Transformed Circuit

These basic relationships may also be represented in the complex-frequency domain. Ideal energy sources, for example, which were given in time domain as $v(t)$ and $i(t)$, may now be represented by their transforms $V(s) = \mathfrak{L}[v(t)]$ and $I(s) = \mathfrak{L}[i(t)]$. The resistor, defined by the v-i relationship

$$v(t) = R\,i(t) \tag{7.1}$$

is defined in the frequency domain by the transform of Eq. 7.1, or

$$V(s) = R\,I(s) \tag{7.2}$$

For an inductor, the defining v-i relationships are

$$\begin{aligned} v(t) &= L\frac{di}{dt} \\ i(t) &= \frac{1}{L}\int_{0-}^{t} v(\tau)\,d\tau + i(0-) \end{aligned} \tag{7.3}$$

Transforming both equations, we obtain

$$\begin{aligned} V(s) &= sL\,I(s) - L\,i(0-) \\ I(s) &= \frac{1}{sL}V(s) + \frac{i(0-)}{s} \end{aligned} \tag{7.4}$$

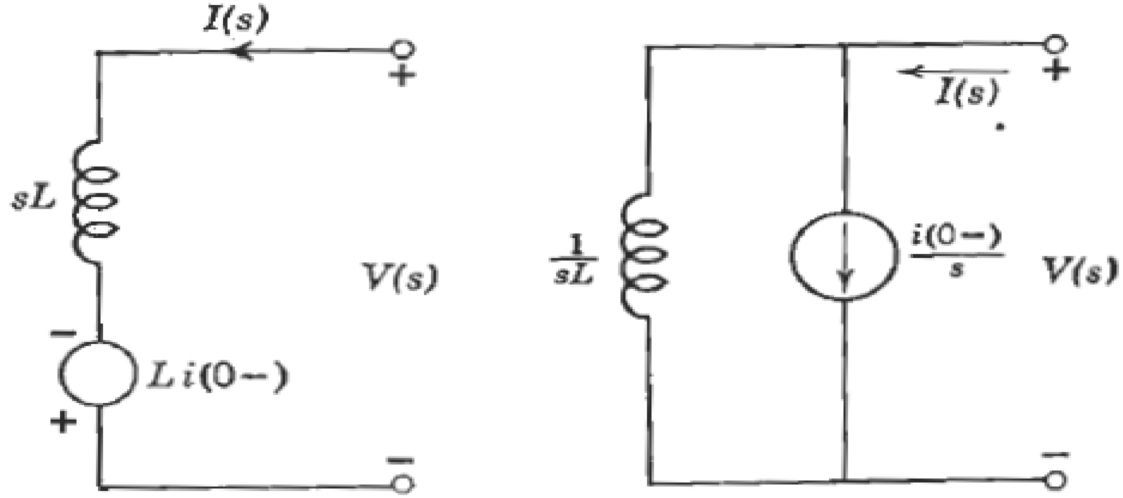


FIG. 7.1. Inductor.

The transformed circuit representation for an inductor is depicted in Fig. 7.1. For a capacitor, the defining equations are

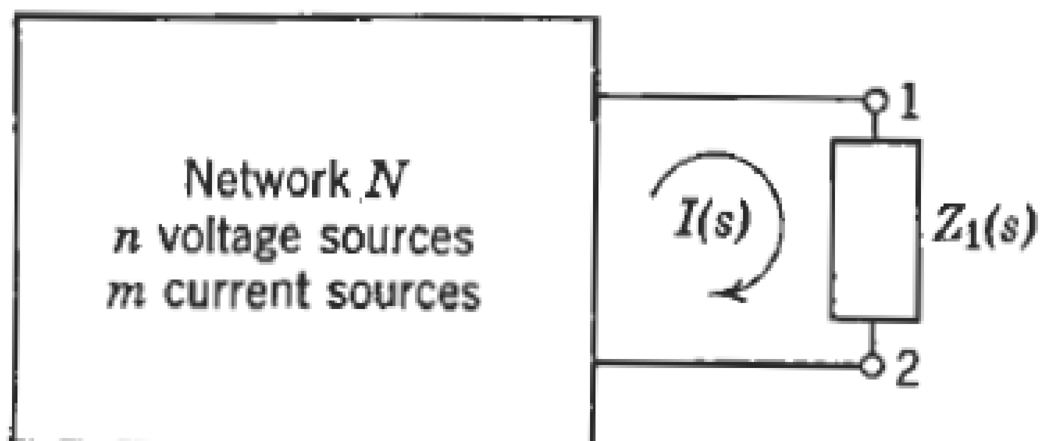
$$v(t) = \frac{1}{C}\int_{0-}^{t} i(\tau)\, d\tau + v(0-)$$
$$i(t) = C\frac{dv}{dt} \tag{7.5}$$

The frequency domain counterparts of these equations are then

$$V(s) = \frac{1}{sC} I(s) + \frac{v(0-)}{s}$$
$$I(s) = sC\, V(s) - C\, v(0-) \tag{7.6}$$

Thevenin's and Nortons Theorem

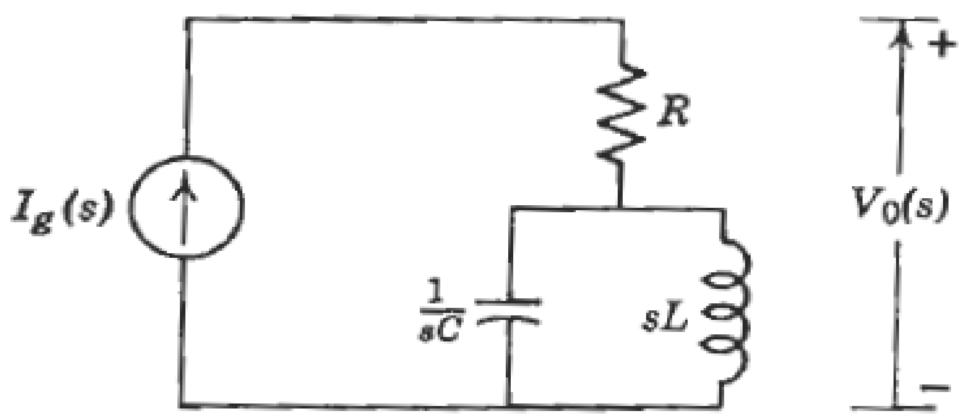
In network analysis, the objective of a problem is often to determine a *single* branch current through a given element or a *single* voltage across an element. In problems of this kind, it is generally not practicable to write a complete set of mesh or node equations and to solve a system of equations for this one current or voltage. It is then convenient to use two very important theorems on equivalent circuits, known as *Thévenin's and Norton's theorems.*



The System Function

As we discussed earlier, a linear system is one in which the excitation $e(t)$ is related to the response $r(t)$ by a linear differential equation. When the Laplace transform is used in describing the system, the relation between the excitation $E(s)$ and the response $R(s)$ is an algebraic one. In particular, when we discuss initially inert systems, the excitation and response are related by the system function $H(s)$ as given the relation

$$R(s) = E(s)H(s) \tag{7.42}$$



the system function may assume many forms and may have special names such as *driving-point admittance*, *transfer impedance*, *voltage* or *current-ratio transfer function*. This is because the form of the system function depends on whether the excitation is a voltage or current source, and whether the response is a specified current or voltage. We now discuss some specific forms of system functions.

Impedance

When the excitation is a current source and the response is a voltage, then the system function is an impedance. When both excitation and response are measured between the same pair of terminals, we have a driving-point impedance. An example of a driving-point impedance is given in Fig. 7.25, where

$$H(s) = \frac{V_0(s)}{I_g(s)} = R + \frac{(1/sC)sL}{sL + 1/sC} \tag{7.43}$$

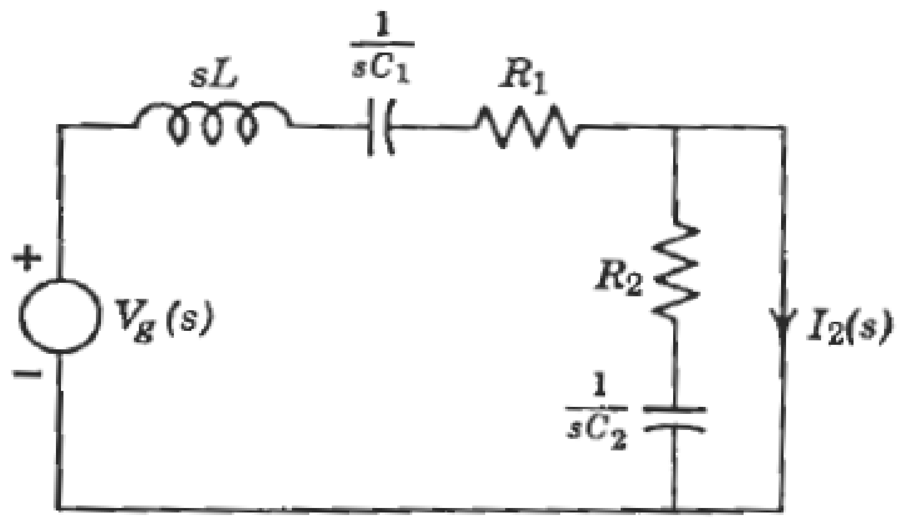
Admittance

When the excitation is a voltage source and the response is a current, $H(s)$ is an admittance. In Fig. 7.26, the transfer admittance I_2/V_g is obtained from the network as

$$H(s) = \frac{I_2(s)}{V_g(s)} = \frac{1}{R_1 + sL + 1/sC_1} \tag{7.44}$$

Voltage-ratio transfer function

When the excitation is a voltage source and the response is also a voltage, then $H(s)$ is a voltage-ratio transfer function. In Fig. 7.27, the voltage-ratio transfer function $V_0(s)/V_g(s)$ is obtained as follows. We first find the



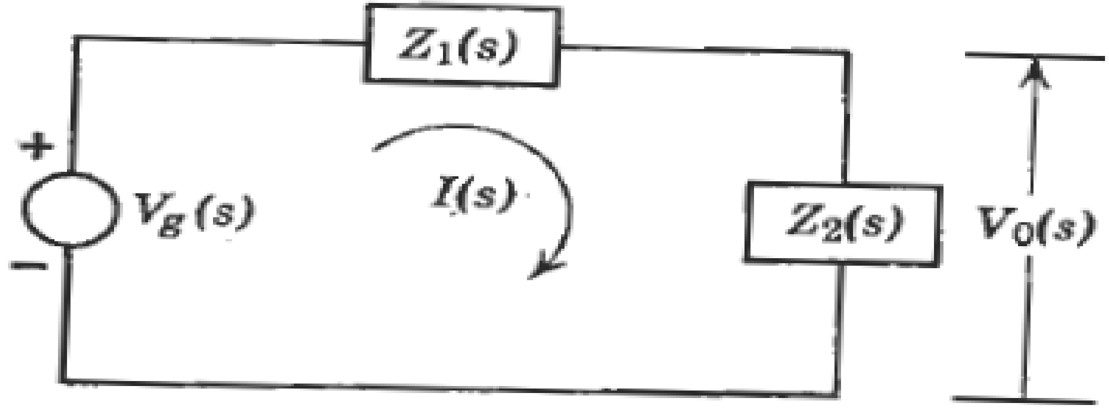


FIG. 7.27

current

$$I(s) = \frac{V_g(s)}{Z_1(s) + Z_2(s)}$$

Since

$$V_0(s) = Z_2(s)\, I(s)$$

then

$$\frac{V_0(s)}{V_g(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

Current-ratio transfer function

When the excitation is a current source and the response is another current in the network, then $H(s)$ is called a current-ratio transfer function. As an example, let us find the ratio I_0/I_g for the network given in Fig. 7.28. Referring to the depicted network, we know that

$$I_g(s) = I_1(s) + I_0(s), \qquad I_1(s)\frac{1}{sC} = I_0(s)(R + sL) \qquad (7.48, 7.49)$$

Eliminating the variable I_1, we find

$$I_g(s) = I_0(s)\left(1 + \frac{R + sL}{1/sC}\right) \qquad (7.50)$$

so that the current-ratio transfer function is

$$\frac{I_0(s)}{I_g(s)} = \frac{1/sC}{R + sL + 1/sC} \qquad (7.51)$$