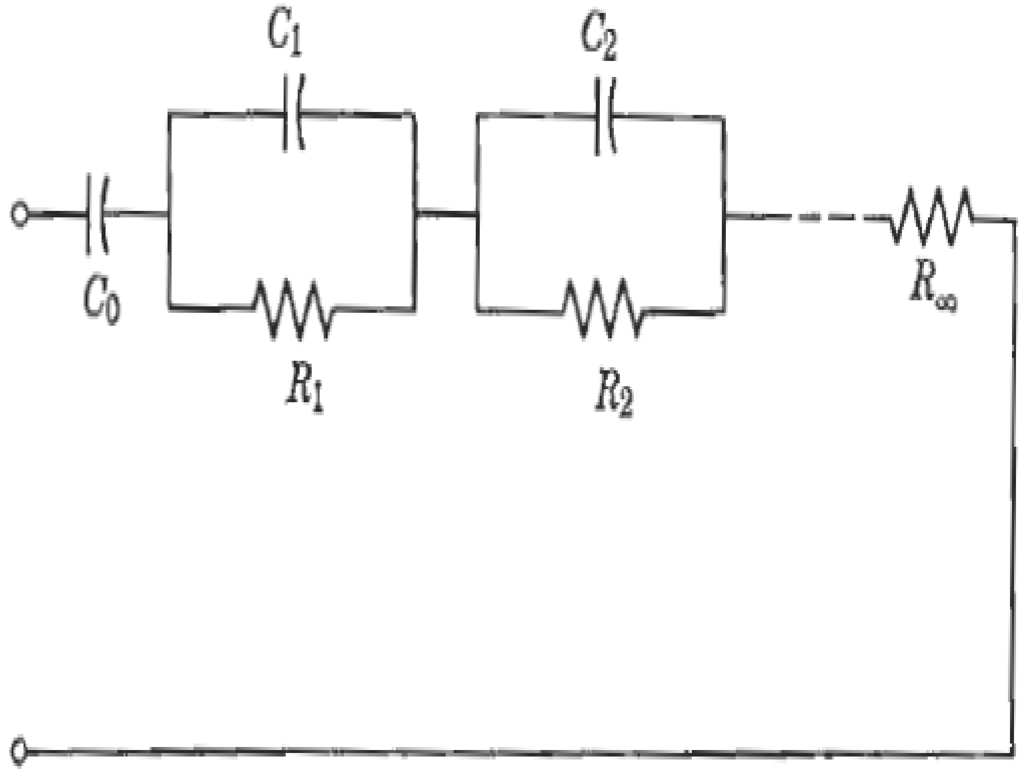


FIG. 11.5

The partial fraction expansion of $Y(s)$ is

$$Y(s) = s + \frac{\frac{1}{2}s}{s^2 + 3} + \frac{\frac{3}{2}s}{s^2 + 1} \tag{11.26}$$

two major properties of *R-C* impedances are obtained, and are listed in the following.



Cont…

1. The poles of an *R-C* driving-point impedance are on the negative real ($-\sigma$) axis. It can be shown from a parallel Foster form that the poles of an *R-C* admittance function are also on the axis. We can thus conclude that the zeros of an *R-C* impedance are also on the $-\sigma$ axis.

2. The residues of the poles, K_i, are real and positive. We shall see later that this property does not apply to *R-C* admittances.

To summarize, the three properties we need to recognize an *R-C* impedance are:

1. Poles and zeros lie on the negative real axis, and they alternate.
2. The singularity nearest to (or at) the origin must be a pole whereas the singularity nearest to (or at) $\sigma = -\infty$ must be a zero.
3. The residues of the poles must be real and positive.

An example of an *R-C* impedance is:

$$Z(s) = \frac{(s+1)(s+4)(s+8)}{s(s+2)(s+6)} \tag{11.45}$$

The following impedances are not *R-C*.

$$Z(s) = \frac{(s+1)(s+8)}{(s+2)(s+4)}$$

$$Z(s) = \frac{(s+2)(s+4)}{(s+1)} \tag{11.46}$$

$$Z(s) = \frac{(s+1)(s+2)}{s(s+3)}$$

RL n/w in Cauer Form

- **Cauer –I form**

1. If $Z(s) \rightarrow \infty$ for $s \rightarrow \infty$ the first element L_1 exists

   If $Z(s)$ is constant for $s \rightarrow \infty$ L_1 does not exist exist.First element is R_2 .

Cauer –II form

2. If $Z(s) \rightarrow$constant for $s \rightarrow 0$ the first element R_1 exists

   If $Z(s) \rightarrow 0$ for $s \rightarrow 0$ R_1 does not exist.First element is $L_{2.}$.