PROPERTIES OF *L-C* IMMITTANCE FUNCTIONS

Consider the impedance $Z(s)$ of a passive one-port network. Let us represent $Z(s)$ as

$$Z(s) = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)} \tag{11.1}$$

where M_1, M_2 are even parts of the numerator and denominator, and N_1, N_2 are odd parts. The average power dissipated by the one-port is

$$\text{Average power} = \tfrac{1}{2}\,\text{Re}\,[Z(j\omega)]\,|I|^2 \tag{11.2}$$

where I is the input current. For a pure reactive network, it is known that the power dissipated is zero. We therefore conclude that the real part of $Z(j\omega)$ is zero; that is

$$\text{Re}\,Z(j\omega) = \text{Ev}\,Z(j\omega) = 0 \tag{11.3}$$

where

$$\text{Ev}\,Z(s) = \frac{M_1(s)\,M_2(s) - N_1(s)\,N_2(s)}{M_2^2(s) - N_2^2(s)} \tag{11.4}$$

Cont....

In order for Ev $Z(j\omega) = 0$, that is,

$$M_1(j\omega)\, M_2(j\omega) - N_1(j\omega)\, N_2(j\omega) = 0 \tag{11.5}$$

either of the following cases must hold:

$$\begin{aligned} &(a) \qquad M_1 = 0 = N_2 \\ &(b) \qquad M_2 = 0 = N_1 \end{aligned} \tag{11.6}$$

In case (*a*), $Z(s)$ is

$$Z(s) = \frac{N_1}{M_2} \tag{11.7}$$

and in case (*b*)

$$Z(s) = \frac{M_1}{N_2} \tag{11.8}$$

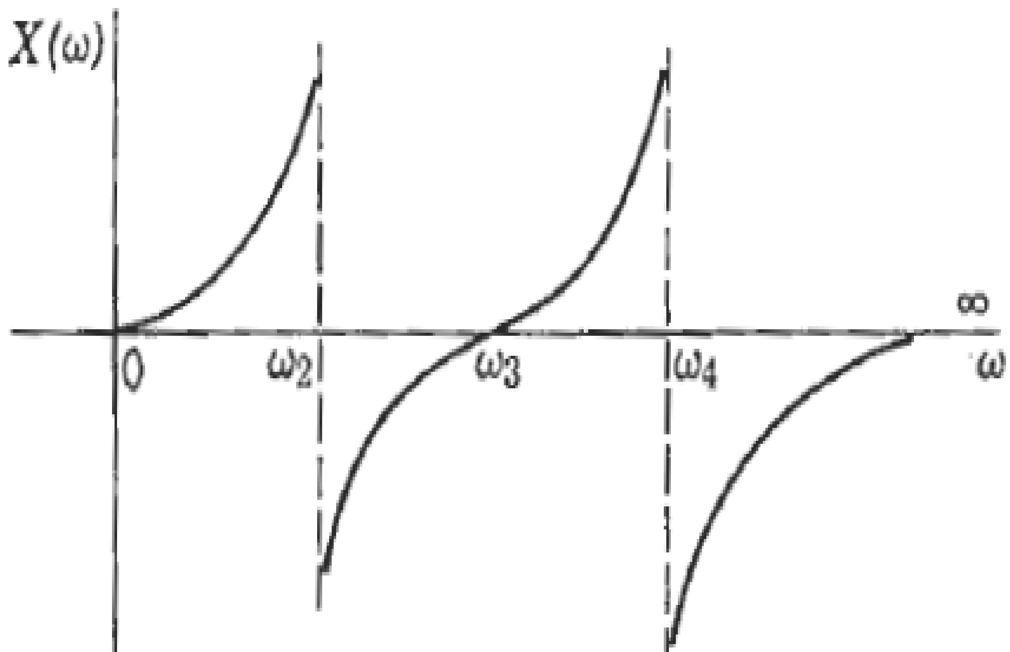
Cont….

We see from this development the following two properties of L-C functions:

1. $Z_{LC}(s)$ or $Y_{LC}(s)$ is the ratio of even to odd or odd to even polynomials.
2. Since both $M_i(s)$ and $N_j(s)$ are Hurwitz, they have only imaginary roots, and it follows that the poles and zeros of $Z_{LC}(s)$ or $Y_{LC}(s)$ are on the imaginary axis.

Finally, let us summarize the properties of *L-C* impedance or admittance functions.

1. $Z_{LC}(s)$ or $Y_{LC}(s)$ is the ratio of odd to even or even to odd polynomials.



2. The poles and zeros are simple and lie on the $j\omega$ axis.

3. The poles and zeros interlace on the $j\omega$ axis.

4. The highest powers of numerator and denominator must differ by unity; the lowest powers also differ by unity.

5. There must be either a zero or a pole at the origin and infinity.

The following functions are not *L-C* for the reasons listed at the left.

3. $$Z(s) = \frac{Ks(s^2 + 4)}{(s^2 + 1)(s^2 + 3)}$$

2. $$Z(s) = \frac{s^5 + 4s^3 + 5s}{3s^4 + 6s^2} \tag{11.20}$$

1. $$Z(s) = \frac{K(s^2 + 1)(s^2 + 9)}{(s^2 + 2)(s^2 + 10)}$$

On the other hand, the function $Z(s)$ in Eq. 11.21, whose pole-zero diagram is shown in Fig. 11.2, is an *L-C* immittance.

$$Z(s) = \frac{2(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)} \tag{11.21}$$

- A function $F(s)= P(s)/ Q(s)$ is positive real if the following condition are satisfied—

1. F(s) is real for s real.
2. Q(s) is hurwitz polynomial.
3. F(s) may have poles on the jω axis.
4. The real part of F(s) is greater than or equal to zero for the real part of s is greater than or equal to zero. ie

$\mathrm{Re}[\ F(s)\ ] \geq 0$ for $\mathrm{Re}\ s \geq 0$

So $\mathrm{Re}[\ F(j\omega)\ ] \geq 0$ for all ω

A simplification of condition 4 is possible

$$F(s) = \frac{p(s)}{Q(s)} = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$$

Where $M_1(s)$ = even function

$M_2(s)$ = odd function

Even part of F(s) is

$$Ev[F(s)] = \frac{M_1M_2 - N_1N_2}{M_2^2 - N_2^2}$$

Odd part of F(s) is

$$Odd\,[F(s)] = \frac{N_1M_2 - M_1N_2}{M_2^2 - N_2^2}$$

- If we let s= jω (since σ =0)

We see that the even part of any polynomial is real, while the odd part of the polynomial is imaginary ie

$$Re\,[\,F(j\omega\,)] = Ev\,[\,F(j\omega)]$$

$$j\,Im\,[\,F(j\omega\,)] = Odd\,[\,F(j\omega)]$$

$$F(s) = Re\,[\,F(j\omega)] + j\,Im\,[\,F(j\omega\,)]$$