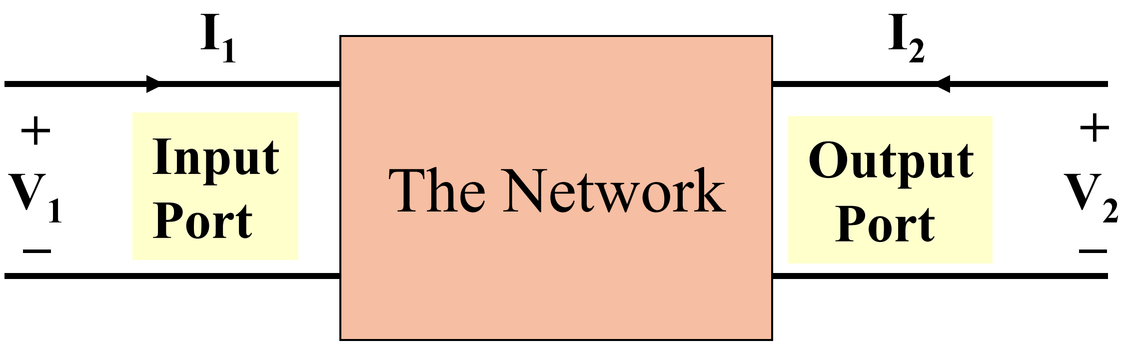
An Introduction To Two – Port Networks

Two Port Networks

Generalities:

The standard configuration of a two port:



The network ?

The voltage and current convention ?

Two Port Networks

Network Equations:

Impedance Z parameters

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$V_2 = b_{11}V_1 - b_{12}I_1$$
$$I_2 = b_{21}V_1 - b_{22}I_1$$

Admittance Y parameters

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

Hybrid H parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

Transmission A, B, C, D parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

Two Port Networks

Z parameters:

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

z_{11} is the impedance seen looking into port 1 when port 2 is open.

$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

z_{12} is a transfer impedance. It is the ratio of the voltage at port 1 to the current at port 2 when port 1 is open.

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

z_{21} is a transfer impedance. It is the ratio of the voltage at port 2 to the current at port 1 when port 2 is open.

$$z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

z_{22} is the impedance seen looking into port 2 when port 1 is open.

Two Port Networks

Y parameters:

$$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

y_{11} is the admittance seen looking into port 1 when port 2 is shorted.

$$y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

y_{12} is a transfer admittance. It is the ratio of the current at port 1 to the voltage at port 2 when port 1 is shorted.

$$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

y_{21} is a transfer impedance. It is the ratio of the current at port 2 to the voltage at port 1 when port 2 is shorted.

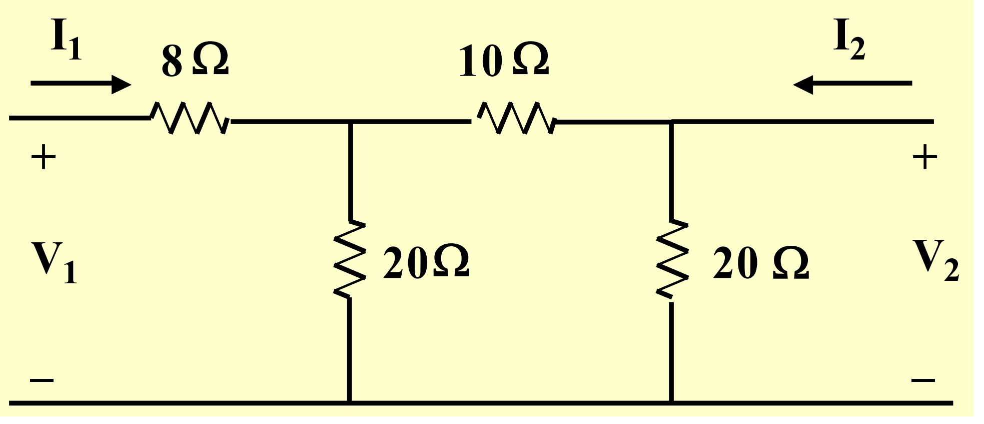
$$y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

y_{22} is the admittance seen looking into port 2 when port 1 is shorted.

Two Port Networks

Z parameters: **Example 1**

Given the following circuit. Determine the Z parameters.



Find the Z parameters for the above network.

Two Port Networks

Z parameters:

Example 1 (cont 1)

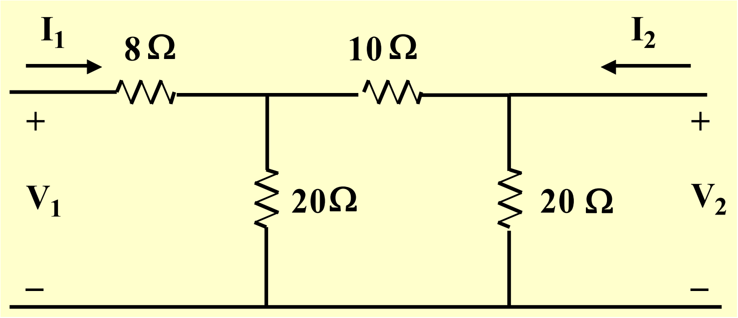
For z_{11}:

$$Z_{11} = 8 + 20||30 = 20\ \Omega$$

For z_{22}:

$$Z_{22} = 20||30 = 12\ \Omega$$

For z_{12}:



$$z_{12} = \frac{V_1}{I_2}\Bigg|_{I_1=0}$$

$$V_1 = \frac{20xI_2x20}{20+30} = 8xI_2$$

Therefore:

$$z_{12} = \frac{8xI_2}{I_2} = 8\ \Omega = z_{21}$$

Two Port Networks

Z parameters: **Example 1 (cont 2)**

The Z parameter equations can be expressed in matrix form as follows.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

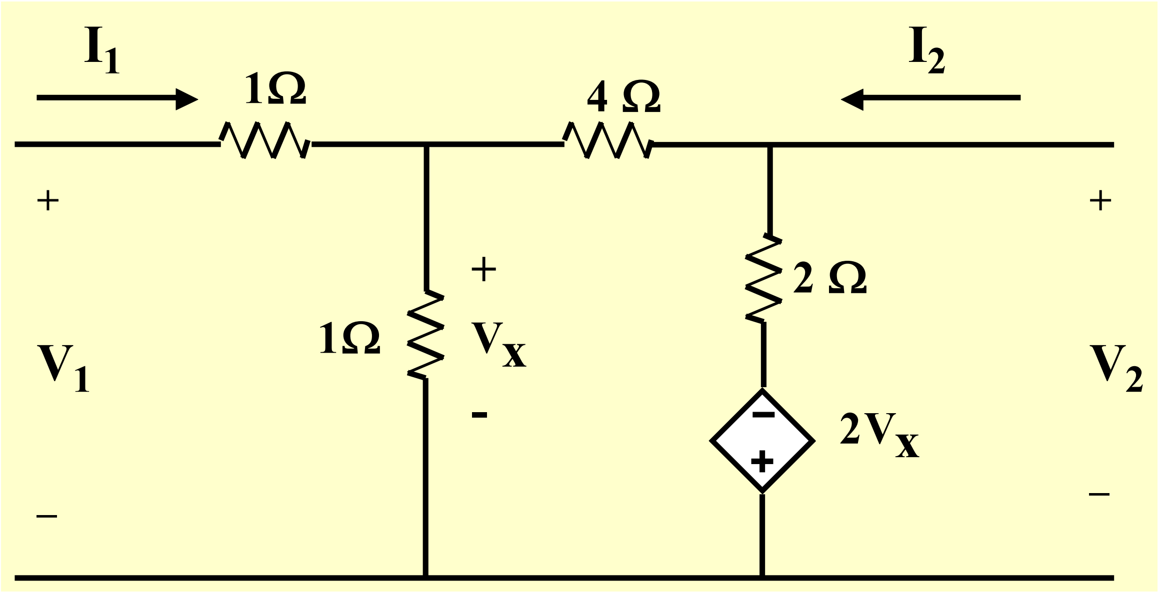
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 8 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Two Port Networks

Z parameters:

Example 2

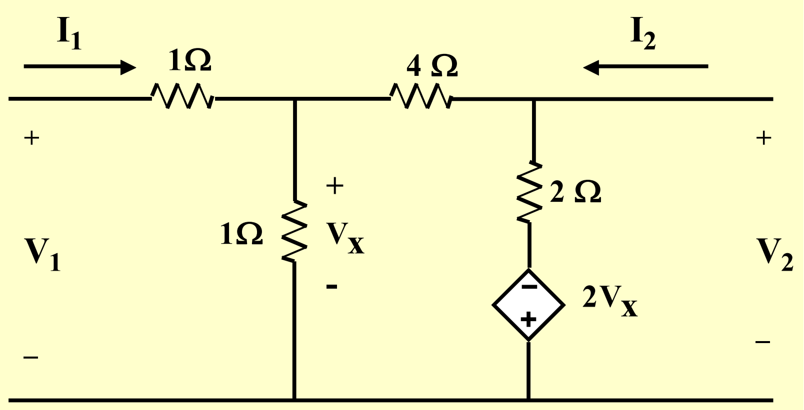
You are given the following circuit. Find the Z parameters.



Two Port Networks

Z parameters:

Example 2 (continue p2)



$$z_{11} = \frac{V_1}{I_1}\Bigg|_{I_2=0}$$

$$I_1 = \frac{V_x}{1} + \frac{V_x + 2V_x}{6} = \frac{6V_x + V_x + 2V_x}{6}$$

$$I_1 = \frac{3V_x}{2} \quad ; \quad \text{but } V_x = V_1 - I_1$$

Substituting gives;

$$I_1 = \frac{3(V_1 - I_1)}{2} \quad \text{or} \quad \frac{V_1}{I_1} = z_{11} = \frac{5}{3}\ \Omega$$

Other Answers

$Z_{21} = -0.667\ \Omega$

$Z_{12} = 0.222\ \Omega$

$Z_{22} = 1.111\ \Omega$

Two Port Networks

Transmission parameters (A,B,C,D):

The defining equations are:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A = \frac{V_1}{V_2} \bigg|_{I_2 = 0} \qquad B = \frac{V_1}{-I_2} \bigg|_{V_2 = 0}$$

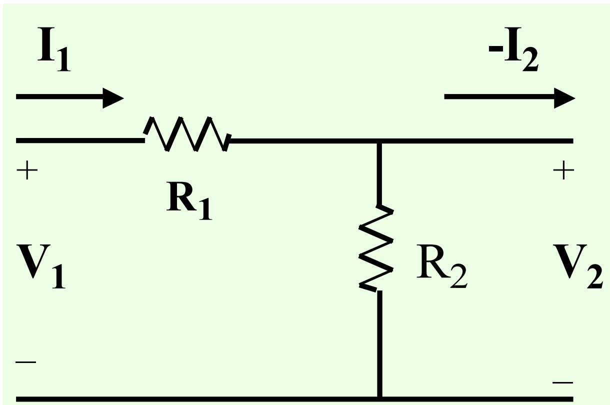
$$C = \frac{I_1}{V_2} \bigg|_{I_2 = 0} \qquad D = \frac{I_1}{-I_2} \bigg|_{V_2 = 0}$$

Two Port Networks

Transmission parameters (A,B,C,D):

Example

Given the network below with assumed voltage polarities and Current directions compatible with the A,B,C,D parameters.



We can write the following equations.

$$V_1 = (R_1 + R_2)I_1 + R_2I_2$$

$$V_2 = R_2I_1 + R_2I_2$$

It is not always possible to write 2 equations in terms of the V's and I's Of the parameter set.

Two Port Networks

Transmission parameters (A,B,C,D):

Example (cont.)

$$V_1 = (R_1 + R_2)I_1 + R_2 I_2$$

$$V_2 = R_2 I_1 + R_2 I_2$$

From these equations we can directly evaluate the A,B,C,D parameters.

$$A = \frac{V_1}{V_2}\Bigg|_{I_2 = 0} = \boxed{}$$

$$B = \frac{V_1}{-I_2}\Bigg|_{V_2 = 0} = \boxed{}$$

$$C = \frac{I_1}{V_2}\Bigg|_{I_2 = 0} = \boxed{}$$

$$D = \frac{I_1}{-I_2}\Bigg|_{V_2 = 0} = \boxed{}$$

Later we will see how to interconnect two of these networks together for a final answer

Two Port Networks

Hybrid Parameters:

The equations for the hybrid parameters are:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

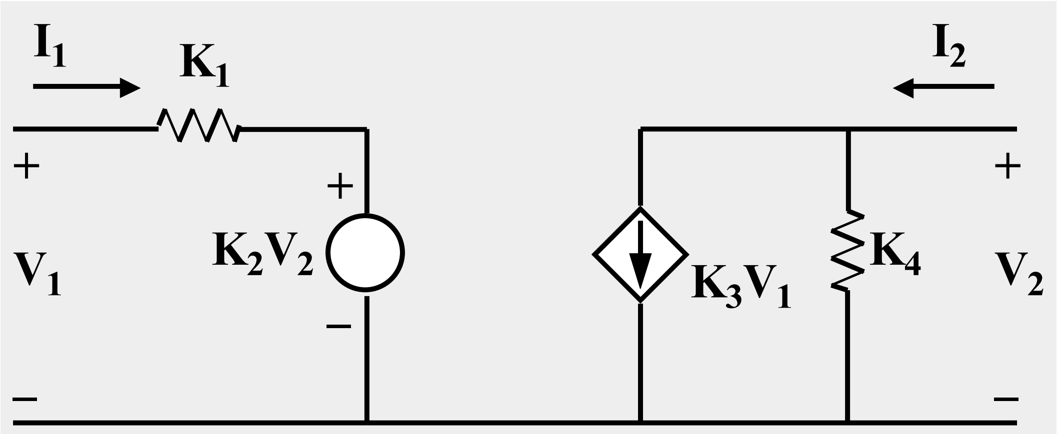
$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0} \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0}$$

Two Port Networks

Hybrid Parameters:

The following is a popular model used to represent a particular variety of transistors.



We can write the following equations:

$$V_1 = AI_1 + BV_2$$

$$I_2 = CI_1 + \frac{V_2}{D}$$

Two Port Networks

Hybrid Parameters:

$$V_1 = AI_1 + BV_2$$

$$I_2 = CI_1 + \frac{V_2}{D}$$

We want to evaluate the H parameters from the above set of equations.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0} = \boxed{}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0} = \boxed{}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0} = \boxed{}$$

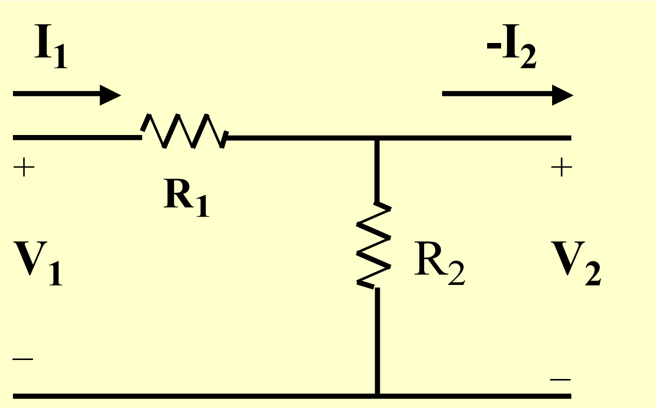
$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0} = \boxed{}$$

Two Port Networks

Hybrid Parameters:

Another example with hybrid parameters.

Given the circuit below.



The equations for the circuit are:

$$V_1 = (R_1 + R_2)I_1 + R_2I_2$$

$$V_2 = R_2I_1 + R_2I_2$$

The H parameters are as follows.

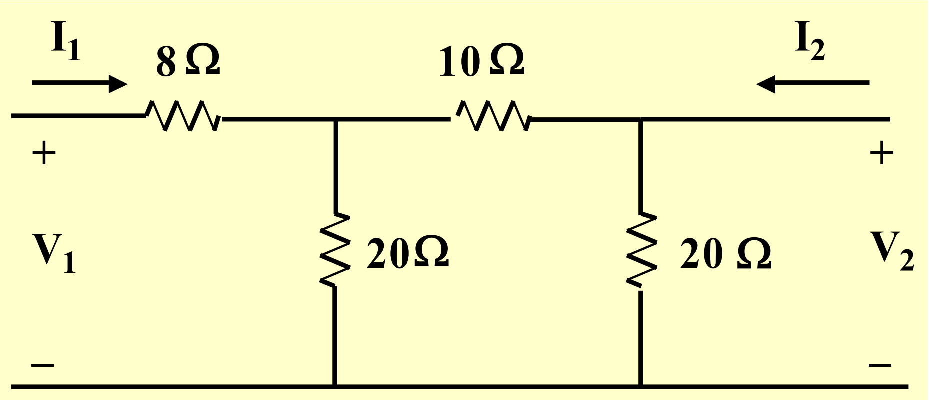
$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = \qquad\qquad h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} =$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = \qquad\qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} =$$

Two Port Networks

Modifying the two port network:

Earlier we found the z parameters of the following network.

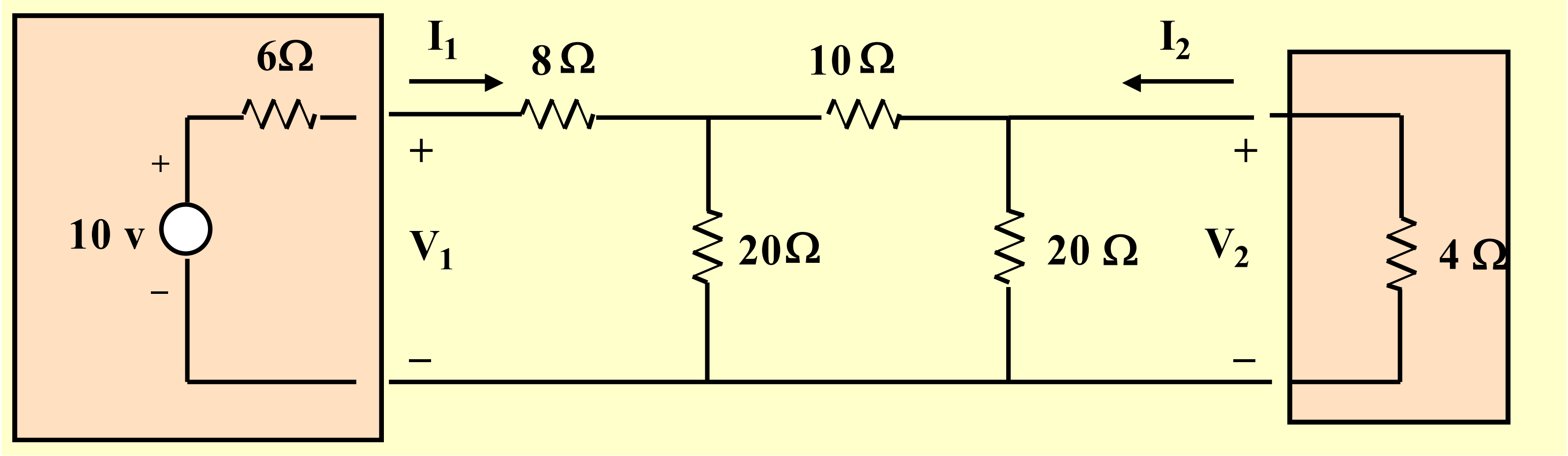


$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 8 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Two Port Networks

Modifying the two port network:

We modify the network as shown be adding elements outside the two ports



We now have:

$$V_1 = 10 - 6I_1$$

$$V_2 = -4I_2$$

Two Port Networks

Modifying the two port network:

We take a look at the original equations and the equations describing the new port conditions.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 8 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = 10 - 6I_1$$

$$V_2 = -4I_2$$

So we have,

$$10 - 6I_1 = 20I_1 + 8I_2$$

$$-4I_2 = 8I_1 + 12I_2$$

Two Port Networks

Modifying the two port network:

Rearranging the equations gives,

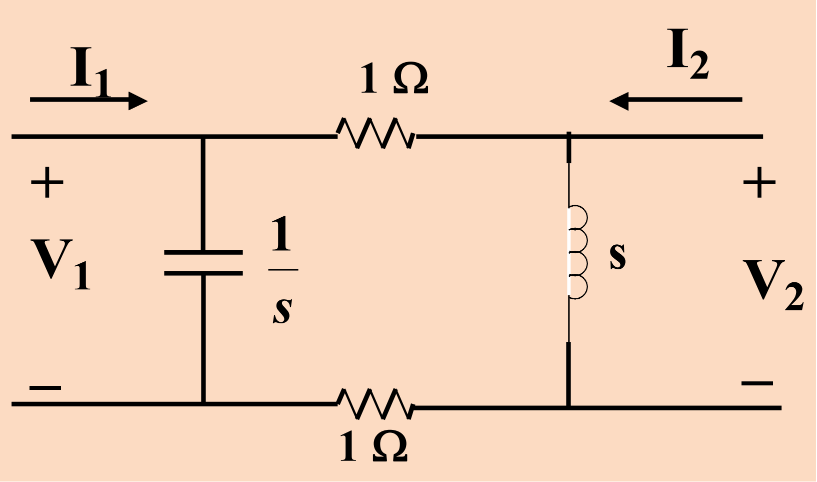
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}^{-1} \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

Two Port Networks

Y Parameters and Beyond:

Given the following network.



(a) Find the Y parameters for the network.

(b) From the Y parameters find the z parameters

Two Port Networks

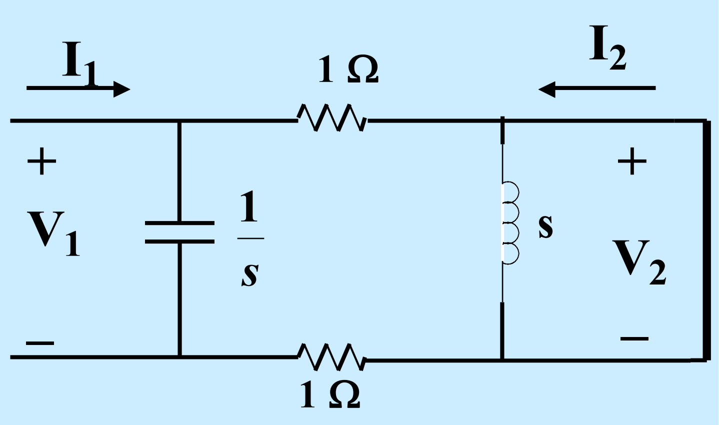
Y Parameter Example

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \qquad y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$



short

We use the above equations to evaluate the parameters from the network.

To find y_{11}

$$V_1 = I_1\left(\frac{2/s}{2+1/s}\right) = I_1\left[\frac{2}{2s+1}\right]$$

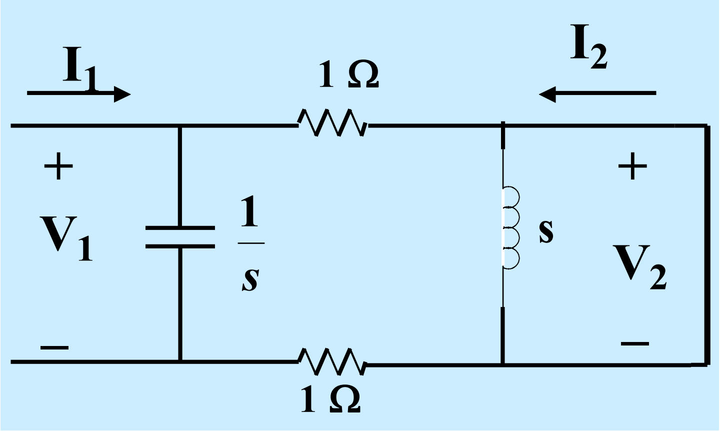
so

$$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = s + 0.5$$

Two Port Networks

Y Parameter Example

$$y_{21} = \frac{I_2}{V_1}\Bigg|_{V_2=0}$$



We see

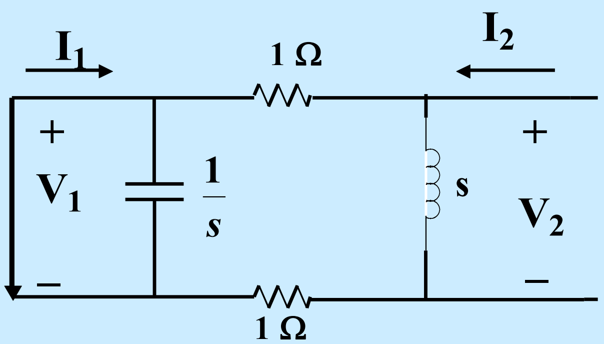
$$V_1 = -2I_2 \longrightarrow y_{21} = \frac{I_2}{V_1} = 0.5\ S$$

Two Port Networks

Y Parameter Example

To find y_{12} and y_{21} we reverse things and short V_1

short



$$y_{12} = \frac{I_1}{V_2}\Bigg|_{V_1=0}$$

We have

$$V_2 = -2I_1$$

$$y_{12} = \frac{I_1}{V2} = 0.5\ \text{S}$$

$$y_{22} = \frac{I_2}{V_2}\Bigg|_{V_1=0}$$

We have

$$V_2 = I_2 \frac{2s}{(s+2)}$$

$$y_{22} = 0.5 + \frac{1}{s}$$

Two Port Networks

Y Parameter Example

Summary:

$$\boxed{\mathbf{Y}} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} s + 0.5 & -0.5 \\ -0.5 & 0.5 + 1/s \end{bmatrix}$$

Now suppose you want the Z parameters for the same network.

Two Port Networks

Going From Y to Z Parameters

For the Y parameters we have:

$$I = Y\,V$$

For the Z parameters we have:

$$V = Z\,I$$

From above;

$$V = Y^{-1} I = Z\,I$$

Therefore

$$Z = Y^{-1} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{y_{22}}{\Delta_Y} & \dfrac{-y_{12}}{\Delta_Y} \\ \dfrac{-y_{21}}{\Delta_Y} & \dfrac{y_{11}}{\Delta_Y} \end{bmatrix}$$

where

$$\Delta_Y = \det|Y|$$

Two Port Parameter Conversions:

$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_H}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{22}}{\Delta_Z} & \frac{-z_{12}}{\Delta_Z} \\ \frac{-z_{21}}{\Delta_Z} & \frac{z_{11}}{\Delta_Z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_H}{h_{11}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_Z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_Y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_H}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
$\begin{bmatrix} \frac{\Delta_Z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_Y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

Two Port Parameter Conversions:

To go from one set of parameters to another, locate the set of parameters you are in, move along the vertical until you are in the row that contains the parameters you want to convert to – then compare element for element

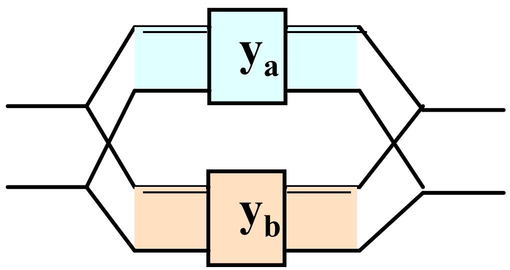
$\begin{bmatrix} \boxed{z_{11}} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \boxed{\frac{\Delta_H}{h_{22}}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{22}}{\Delta_Z} & \frac{-z_{12}}{\Delta_Z} \\ \frac{-z_{21}}{\Delta_Z} & \frac{z_{11}}{\Delta_Z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_H}{h_{11}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_Z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_Y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_H}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
$\begin{bmatrix} \frac{\Delta_Z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_Y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

$$z_{11} = \frac{\Delta_H}{h_{22}}$$

Interconnection Of Two Port Networks

Three ways that two ports are interconnected:

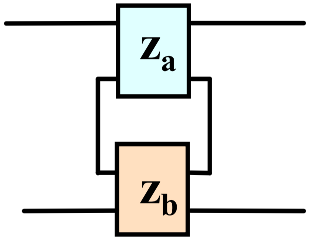
* **Parallel**



Y parameters

$$[y] = [y_a] + [y_b]$$

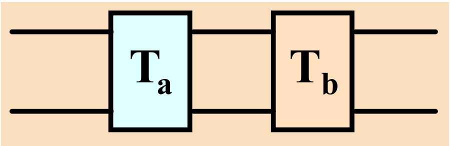
* **Series**



Z parameters

$$[z] = [z_a] + [z_b]$$

* **Cascade**



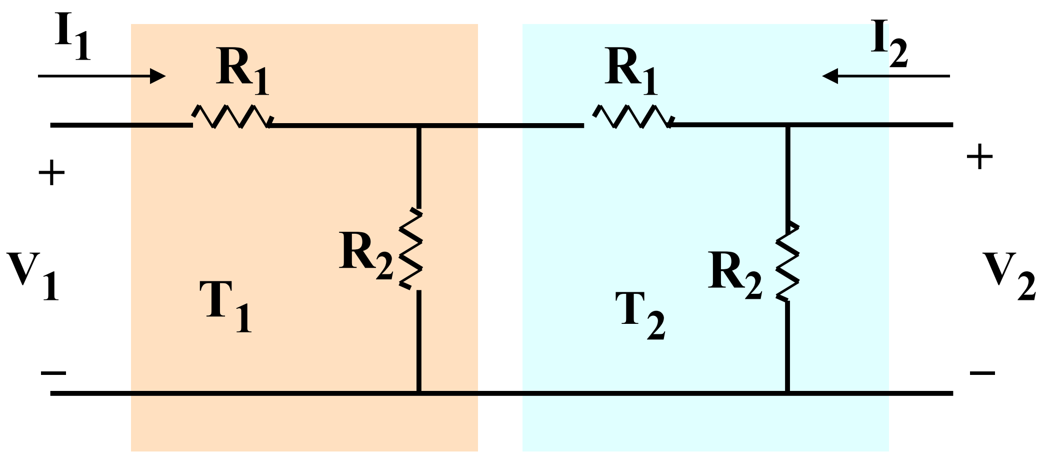
ABCD parameters

$$[T] = [T_a]\,[T_b]$$

Interconnection Of Two Port Networks

Consider the following network:

Find $\dfrac{V_2}{V_1}$



Referring to slide 13 we have;

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & R_1 \\ \dfrac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & R_1 \\ \dfrac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Interconnection Of Two Port Networks

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & R_1 \\ \dfrac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & R_1 \\ \dfrac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Multiply out the first row:

$$V_1 = \left[\left[\left(\frac{R_1 + R_2}{R_2} \right)^2 + \frac{R_1}{R_2} \right] V_2 + \left[\left(\frac{R_1 + R_2}{R_2} \right) R_1 + R_1 \right] (-I_2) \right]$$

Set I_2 = 0 (as in the diagram)

$$\frac{V_2}{V_1} = \frac{R_2{}^2}{R_1{}^2 + 3R_1R_2 \;\; R_2{}^2}$$

Can be verified directly by solving the circuit