CONVOLUTION INTEGRAL

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- Convolution integral finds two very important applications in network theorems :-
- (a) It enables to evaluate the response of a network to an arbitrary input in terms of the impulse response of the network.
- (b) If F(s) is the laplace transform of a function f(t) and if F(s) could be expressed as product of two functions $F_1(s)$ and $F_2(s)$, then convolutions integral can be used to obtain f(t) which makes the calculations of laplace inverse much easier.

Convolution Theorem

- The integrals indicated above are called Convolution integrals.
- The convolution may be interpreted in terms of following four processes :-
- (i) Folding

  (ii) Translating

  (iii) Multiplying

  (iv) Integrating

The convolution integral is also called Faltung Integral.(Faltung is German word)

Proof : Convolution Theorem (contd)

- Now we introduce a new variable
  $y = t - \tau$ or $t = y + \tau$ and eq (2) is re-written as

$$L[f_1(t) * f_2(t)] = \int_0^\infty [\int_0^\infty f_1(y\ ) u(y) f_2(\tau)\, d\tau\ ] e^{-s(y + \tau)} dy$$

$$= \int_0^\infty f_1(y\ ) u(y)\, e^{-sy}\, dy \int_0^\infty f_2(\tau)\, e^{-s\tau}\, d\tau$$

$$= F_1(s)\, F_2(s)$$

thus the convolution theorem is proved

Amplitude and Phase Plot

- The amplitude and phase response of a system provides valuable information in the analysis and design of transmission circuits. .
- Frequency range is taken from 0 to infinity.
- For determining the amplitude /phase response of H(s) , s is replaced by jw.
- Calculate M(w) and φ(w) using knowledge of complex variables
- Amplitude and real parts are even functions of frequencies.
- Phase and imaginary parts are odd functions of frequencies
- From pole – zero plot, Calculate M(w) and φ(w) using knowledge of vector algebra.
- From pole – zero plot ,Amplitude M(w) is given by product of all zero lines to the point on jw axis divided by product of all pole lines to the point on the jw axis..
- From pole – zero plot phase response φ(w) is given by sum of the angles of all zero lines to the point on jw axis minus sum of angles of all pole lines to the point on jw axis .

Amplitude and Phase Plot

- Example

Given $H(s) = (3+4s) / (4+3s)$

Find Amplitude and phase for H(j5)

Put s=jw in the given function

$H(jw) = (3+4jw) / (4+3jw)$

Now $H(j5) = (3+4j5) / (4+3j5) = (3+j20) / (4+j15)$

Amplitude $H(j5) = (\sqrt{(3^2 + 20^2)}) / (\sqrt{(4^2 + 15^2)})$

$= (\sqrt{(409)}) / (\sqrt{(241)}) = 20.22/15.52 = 1.302$

Phase $H(j5) = \tan^{-1} 20/3 – \tan^{-1} 15/4$

$= \tan^{-1} 6.67 – \tan^{-1} 3.75$

$= 81.4^0 – 75.1^0$

$= 6.3s^0$

Amplitude and Phase Plot

- $H(s) = 5s / s^2 + 6s + 25$
- $H(j5) = 5Xj5 / -25 + 30j + 25$

  $= 25j / +j30$

  $M(j5) = j25 / j30$

  $= 5/6$

  $\varphi(j5) = 0^0$