SECOND ORDER CIRCUIT

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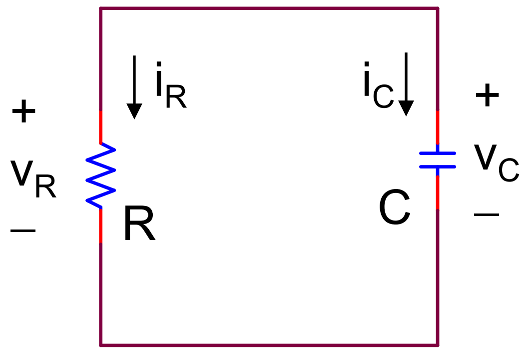
- Revision of 1st order circuit
- Second order circuit
- Natural response (source-free)
- Forced response

Revision of 1st order circuit



NATURAL RESPONSE (SOURCE-FREE)

- initial energy in capacitor
- i.e. $v_C(0) = V_o$

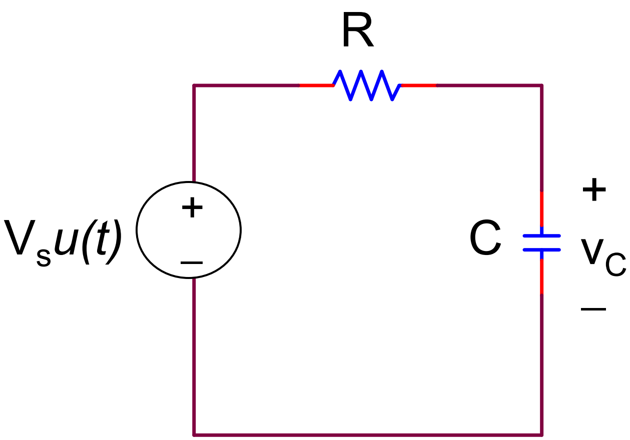
Solution: KCL $\Rightarrow$ $i_C + i_R = 0$

$$\Rightarrow \quad \frac{dv_C}{dt} + \frac{v_C}{RC} = 0$$

Solving this first order differential equation gives:

$$v_C(t) = V_o e^{-t/RC}$$

Revision of 1st order circuit



FORCED RESPONSE

- no initial energy in capacitor
- i.e. $v_C(0) = 0$

Solution: KCL $\Rightarrow$ $i_C + i_R = 0$

$$\Rightarrow \quad C\frac{dv_C}{dt} + \frac{v_C - V_s}{R} = 0 \quad \Rightarrow \quad \frac{dv_C}{dt} + \frac{v_C}{RC} = \frac{V_s}{RC}$$

Solving this first order differential equation gives:

$$v_C(t) = V_s\left(1 - e^{-t/RC}\right)$$

Revision of 1st order circuit

COMPLETE RESPONSE

Complete response = natural response + forced response

$$v(t) = v_n(t) + v_f(t)$$

$$v_C(t) = V_o e^{-t/RC} + V_s\left(1 - e^{-t/RC}\right)$$

Complete response = Steady state response + transient response

$$v(t) = v_{ss}(t) + v_t(t)$$

$$v_C(t) = V_s + \left(V_o - V_s\right)e^{-t/RC}$$

Revision of 1st order circuit

COMPLETE RESPONSE

In general, this can be written as:

$$x(t) = x(\infty) + [x(0) - x(\infty)]e^{-t/\tau}$$

- can be applied to voltage or current

- $x(0)$: initial value

$$v_C(t) = V_s + (V_o - V_s)e^{-t/RC}$$

- $x(\infty)$: steady state value

For the 2nd order circuit, we are going to adopt the same approach

Before we begin …..

To successfully solve 2nd order equation, need to know how to get the initial condition and final values CORRECTLY

INCORRECT initial conditions /final values will result in a wrong solution

In 1st order circuit

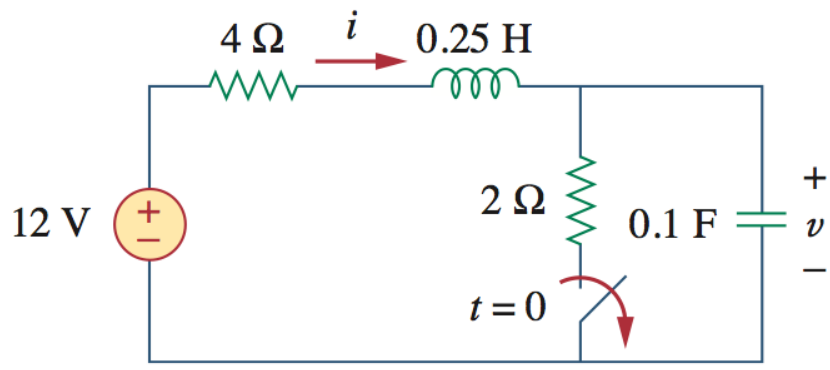
- need to find initial value of inductor current (RL circuit) OR capacitor voltage (RC circuit): $i_L(0)$ or $v_C(0)$
- Need to find final value of inductor current OR capacitor voltage: $i_L(\infty)$ or $v_C(\infty)$

In 2nd order circuit

- need to find initial values of i_L and/or v_C : $i_L(0)$ or $v_C(0)$
- Need to find final values of inductor current and/or capacitor voltage: $i_L(\infty)$, $v_C(\infty)$
- Need to find the initial values of first derivative of i_L or v_C : $di_L(0)/dt$ $dv_C(0)/dt$

Finding initial and final values

Example



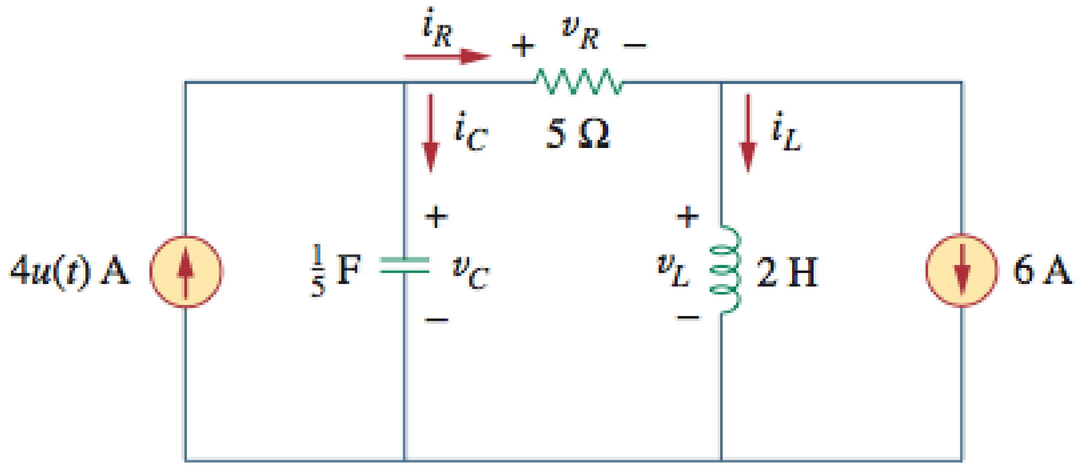
Switch closed for a long time and open at t=0. Find:

$i(0^+)$, $v(0^+)$,

$di(0^+)/dt$, $dv(0^+)/dt$,

$i(\infty)$, $v(\infty)$

Finding initial and final values



Find:

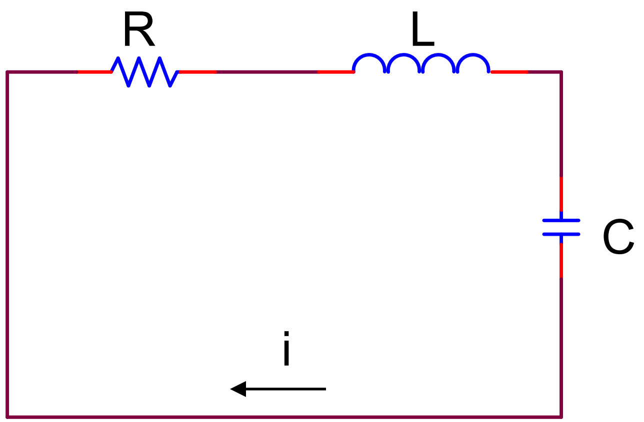
$i_L(0^+)$, $v_C(0^+)$, $v_R(0^+)$

$di_L(0^+)/dt$, $dv_C(0^+)/dt$, $dv_R(0^+)/dt$,

$i_L(\infty)$, $v_C(\infty)$, $v_R(\infty)$

Second order circuit

Natural Response of Series RLC Circuit
(Source-Free Series RLC Circuit)



We want to solve for i(t).

Applying KVL,

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt = 0$$

Differentiate once,

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

$\Leftarrow$ This is a second order differential equation with constant coefficients

Second order circuit

Assuming $i(t) = Ae^{st}$

$$As^2e^{st} + \frac{AR}{L}se^{st} + \frac{A}{LC}e^{st} = 0$$

$$Ae^{st}\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) = 0$$

Since Ae^{st} cannot become zero,

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

This is known as the **CHARACTERISTIC EQUATION** of the diff. equation

Second order circuit

Solving for s,

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Which can also be written as

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

where $\alpha = \frac{R}{2L}$, $\omega_o = \frac{1}{\sqrt{LC}}$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) = 0$$

s_1, s_2 – *known as natural frequencies (nepers/s)*

α – *known as neper frequency*, ω_o – *known as resonant frequency*

Second order circuit

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

A_1 and A_2 are determined from initial conditions

Case 1

$\alpha > \omega_o$

Case 2 Overdamped solution

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

$\alpha = \omega_o$ Critically damped solution

Case 3

$\alpha < \omega_o$ Underdamped solution