SECTION- C

NETWORK SYNTHESIS AND POSITIVE REAL FUNCTIONS

SYNTHESIS AND POSITIVE REAL FUNCTIONS

- T(s) = R(s) / E(s)   ratio of response to excitation

Our task is - synthesize a network from a given network function.

STEP 1 Determine whether T(s) can be realised as a physical passive network.

It should be **stable** and satisfy **causality**.

Causality means , a voltage cannot appear between any pair of terminals in the network before a current is impressed or vice-versa.It means response of the network must be zero for t < o .

Stability Network function T(s) must satisfy following three conditions : -

(a) T(s) can not have poles in the right half of s – plane .

(b) T(s) can not have multiple poles on the imaginary (jw) - axis .

(c) The degree of the numerator of T(s) can not exceed the degree of denominator by more than one .

STEP 2 Determine whether the denominator of T(s) is a HURWITZ polynomial.

A polynomial is said to be Hurwitz if it satisfy following two conditions :-

(i) P(s) is real when s is real .

(ii) The roots of P(s) have real parts which are zero are negative .

Properties of Hurwitz Polynomial

- As a result of above conditions (1) and (2) , Hurwitz polynomial has the following properties : -

  (i) All the coefficients of P(s) must be positive .

  (ii) Both the even and odd parts of the polynomial have roots on the jw –axis only . M(s) → even parts and N(s) → odd parts. P(s) = M(s) + N(s).

  Then M(s) and N(s) both have roots on the jw – axis .

  (iii) If P(s) is either even or odd , then all its roots are on jw axis including origin .

  (iv) The continued fraction expansion of the ratio ψ(s) of the odd to even parts [N(s) / M(s)] or the even to odd parts [M(s) / N(s)] of P(s) yields all positive quotient terms .

  (v) If P(s) is either only even or only odd , then the ratio of P(s) and its derivative gives positive quotients in its continued fraction expansion.

  (vi) If P(s) is Hurwitz polynomial and W(s) is a multiplicative factor, then

  P1(s) = P(s).W(s) is also Hurwitz polynomial , if W(s) is Hurwitz polynomial.

Hurwitz Polynomial

- The denominator of polynomial of the system function belongs to a class of polynomial known as Hurwitz

polynomials. A polynomial $P(s)$ is said to be Hurwitz if the following conditions are satisfied:

1. $P(s)$ is real when s is real.
2. The roots of $P(s)$ have real parts which are zero or negative.

As a result of these conditions, if $P(s)$ is a Hurwitz polynomial given by

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 \qquad (10.15)$$

then all the coefficients a_i must be real; if $s_i = \alpha_i + j\beta$ is a root of $P(s)$, then α_i must be negative. The polynomial

$$P(s) = (s + 1)(s + 1 + j\sqrt{2})(s + 1 - j\sqrt{2}) \qquad (10.16)$$

is Hurwitz because all of its roots have negative real parts. On the other hand,

$$G(s) = (s - 1)(s + 2)(s + 3) \qquad (10.17)$$

is not Hurwitz because of the root $s = 1$, which has a positive real part. Hurwitz polynomials have the following properties:

1. All the coefficients a_i are nonnegative. This is readily seen by examining the three types of roots that a Hurwitz polynomial might have. These are

$s = -\gamma_i$	γ_i real and positive
$s = \pm j\omega_i$	ω_i real
$s = -\alpha_i \pm j\beta_i$	α_i real and positive

Continue....

The polynomial $P(s)$ which contains these roots can be written as

$$P(s) = (s + \gamma_i)(s^2 + \omega_i^2)[(s + \alpha_i)^2 + \beta_i^2] \cdots \qquad (10.18)$$

2. Both the odd and even parts of a Hurwitz polynomial $P(s)$ have roots on the $j\omega$ axis only. If we denote the odd part of $P(s)$ as $n(s)$ and the even part as $m(s)$, so that

$$P(s) = n(s) + m(s) \qquad (10.19)$$

then $m(s)$ and $n(s)$ both have roots on the $j\omega$ axis only. The reader is referred to a proof of this property by Guillemin.[5]

3. As a result of property 2, if $P(s)$ is either even or odd, all its roots are on the $j\omega$ axis.

Cont.....

4. The continued fraction expansion of the ratio of the odd to even parts or the even to odd parts of a Hurwitz polynomial yields all positive quotient terms. Suppose we denote the ratios as $\psi(s) = n(s)/m(s)$ or $\psi(s) = m(s)/n(s)$, then the continued fraction expansion of $\psi(s)$ can be written as

$$\psi(s) = q_1 s + \cfrac{1}{q_2 s + \cfrac{1}{q_3 s + \cfrac{1}{\cdots + \cfrac{1}{q_n s}}}} \qquad (10.20)$$

Positive Real Functions

- These functions are important Because they represent physically realizable passive driving point immittances. A function T(s) = N(s) / D(s) is positive real (p. r) if the following conditions are satisfied : -

1). **T(s) is real for s real , i.e. T(σ) is purely real .**

2). D(s) is Hurwitz polynomial.

3). T(s) may have poles on the jw – axis . These poles are simple and the residues there-of are real and positive.

4). The real part of T(s) is greater than or equal to zero for the real partof of s is greater than or equal to zero ,i.e.,

$$\mathrm{Re}\,[\,T(s)\,] \geq 0 \qquad \text{for } \mathrm{Re}\ s \geq 0$$

$$\mathrm{Re}\,[\,T(jw)\,] \geq 0 \qquad \text{for all } w$$

this leads to $A(w^2) \equiv M_1(jw).\, M_2(jw) - N_1(jw).\, N_2(jw) \geq 0$

$$T(s) = \frac{N(s)}{D(s)} = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)} = \frac{M_1(jw) + N_1(jw)}{M_2(jw) + N_2(jw)}$$

$M_1(s)$ & $N_1(s)$ are the even and odd parts of N(s).

$M_2(s)$ & $N_2(s)$ are the even and odd parts of D(s)