Dual Simplex Algorithm

- In "primal" simplex, RHS column in always non-negative, hence basic solution is feasible at every iteration
- >What if some elements of the RHS column are negative?
- >In such a case, primal is infeasible
- >Dual Simplex Algorithm (DSA) : addresses such a scenario
- DSA: particularly useful for re-optimizing a problem after a constraint has been added or some problem parameter has been changed (sensitivity analysis), such that a previously optimal basis is no longer feasible

Dual Simplex Algorithm : Concept

>At each iteration of "primal" simplex:

- ✤always maintain primal feasibility (RHS≥0)
- corresponding dual is always infeasible

≻At each iteration of "dual" simplex

- ◆always maintain primal optimality, i.e., coefficients of variables in (-z) row≤0
- ✤In other words, always maintain dual feasibility
- ♦ drive towards primal feasibility (RHS≥0)
- ★terminate when primal feasibility is attained, i.e., all elements in RHS column ≥ 0

Dual Canonical Form

- ≻ All decision variables ≥ 0
- All RHS coefficients negative (only difference with "primal" simplex)
- All constraints, except non-negativity stated as equalities

Isolate one decision variable from each constraint with +1 coefficient, which does not appear in any other constraint and appears with a zero coefficient in the objective function

Procedure of Dual Simplex method

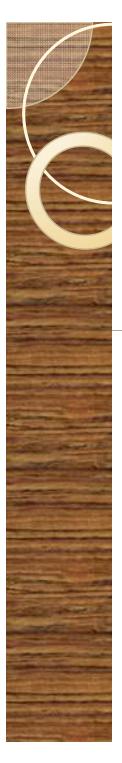
> Convert any functional constraint in \geq form to \leq form by

multiplying both sides by -1

- Introduce slack variables as needed
- > Identify leaving variable

➤ variable to leave is the basic variable associated with the constraint with most negative RHS value

Row corresponding to leaving variable called "pivot row"



Perform ratio test to identify entering variable

- *Pick all negative coefficients in pivot row (a_{lj})
- $Let x_1$ be leaving variable
- Compute the ratios (c_j / a_{lj}) where all $a_{lj} < 0$
- Column(variable) that gives smallest ratio enters basis
- Identify pivot element (as in "primal" simplex)
- Divide it by itself to make it 1
- Make other elements in the column of the pivot element
 - = 0 by performing row operations
- ♦ Continue till all elements in RHS column become ≥ 0

Example 1

➤ Consider the following LP $Max -3x_1-4x_2$ $s.t. -2x_1+x_2 \le -2$ $x_1+2x_2 \ge 4$ $x_1, x_2 \ge 0$ Step 1: Multiply second constraint by -1

Step 1:Multiply second constraint by -1 to convert to \leq Form

$$Max - 3x_{1} - 4x_{2}$$

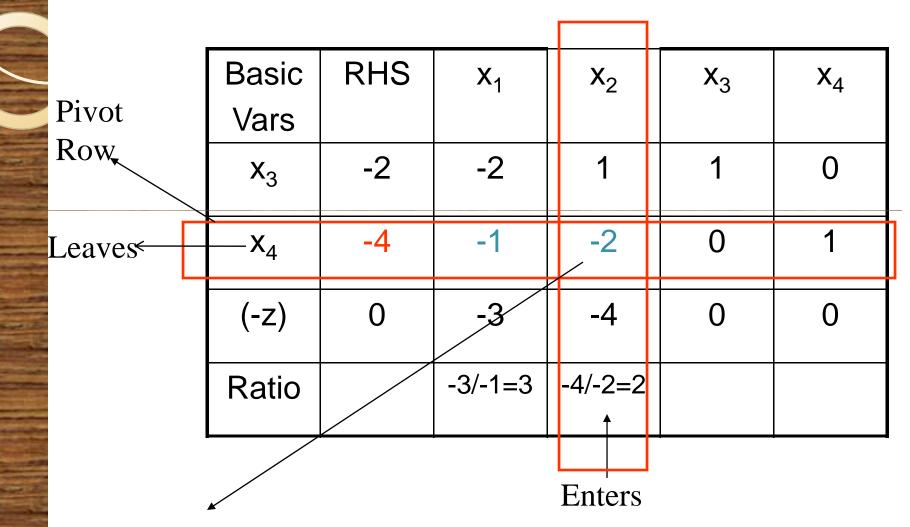
s.t. $-2x_{1} + x_{2} \le -2$
 $-x_{1} - 2x_{2} \le -4$
 $x_{1}, x_{2} \ge 0$

Example 1-Contd...

Step 2: Add slack variables, convert into dual canonical form

Max $-3x_1 - 4x_2$ s.t. $-2x_1 + x_2 + x_3 = -2$ $-x_1 - 2x_2 + x_4 = -4$ $x_1, x_2, x_3, x_4 \ge 0$ Canonical form shown below $-2x_1 + x_2 + x_3 + 0x_4 = -2$ $-x_1 - 2x_2 + 0x_3 + x_4 = -4$ $-3x_1 - 4x_2 + 0x_3 + 0x_4 = 0$

1st Tableau



Pivot Element, make it 1 and other elements in column of $x_2 = 0$ by row operations

x₄ Leaves

2nd Tableau

Basic Vars	RHS	X ₁	x ₂	X ₃	х ₄	
X ₃	-4	-5/2	0	1	1/2	
x ₂	2	1/2	1	0	-1/2	_
(-z)	8	-1	0	0	-2	
Ratio		-1/(-5/2)=2/5				

 x_3 leaves, x_1 enters

3rd Tableau

Basic	RHS	x ₁	X ₂	Х ₃	x ₄
Vars					
X ₁	8/5	1	0	-2/5	-1/5
X ₂	6/5	0	1	1/5	-2/5
(-z)	48/5	0	0	-2/5	-11/5
Ratio					

Note:

All RHS elements are now ≥ 0

Hence we are done

Optimal solution: z=-48/5, $x_1=8/5$, $x_2=6/5$

Points to Note

- In "primal" simplex, first identify entering variable, then leaving variable
- In "dual" simplex, first identify leaving variable, then entering variable
- → At each iteration, all elements of (-z) row ≤ 0
- At each iteration, the dual to the original problem is always feasible
 - > Verify this by writing the dual to the original problem
 - ➢ Obtain values form dual multipliers from each tableau
 - *At each iteration, dual multipliers = values of slacks in
 - (z) row, e.g. at 2^{nd} iteration, dual multipliers are 0 and 2



Example 2

Max $-x_1 - 2x_2$ s.t. $-x_1 + 2x_2 - x_3 \le -2$ $-2x_1 - x_2 + x_3 \le -6$ $x_1, x_2, x_3 \ge 0$



1st Tableau

	[
Basic Vars	RHS	х ₁	X ₂	Х ₃	X ₄	X 5	
x ₄	-4	-1	2	-1	1	0	
Х ₅	-6	-2	-1	1	0	1	
(-z)	0	-1	-2	0	0	0	
Ratio		-1/- 2=1/2	2 -2/-1=2				

 x_5 leaves, x_1 enters

2nd Tableau

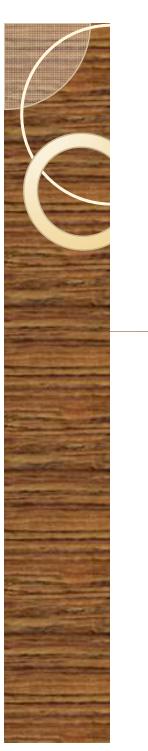
Basic Vars	RHS	X ₁	x ₂	X ₃	X ₄	X 5	
X ₄	-1	0	5/2	-3/2	1	-1/2	
X ₁	3	1	1⁄2	-1/2	0	-1/2	
(-z)	3	0	3/2	-1/2	0	-1/2	
Ratio				(-1/2)/(3/2)=1/3			

 x_4 leaves, x_3 enters

3rd Tableau

Basic Vars	RHS	x ₁	x ₂	Х ₃	X ₄	X ₅	
х ₃	2/3	0	-5/3	1	-2/3	1/3	
X ₁	10/3	1	-1/3	0	-1/3	-1/3	
(-z)	10/3	0	-7/3	0	-1/3	-1/3	
Ratio							

Optimal solution obtained: z = -10/3, $x_1 = 10/3$, $x_2 = 0$



Assignment

• Try yourself

Q.1 Obtain the Dual of

Maximize $z = 5x_1 + 4x_2 + 3x_3$

Subject to the constraints

 $3x_1 + 2x_2 + x_2 \le 10, \ 2x_1 + x_2 + 2x_3 \le 12, \ x_1 + x_2 + 3x_2 \le 15$ $x_1, x_2, x_3 \ge 0$

Q.2 Solve the LPP by dual simplex method:

Maximize $z = -2x_1 - x_2$

Subject to the constraints:

 $3x_1 + x_2 \ge 3, 4x_1 + 3x_2 \ge 6, x_1 + 2x_2 \ge 3 \quad x_1, x_2 \ge 0$

Q.3 Solve the LPP by dual simplex method:

Minimize $z = 2x_1 + 2x_2 + 4x_3$

Subject to the constraints:

 $\begin{array}{l} 2x_1 + 3x_2 + 5x_3 \geq 2 \;, 3x_1 + x_2 + 7x_3 \leq 3, x_1 + 4x_2 + 6x_3 \leq 5 \\ x_1, x_2, \; x_3 \geq 0 \end{array}$

Thank you