Ideal Models

Ideal models

Let us now examine some idealized models of linear systems. The systems given in the following all have properties which make them very useful in signal processing.

FIG. 1.15. Amplifier.

FIG. 1.16. Differentiator.

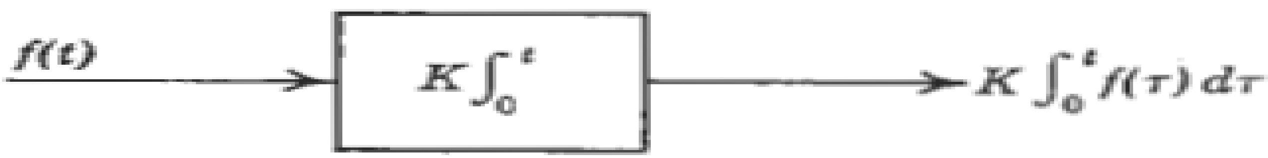

FIG. 1.17. Integrator.

FIG. 1.18. Time-delay network.

1. *Amplifier:* An amplifier scales up the magnitude of the input, i.e., $r(t) = Ke(t)$, where K is a constant (Fig. 1.15).

2. *Differentiator:* The input signal is differentiated and possibly scaled up or down (Fig. 1.16).

3. *Integrator:* The output is the integral of the input, as shown in Fig. 1.17.

4. *Time delayer:* The output is delayed by an amount T, but retains the same wave shape as the input (Fig. 1.18).

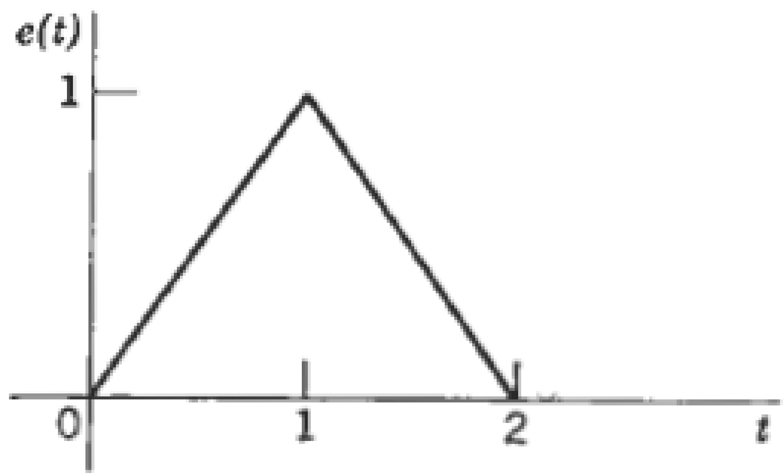

FIG. 1.19. Excitation function.

Suppose we take the triangular pulse in Fig. 1.19 as the input signal. Then the outputs for each of the four systems just described are shown in Figs. 1.20*a*–1.20*d*.

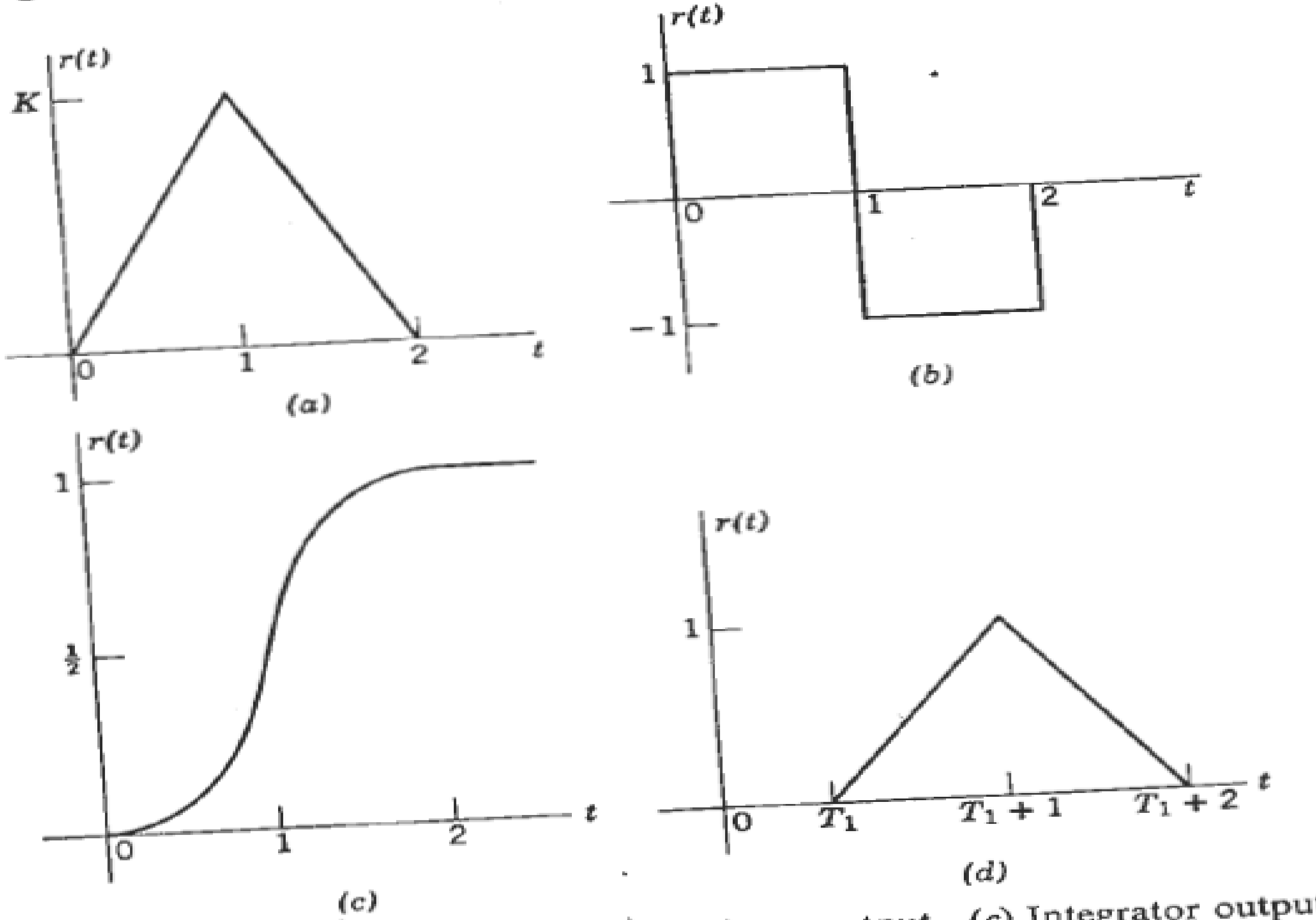

FIG. 1.20. (*a*) Amplifier output. (*b*) Differentiator output. (*c*) Integrator output. (*d*) Delayed output.

Ideal elements

In the analysis of electric networks, we use idealized linear mathematical models of physical circuit elements. The elements most often encountered are the resistor R, given in ohms, the capacitor C, given in farads, and the inductor L, expressed in henrys. The endpoints of the elements are called *terminals*. A *port* is defined as any pair of two terminals into which energy is supplied or withdrawn or where network variables may be measured or observed. In Fig. 1.21 we have an example of a two-port network.

The energy sources that make up the excitation functions are ideal *current* or *voltage sources*, as shown in Figs. 1.22*a* and *b*. The polarities indicated for the voltage source and the direction of flow for the current source are arbitrarily assumed for reference purposes only. An ideal voltage source is an energy source that provides, at a given port, a voltage signal that is independent of the current at that port. If we interchange the words "current" and "voltage" in the last definition, we then define an ideal current source.

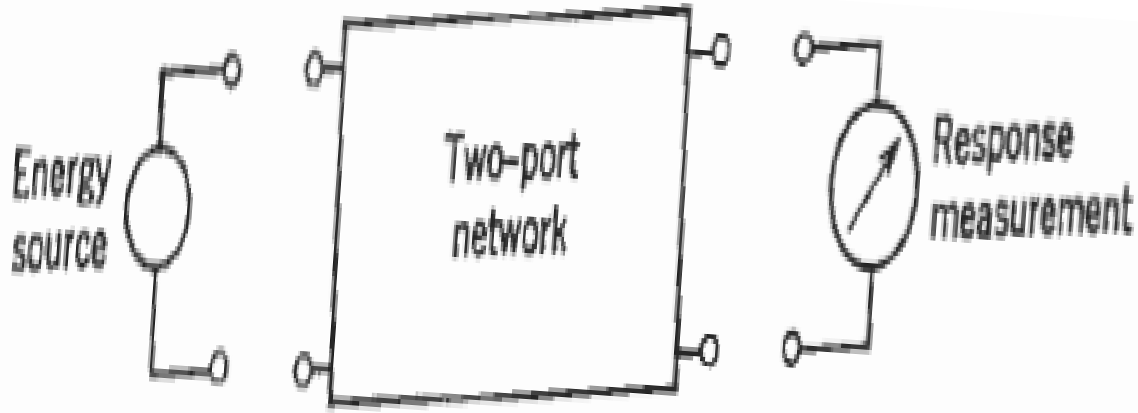

FIG. 1.21. Two-port network.

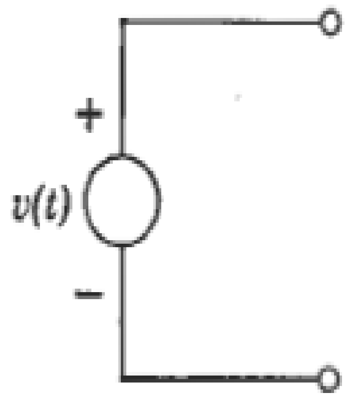

FIG. 1.22a. Voltage source.

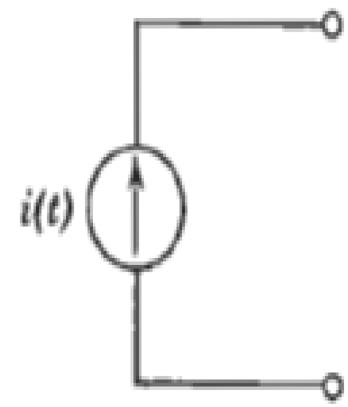

FIG. 1.22b. Current source.

In network analysis, the principal problem is to find the relationships that exist between the currents and voltages at the ports of the network. Certain simple voltage-current relationships for the network elements also serve as defining equations for the elements themselves. For example, when the currents and voltages are expressed as functions of time, then the *R*, *L*, and *C* elements, shown in Fig. 1.23, are defined by the equations

$$v(t) = Ri(t) \quad \text{or} \quad i(t) = \frac{1}{R}v(t)$$

$$v(t) = L\frac{di(t)}{dt} \quad \text{or} \quad i(t) = \frac{1}{L}\int_0^t v(x)\,dx + i(0)$$

$$v(t) = \frac{1}{C}\int_0^t i(x)\,dx + v(0) \quad \text{or} \quad i(t) = C\frac{dv(t)}{dt}$$

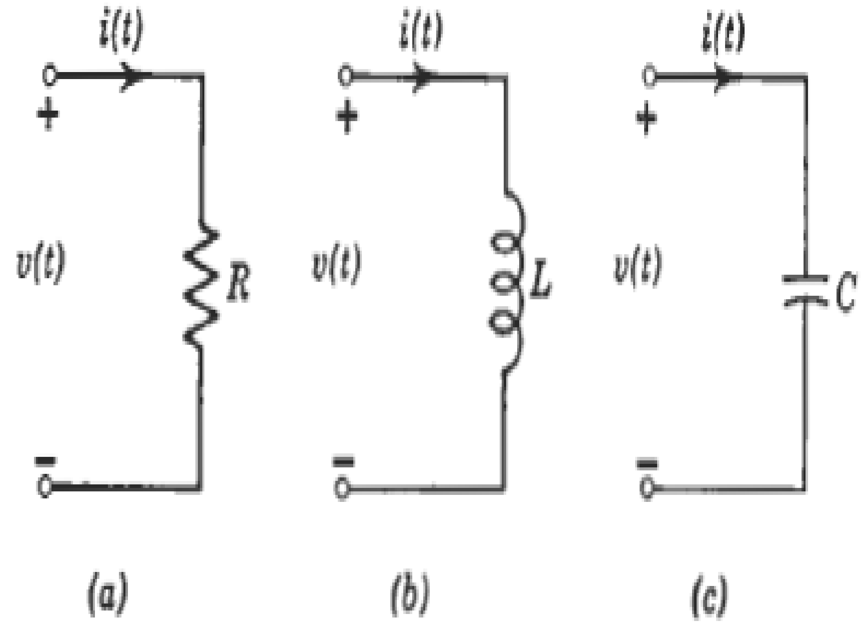

FIG. 1.23. (*a*) Resistor. (*b*) Inductor. (*c*) Capacitor.

defining the R, L, and C elements, shown in Fig. 1.24, are (ignoring initial conditions for the moment)

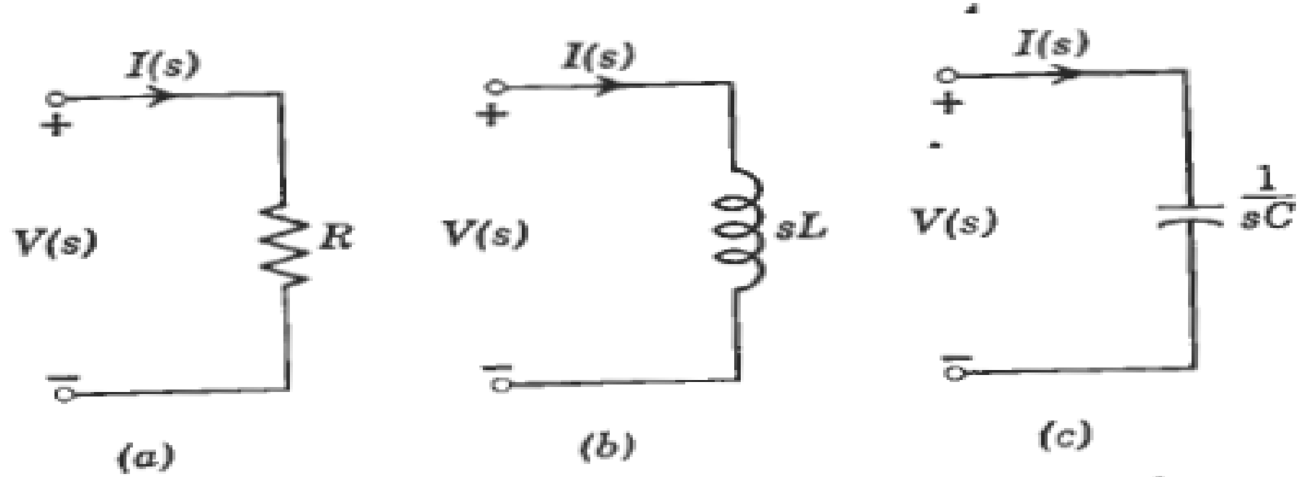

FIG. 1.24. (*a*) Resistor. (*b*) Inductor. (*c*) Capacitor.

$$V(s) = RI(s) \qquad \text{or} \qquad I(s) = \frac{1}{R} V(s)$$

$$V(s) = sLI(s) \qquad \text{or} \qquad I(s) = \frac{1}{sL} V(s)$$

$$V(s) = \frac{1}{sC} I(s) \qquad \text{or} \qquad I(s) = sCV(s)$$

Network synthesis

in the network synthesis, the response R(s) and Excitation E(s) is given and we required to synthesize the network from the system function

$$H(s) = \frac{R(s)}{E(s)}$$

Since $R(s)$ and $E(s)$ are voltages or currents, then $H(s)$ is denoted generally as an *immittance* if $R(s)$ is a voltage and $E(s)$ is a current, or vice versa. A *driving-point immittance*[3] is defined to be a function for which the variables are measured at the same port. Thus a *driving-point impedance* $Z(s)$ at a given port is the function

$$Z(s) = \frac{V(s)}{I(s)}$$

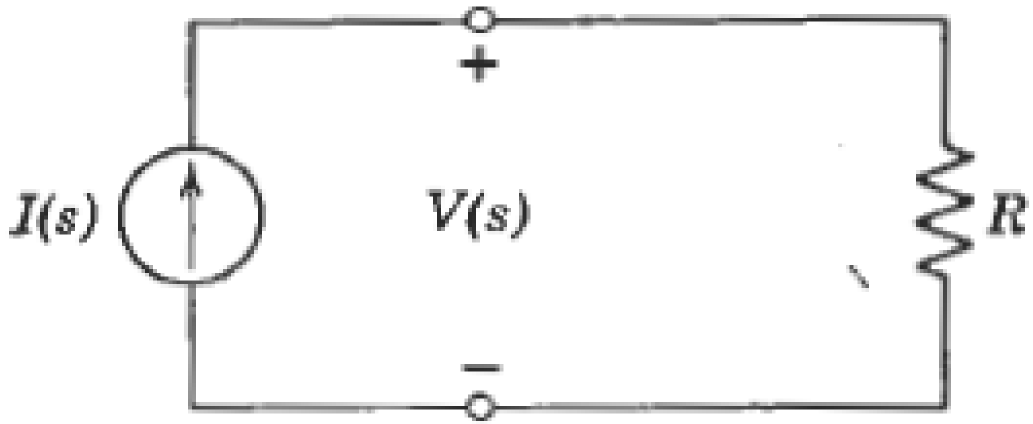

FIG. 1.25. Driving-point impedance $Z(s) = R$.

where the excitation is a current $I(s)$ and the response is a voltage $V(s)$, as shown in Fig. 1.25. When we interchange the words "current" and "voltage" in the last definition, we then have a *driving-point admittance*. An example of a driving-point impedance is the network in Fig. 1.25, where

$$Z(s) = \frac{V(s)}{I(s)} = R$$