System Function

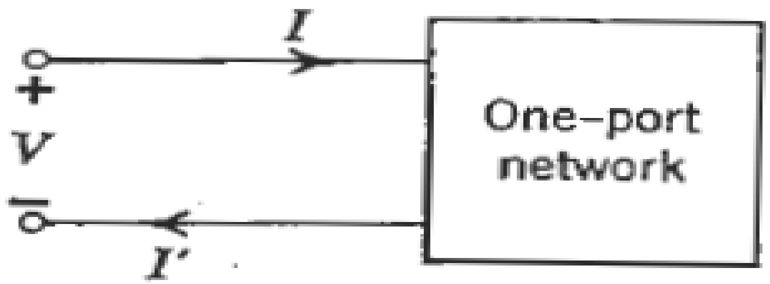


FIG. 9.1

In electric network theory, the word *port* has a special meaning. A port may be regarded as a pair of terminals in which the current into one terminal equals the current out of the other. For the one-port network shown in Fig. 9.1, $I = I'$. A one-port network is completely specified when the voltage-current relationship at the terminals of the port is given. For example, if $V = 10$ v and $I = 2$ amp, then we know that the *input or driving-point* impedance of the one-port is

$$Z_{\text{in}} = \frac{V}{I} = 5\,\Omega \tag{9.1}$$

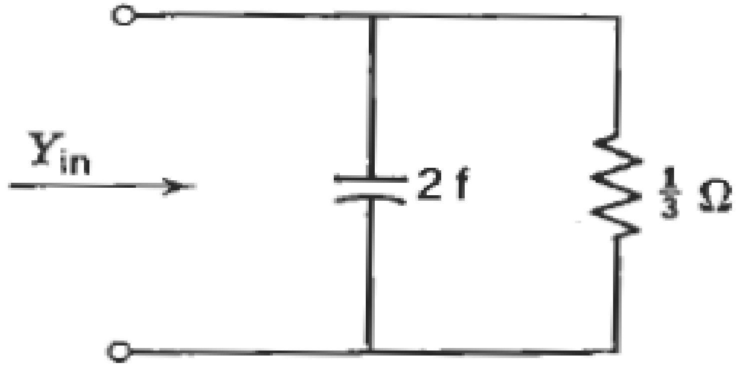


FIG. 9.2

Whether the one-port is actually a single 5-Ω resistor, two 2.5-Ω resistors in series, or two 10-Ω resistors in parallel, is of little importance because the primary concern is the current-voltage relationship at the port. Consider the example in which $I = 2s + 3$ and $V = 1$; then the input admittance of the one-port is

$$Y_{in} = \frac{I}{V} = 2s + 3 \tag{9.2}$$

which corresponds to a 2-f capacitor in parallel with a $\frac{1}{3}$-Ω resistor i its simplest case (Fig. 9.2).

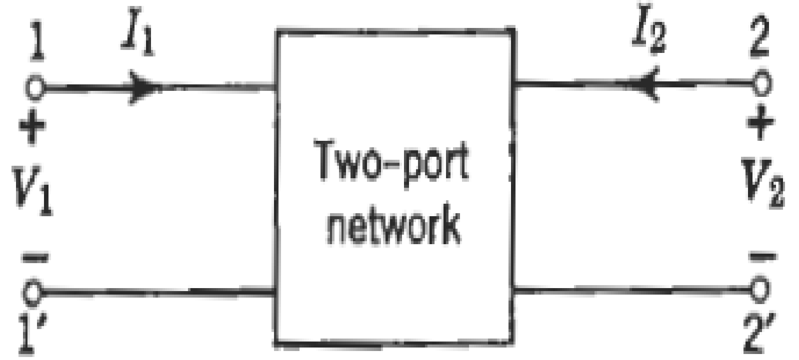


FIG. 9.3

Two-port parameters

A general *two-port* network, shown in Fig. 9.3, has two pairs of voltage-current relationships. The variables are V_1, V_2, I_1, I_2. Two of these are *dependent* variables; the other two are *independent* variables. The number of possible combinations generated by four variables taken two at a time is six. Thus there are six possible sets of equations describing a two-port network. We will discuss the four most useful descriptions here.

The z parameters

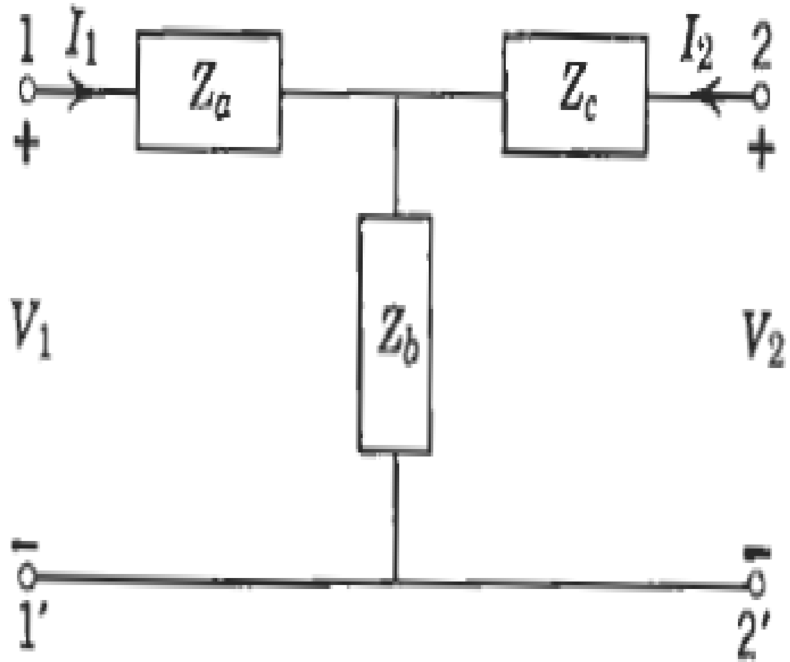
A particular set of equations that describe a two-port network are the z-parameter equations

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2 \qquad (9.3)$$

In these equations the variables V_1 and V_2 are dependent, and I_1, I_2 are independent. The individual z parameters are defined by

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$
$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \qquad (9.4)$$

It is observed that all the z parameters have the dimensions of impedance. Moreover, the individual parameters are specified only when the current in one of the ports is zero. This corresponds to one of ports being *open circuited*, from which the z parameters also derive the name *open-circuit*



As an example, let us find the z parameters of the *Pi* circuit in Fig. 9.5. First, the node equations are

$$\begin{aligned} I_1 &= (Y_A + Y_C)V_1 - Y_C V_2 \\ I_2 &= -Y_C V_1 + (Y_B + Y_C)V_2 \end{aligned} \tag{9.10}$$

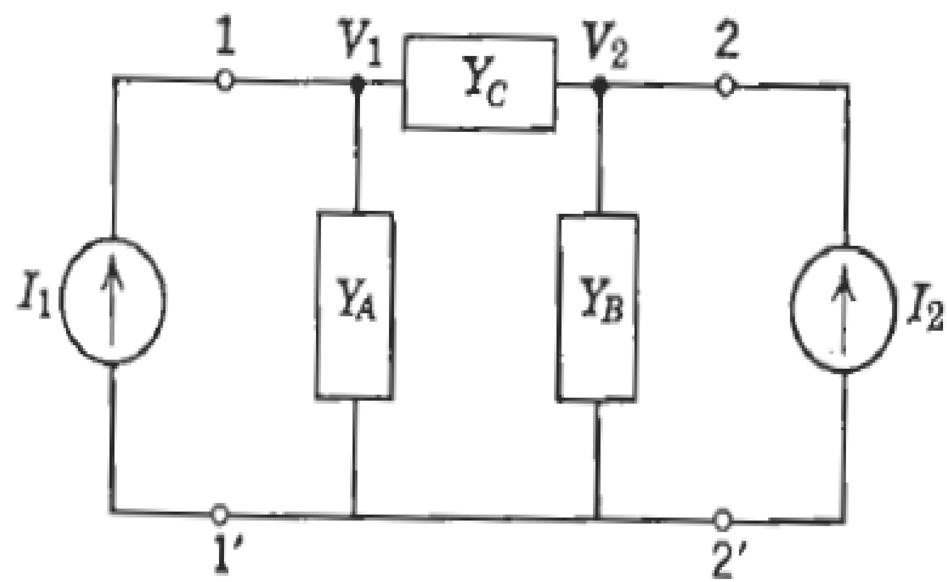


FIG. 9.5

The determinant for this set of equations is

$$\Delta Y = Y_A Y_B + Y_A Y_C + Y_B Y_C \tag{9.11}$$

In terms of ΔY, the open-circuit parameters for the *Pi* circuit are

$$z_{11} = \frac{Y_B + Y_C}{\Delta Y} \qquad z_{21} = \frac{Y_C}{\Delta Y}$$

$$z_{12} = \frac{Y_C}{\Delta Y} \qquad z_{22} = \frac{Y_A + Y_C}{\Delta Y} \tag{9.12}$$

Now let us perform a *delta-wye* transformation for the circuits in Figs. 9.4 and 9.5. In other words, let us find relationships between the immittances of the two circuits so that they both have the same z parameters. We readily obtain

$$
\begin{aligned}
z_{12} &= Z_b = \frac{Y_C}{\Delta Y} \\
z_{22} &= Z_b + Z_c = \frac{Y_A + Y_C}{\Delta Y} \\
z_{11} &= Z_a + Z_b = \frac{Y_B + Y_C}{\Delta Y}
\end{aligned}
\tag{9.13}
$$

We then find

$$
\begin{aligned}
Z_a &= \frac{Y_B}{\Delta Y} \\
Z_c &= \frac{Y_A}{\Delta Y}
\end{aligned}
\tag{9.14}
$$