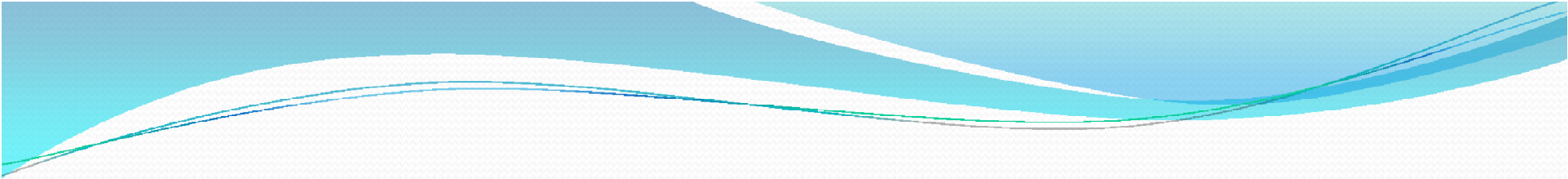
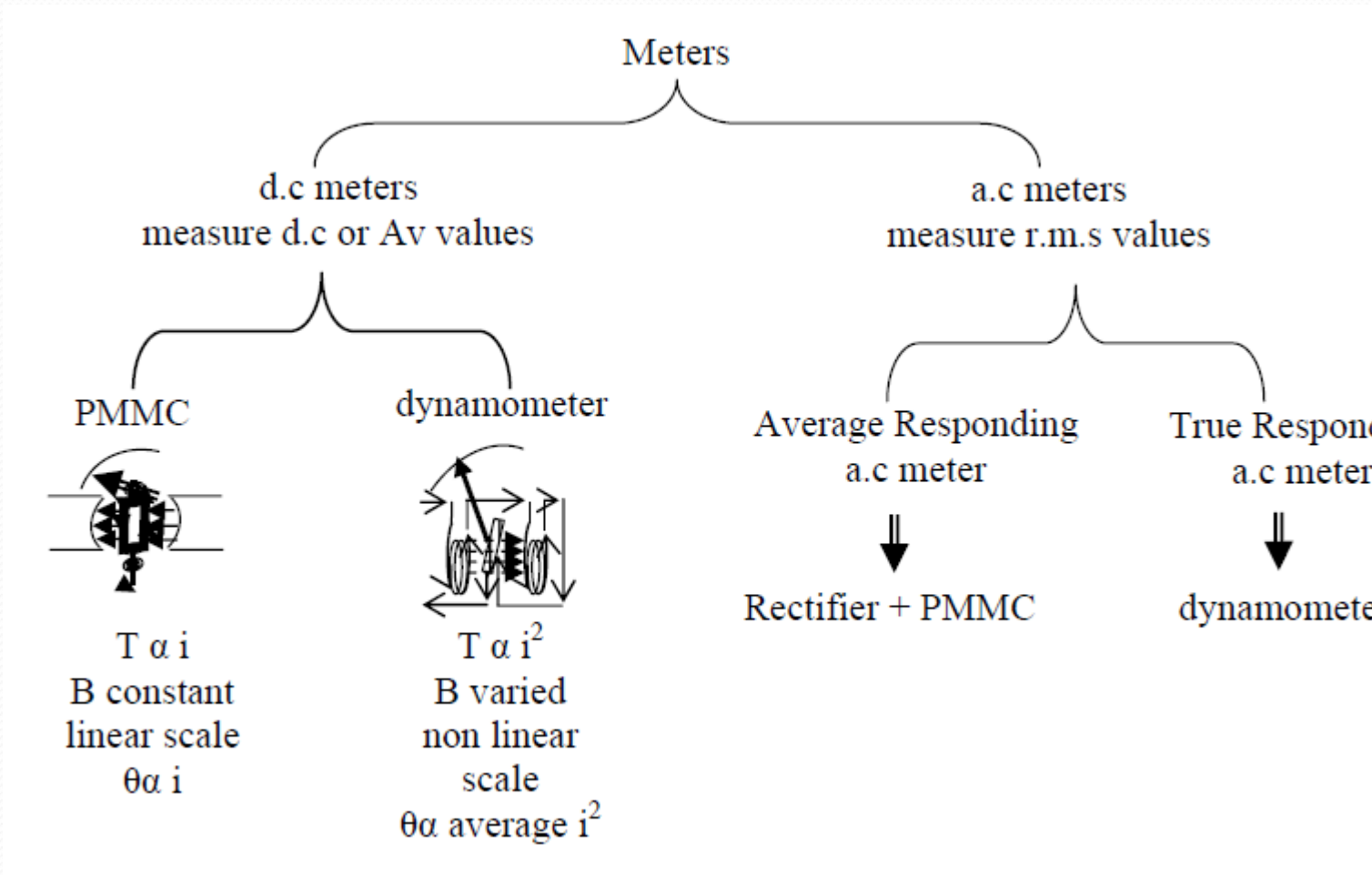
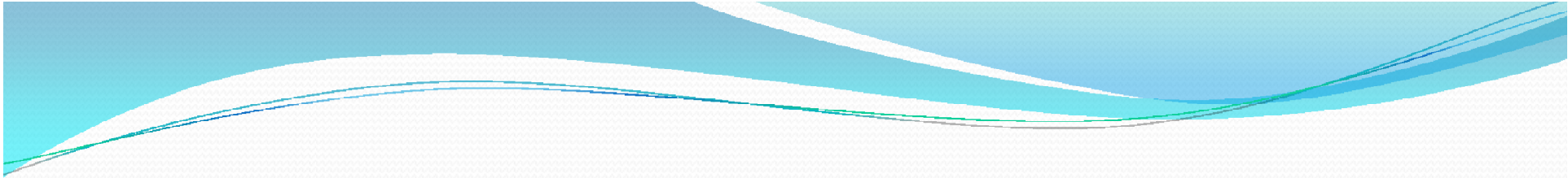


SECTION B

**MEASURING SYSTEM FUNDAMENTALS and
MEASURING INSTRUMENTS**

- 
- **MEASURING SYSTEM FUNDAMENTALS:** Classification of Instruments (Absolute & Secondary Instruments; Indicating, Recording & Integrating instruments; Based upon Principle of operation).
 - **MEASURING INSTRUMENTS:** Construction, operating principle, Torque equation, Shape of scale, use as Ammeter or as Voltmeter (Extension of Range), Use on AC/DC or both, Advantages & disadvantages, Errors (Both on AC/DC) of PMMC types, Electrodynamical Type, Moving iron type (attraction, repulsion & combined types), Hot wire type & Induction type, Electrostatic type Instruments.





Classification of instruments

Electrical measuring instruments may be classified into two groups:

- (a) Absolute (or primary) instruments.
- (b) Secondary instruments.

Absolute Instruments

These instruments give the value of the electrical quantity in terms of absolute quantities (or some constants) of the instruments and their deflections. In this type of instruments no calibration or comparison with other instruments is necessary. They are generally not used in laboratories and are seldom used in practice by electricians and engineers. They are mostly used as means of standard measurements and are maintained in national laboratories and similar institutions. Some of the examples of absolute instruments are:

- Tangent galvanometer
- Raleigh current balance
- Absolute electrometer.



Secondary Instruments

They are direct reading instruments. The quantity to be measured by these instruments can be determined from the deflection of the instruments. They are often calibrated by comparing them with either some absolute instruments or with those which have already been calibrated. The deflections obtained with secondary instruments will be meaningless until it is not calibrated. These instruments are used in general for all laboratory purposes. Some of the very widely used secondary instruments are: ammeters, voltmeter, wattmeter, energy meter (watt-hour meter), ampere-hour meters etc.

Classification of Secondary Instruments

Classification based on the various effects of electric current (or voltage) upon which their operation depend. They are:

- **Magnetic effect:** Used in ammeters, voltmeters, watt-meters, integrating meters etc.
- **Heating effect:** Used in ammeters and voltmeters.
- **Chemical effect:** Used in dc ampere hour meters.
- **Electrostatic effect:** Used in voltmeters.
- **Electromagnetic induction effect:** Used in ac ammeters, voltmeters, watt meters and integrating meters. Generally the magnetic effect and the electromagnetic induction effect are utilized for the construction of the commercial instruments. Some of the instruments are also named based on the above effect such as electrostatic voltmeter, induction instruments, etc.

Both direct current and alternating current instruments (dc/ac instruments).



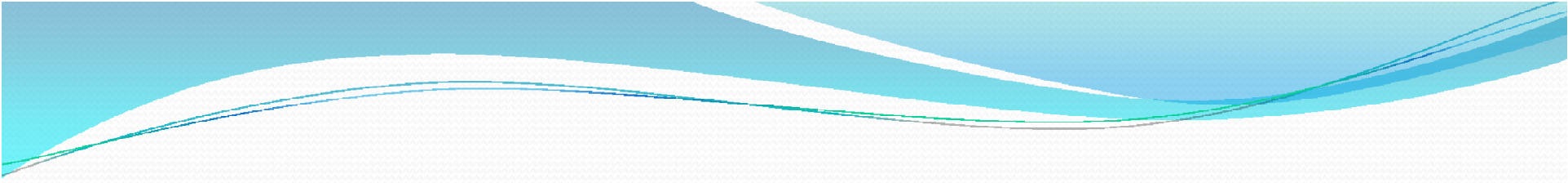
Classification based on the Nature of their Operations

Classification based on the Nature of their Operations

We have the following instruments.

Indicating instruments: Indicating instruments indicate, generally the quantity to be measured by means of a pointer which moves on a scale. Examples are ammeter, voltmeter, wattmeter etc.

Recording instruments: These instruments record continuously the variation of any electrical quantity with respect to time. In principle, these are indicating instruments but so arranged that a permanent continuous record of the indication is made on a chart or dial. The recording is generally made by a pen on a graph paper which is rotated on a disc or drum at a uniform speed. The amount of the quantity at any time (instant) may be read from the traced chart. Any variation in the quantity with time is recorded by these instruments. Any electrical quantity like current, voltage, power etc., (which may be measured by the indicating instruments) may be arranged to be recorded by a suitable recording mechanism.



Integrating instruments: These instruments record the consumption of the total quantity of electricity, energy etc., during a particular period of time. That is, these instruments totalize events over a specified period of time. No indication of the rate or variation or the amount at a particular instant are available from them. Some widely used integrating instruments are: Ampere-hour meter: kilowatthour (kWh) meter, kilovolt-ampere-hour (kVARh) meter.

(c) Classification based on the Kind of Current that can be Measurand. Under this heading, we have:

- Direct current (dc) instruments
- Alternating current (ac) instruments



GENERAL CONSIDERATIONS OF ELECTRICAL MEASURING INSTRUMENTS

Classification based on the method used.

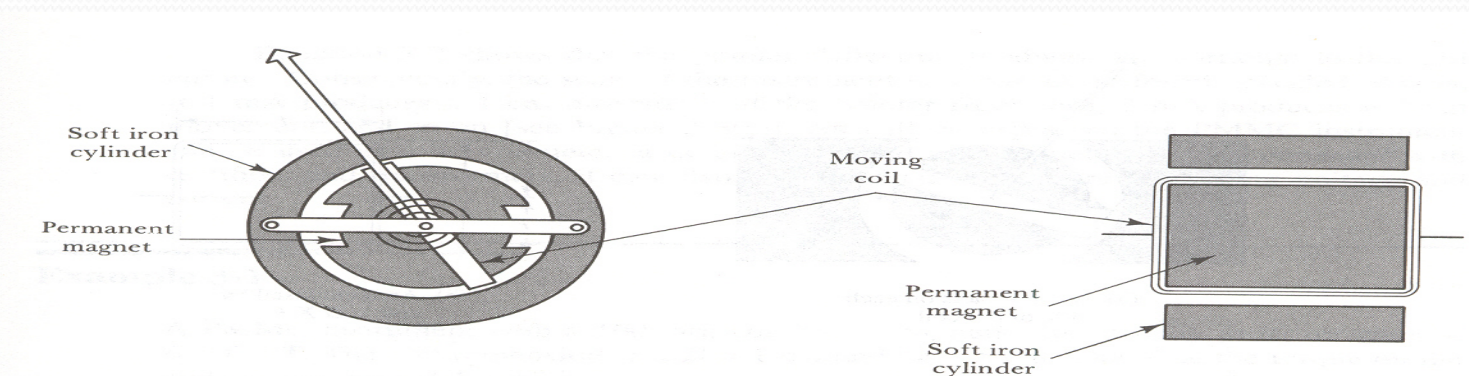
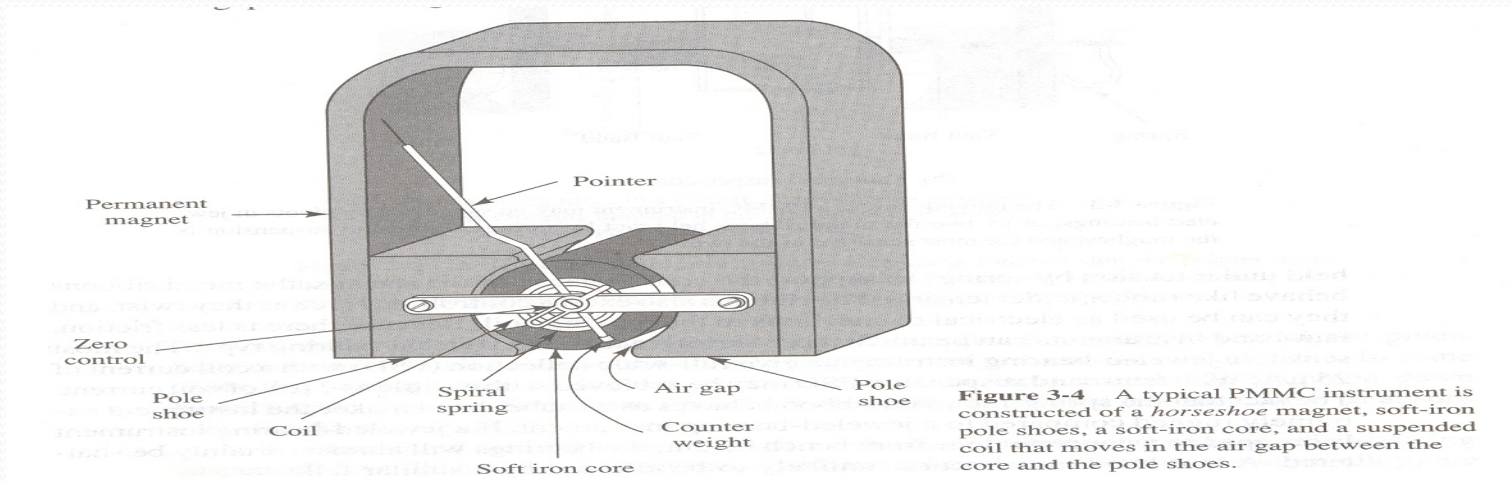
Under this category, we have:

- **Direct measuring instruments:** These instruments convert the energy of the measured quantity directly into energy that actuates the instrument and the value of the unknown quantity is measured or displayed or recorded directly. These instruments are most widely used in engineering practice because they are simple and inexpensive. Also, time involved in the measurement is shortest. Examples are Ammeter, Voltmeter, Watt meter etc.
- **Comparison instruments:** These instruments measure the unknown quantity by comparison with a standard. Examples are dc and ac bridges and potentiometers. They are used when a higher accuracy of measurements is desired.

Permanent Magnet Moving Coil Mechanism (PMMC)

In PMMC meter or (D'Arsonval) meter or galvanometer all are the same instrument, a coil of fine wire is suspended in a magnetic field produced by permanent magnet. According to the fundamental law of electromagnetic force, the coil will rotate in the magnetic field when it carries an electric current by electromagnetic (EM) torque effect. A pointer which attached the movable coil will deflect according to the amount of current to be measured which applied to the coil. The (EM) torque is counterbalance by the mechanical torque of control springs attached to the movable coil also. When the torques are balanced the moving coil will stopped and its angular deflection represent the amount of electrical current to be measured against a fixed reference, called a scale. If the permanent magnet field is uniform and the spring linear, then the pointer deflection is also linear.

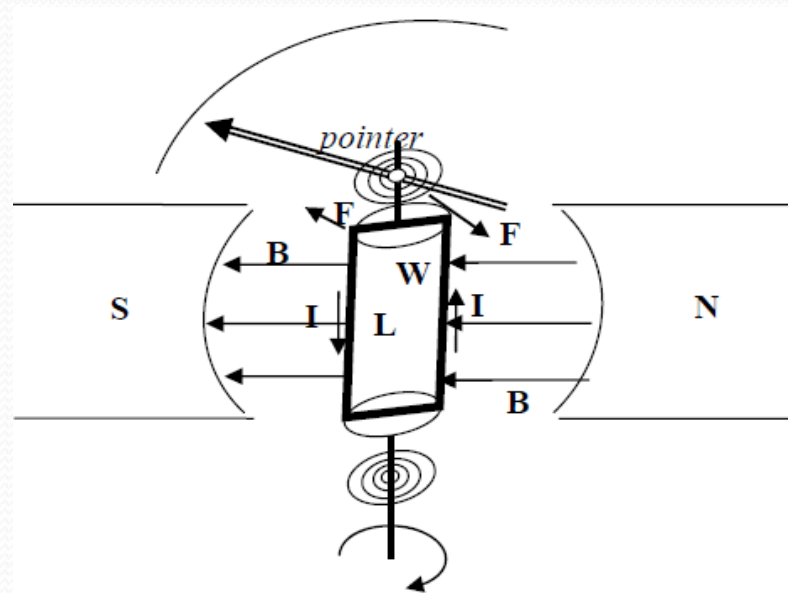
- PMMC Construction
 - D'Arsonval or horseshoe magnet
 - Core-magnet

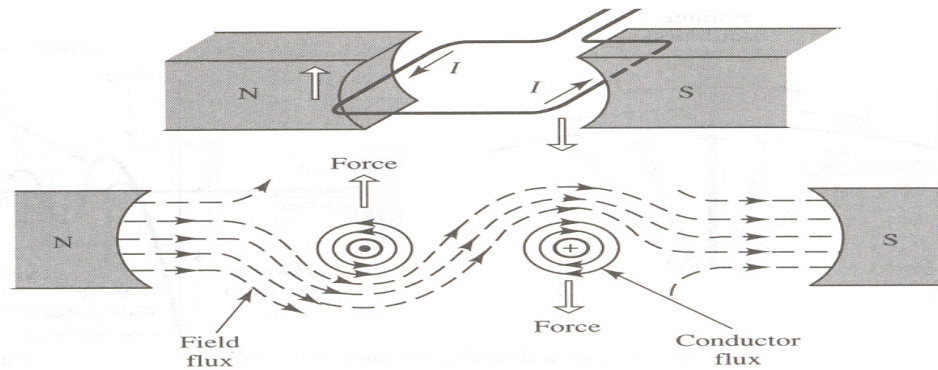


Working Principle

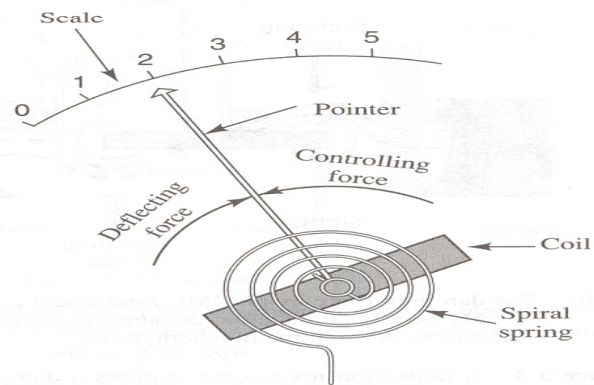
Deflecting force

It causes the pointer to move from its zero position when a current flows in the coil; the current sets up a magnetic field that interacts with the field of the permanent magnet.





(a) The deflecting force in a PMMC instrument is provided by a current-carrying coil pivoted in a magnetic field.



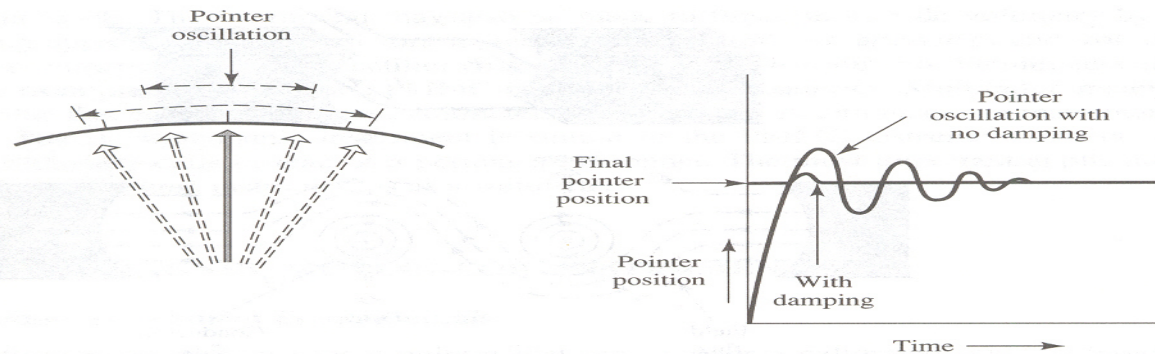
(b) The controlling force from the springs balances the deflecting force.

Figure 3-1 The deflecting force in a PMMC instrument is produced by the current in the moving coil. The controlling force is provided by spiral springs. The two forces are equal when the pointer is stationary.

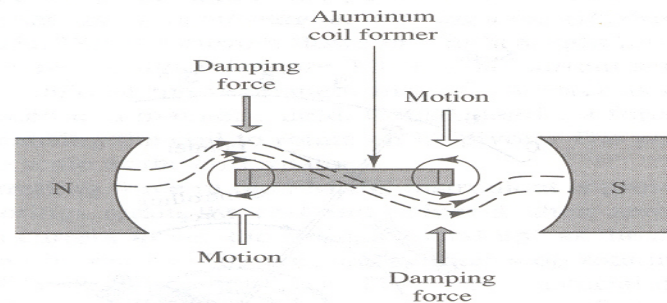


Controlling force

It is provided by spiral springs retain the coil and pointer at their zero position when no current is flowing. When current flows, the springs wind up as the coil rotates, and the force they exert on the coil increases. The coil and pointer stop rotating when the controlling force becomes equal to the deflecting force. The spring material must be nonmagnetic to avoid any magnetic field influence on the controlling force. Since the springs are used to make electrical connection to the coil, they must have a low resistance.



(a) Lack of damping causes the pointer to oscillate.



(b) The damping force in a PMMC instrument is provided by eddy currents induced in the aluminum coil former as it moves through the magnetic field.

Figure 3-2 A deflection instrument requires a damping force to stop the pointer oscillating about the indicated reading. The damping force is usually produced by eddy currents in a nonmagnetic coil former. These exist only when the coil is in motion.

- Eddy currents induced in the coil former set up a magnetic flux that opposes the coil motion, thus damping the oscillations of the coil (see Figure 3.2 (b)).



Damping force

It is required to minimize (or damp out) the oscillations. It must be present only when the coil is in motion, thus it must be generated by the rotation of the coil. In PMMC instruments, the damping force is normally provided by eddy currents.

Mathematical Representation of PMMC Mechanism

Assume there are (N) turns of wire and the coil is (L) in long by (W) in wide. The force (F) acting perpendicular to both the direction of the current flow and the direction of magnetic field is given by:

$$F = N \cdot B \cdot I \cdot L \quad \text{where } N: \text{ turns of wire on the coil} \quad I: \text{ current in the movable coil}$$

B: flux density in the air gap L: vertical length of the coil

Electromagnetic torque is equal to the multiplication of force with distance to the point of suspension

$$T_{I1} = NBIL \frac{W}{2} \quad \text{in one side of cylinder} \quad T_{I2} = NBIL \frac{W}{2} \quad \text{in the other side of cylinder}$$

The total torque for the two cylinder sides

$$T_I = 2 \left(NBIL \frac{W}{2} \right) = NBILW = NBA \quad \text{where } A: \text{ effective coil area}$$

This torque will cause the coil to rotate until an equilibrium position is reached at an angle θ with its original orientation. At this position

Electromagnetic torque = control spring torque

$$T_I = T_s$$

Since $T_s = K\theta$

So $\theta = \frac{NBA}{K} I$ where $C = \frac{NBA}{K}$ Thus $\theta = CI$

The angular deflection proportional linearly with applied current

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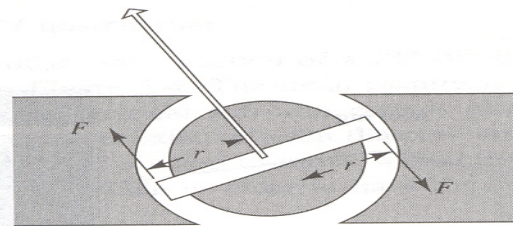
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The angular deflection proportional linearly with applied current

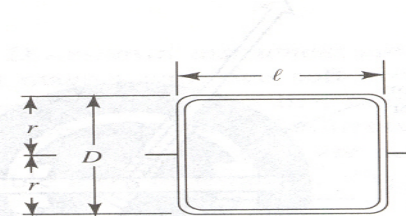
- Torque Equation and Scale

When a current I flows through a one-turn coil situated in a magnetic field, a force F is exerted on each side of the coil

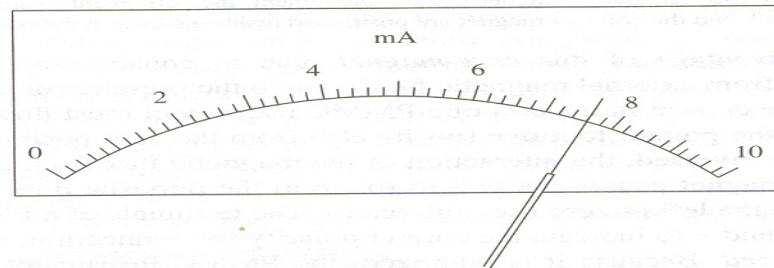
$$F = BIl \quad \text{newtons}$$



(a) Force F acts on each side of the coil



(b) Area enclosed by coil is $D \times l$



(c) Linear scale on a PMMC instrument



Since the force acts on each side of the coil, the total force for a coil of N turns is

$$F = 2 B I N r$$

The force on each side acts at a radius r , producing a deflecting torque:

$$\begin{aligned} T_D &= 2 B I N r \quad = B I N (2 r) \\ &= B I N D \\ &= B A I N \end{aligned}$$

The controlling torque exerted by the spiral springs is directly proportional to the deformation or windup of the springs. Thus, the controlling torque is proportional to the actual angle of deflection of the pointer.

$$T_c = K \theta \quad \text{where} \quad K \text{ is a constant}$$

For a given deflection, the controlling and deflecting torque are equal:

$$K \theta = BIIND$$

$$\theta = CI \quad \text{where } C \text{ is a constant}$$

Example A PMMC instrument with a 100-turn coil has a magnetic flux density in its air gaps of $B = 0.2 \text{ T}$. The coil dimension are $D = 1 \text{ cm}$ and $l = 1.5 \text{ cm}$. Calculate the torque on the coil for a current of 1 mA .

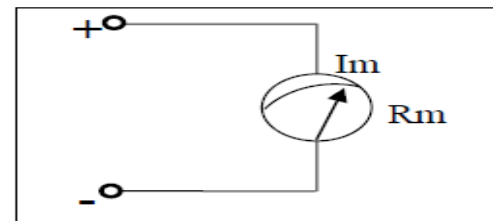
Solution

$$\begin{aligned} T_d = BIIND &= (0.2T)(1.5 \times 10^{-2})(1 \times 10^{-3})(100)(1 \times 10^{-2}) \\ &= 3 \times 10^{-6} \text{ Nm} \end{aligned}$$

Extension of Ranges

1- D.c Ammeter:

An Ammeter is always connected in series with a circuit branch and measures the current flowing in it. Most d.c ammeters employ a d'Arsonval movement, an ideal ammeter would be capable of performing the measurement without changing or distributing the current in the branch but real ammeters would possess some internal resistance.



Extension of Ammeter Range:

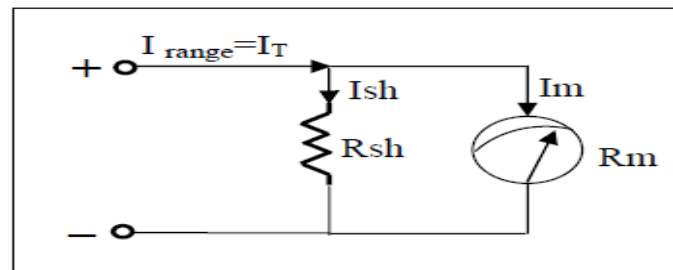
Since the coil winding in PMMC meter is *small and light*, they can carry only small currents (μA - 1mA). Measurement of large current requires *a shunt external resistor* to connect with the meter movement, so only a fraction of the total current will pass through the meter.

$$V_m = V_{sh}$$

$$I_m R_m = I_{sh} R_{sh}$$

$$I_{sh} = I_T - I_m$$

$$R_{sh} = \frac{I_m R_m}{I_T - I_m}$$



Example:

If PMMC meter have internal resistance of 10Ω and full scale range of 1mA . Assume we wish to increase the meter range to 1A .

Sol.

So we must connect shunt resistance with the PMMC meter of

$$R_{sh} = \frac{I_m R_m}{I_T - I_m}$$

$$R_{sh} = \frac{1 \times 10^{-3} \cdot 10}{1 - 1 \times 10^{-3}} = 0.01001\Omega$$

Extension of Ranges

A voltmeter is always connect in parallel with the element being measured, and measures the voltage between the points across which its' connected. Most d.c voltmeter employ PMMC meter with series resistor as shown. The series resistance should be much larger than the impedance of the circuit being measured, and they are usually much larger than R_m .

$$R_s = R_T - R_m$$

$$R_s = \frac{V_{range}}{I_m} - R_m$$

$$I_m = I_{FSD}$$

The ohm/volt sensitivity of a voltmeter

Is given by:

$$S_v = \frac{R_m}{V_{FSD}} = \frac{1}{I_{FSD}} = \frac{\Omega}{V} \text{ rating}$$

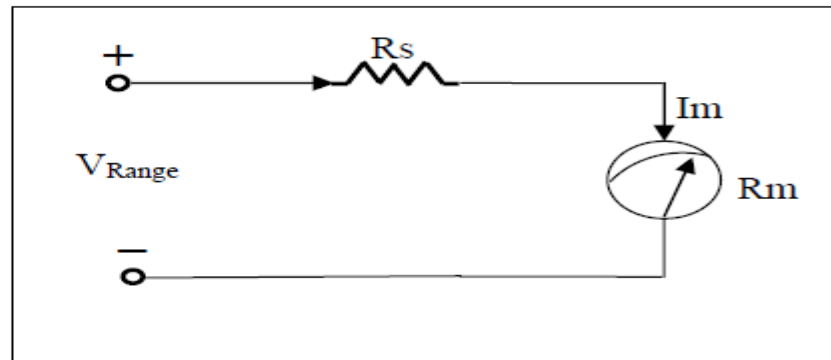
$$S_{Range} = \frac{R_m + R_s}{V_{Range}} = \frac{1}{I_{Range}} = \frac{\Omega}{V}$$

So the internal resistance of voltmeter or the input resistance of voltmeter is

$$R_v = V_{FSD} \times \text{sensitivity}$$

Example:

We have a micro ammeter and we wish to adapted it so as to measure 1 volt full scale, the meter has internal resistance of 100Ω and I_{FSD} of $100\mu A$.



Sol.:

$$R_s = \frac{V}{I_m} - R_m$$

$$R_s = \frac{1}{0.0001} - 100 = 9900\Omega = 9.9K\Omega$$

So we connect with PMMC meter a series resistance of 9.9KΩ to convert it to voltmeter



Advantages and Limitations

Advantages:

- The scale is uniformly divided.
- The power consumption can be made very low ($25200W$ to $W\mu\mu$).
- The torque-weight ratio can be made high with a view to achieve high accuracy.
- A single instrument can be used for multi range ammeters and voltmeters.
- Error due to stray magnetic field is very small.

Limitations:

- They are suitable for direct current only.
- The instrument cost is high.
- Variation of magnet strength with time.



Sources of errors

The Errors are due to:

Frictional error

Magnetic decay

Thermo electric error

Temperature error.



Minimization of Errors

Errors can be reduced by following the steps given below:

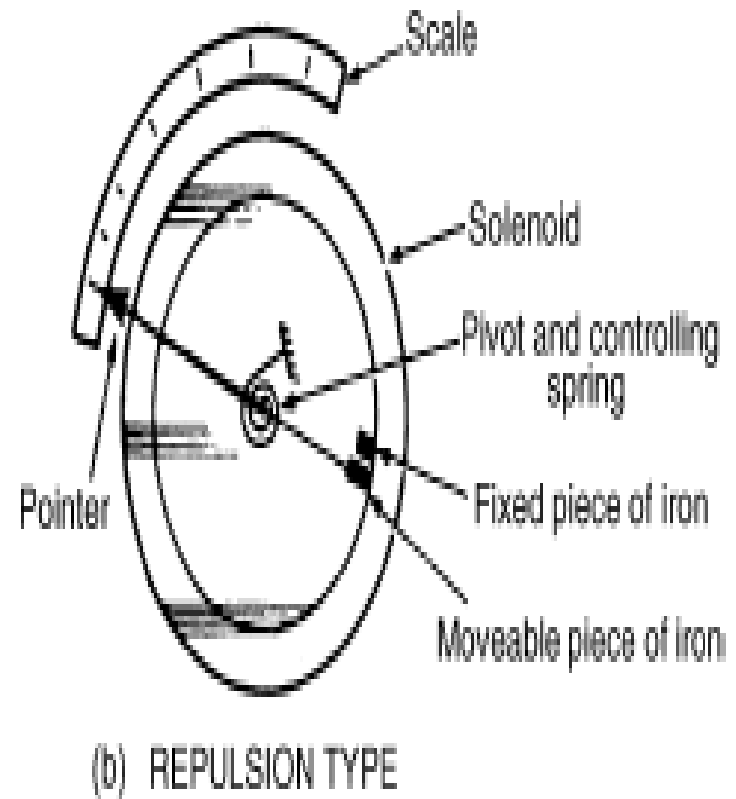
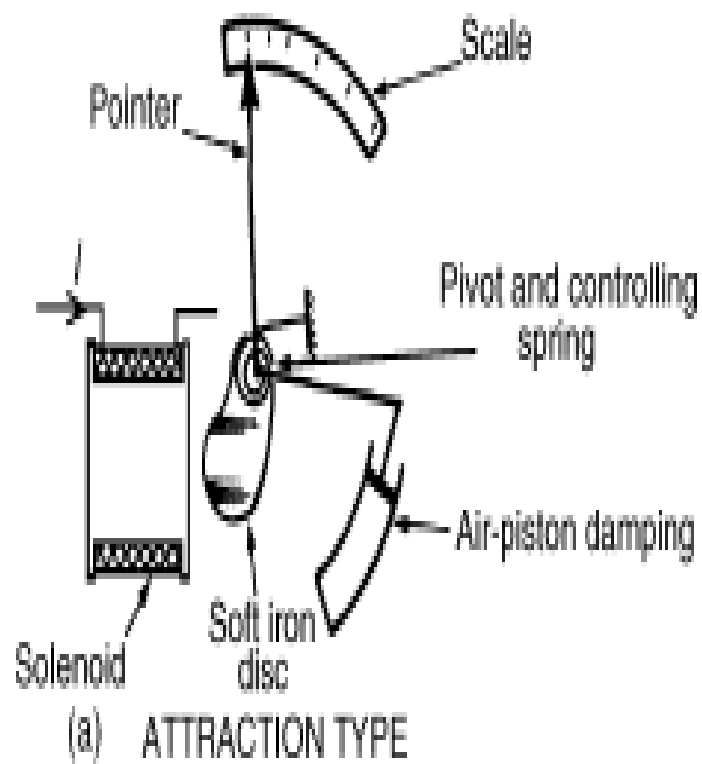
- Proper pivoting and balancing weight may reduce the frictional error.
- Suitable aging can reduce the magnetic decay.
- Use of manganin resistance in series (swamping resistance) can nullify the effect of variation of resistance of the instrument circuit due to temperature variation.
- The stiffness of spring, permeability of magnetic core (Magnetic core is the core of electromagnet or inductor which is typically made by winding a coil of wire around a ferromagnetic material) decreases with increases in temperature.



Moving Iron

The basic principle of moving-iron instruments that are generally used to measure alternating voltages and currents. In moving iron instruments the movable system consists of one or more pieces of specially-shaped soft iron, which are so pivoted as to be acted upon by the magnetic field produced by the current in coil. There are two general types of moving-iron instruments namely

- (i) Repulsion (or double iron) type
- (ii) Attraction (or single-iron) type.





Construction

The brief description of different components of a moving-iron instrument is given below-

Moving element: a small piece of soft iron in the form of a vane or rod

Coil: to produce the magnetic field due to current flowing through it and also to magnetize the iron pieces.

In repulsion type, a fixed vane or rod is also used and magnetized with the same polarity.

Control torque is provided by spring or weight (gravity).

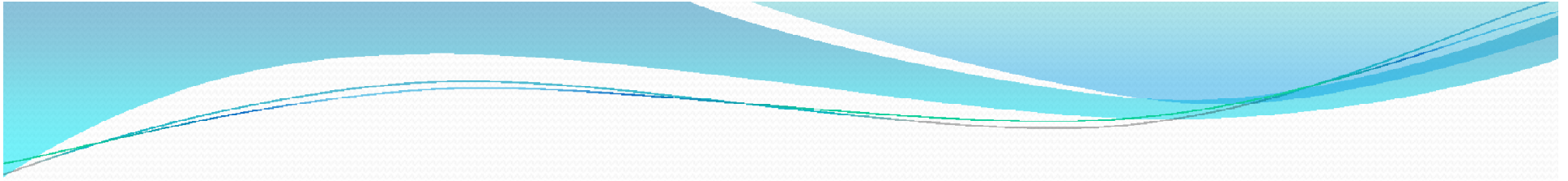
Damping torque is normally pneumatic, the damping device consisting of an air chamber and a moving vane attached to the instrument spindle.

Deflecting torque produces a movement on an aluminum pointer over a graduated scale.

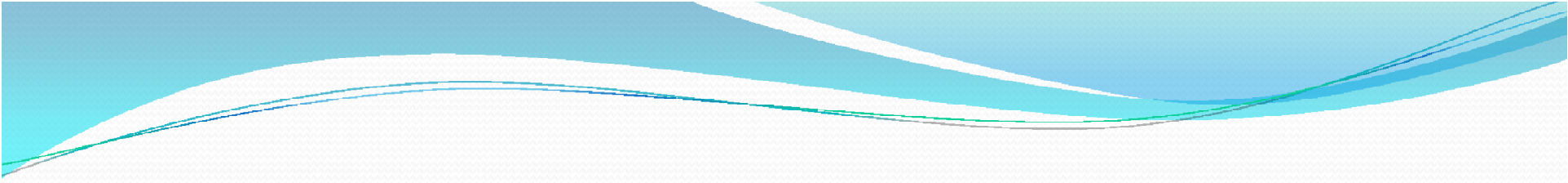


Working Principle

The deflecting torque in any moving-iron instrument is due to forces on a small piece of magnetically 'soft' iron that is magnetized by a coil carrying the operating current. In repulsion type moving-iron instrument consists of two cylindrical soft iron vanes mounted within a fixed current-carrying coil. One iron vane is held fixed to the coil frame and other is free to rotate, carrying with it the pointer shaft. Two irons lie in the magnetic field produced by the coil that consists of only few turns if the instrument is an ammeter or of many turns if the instrument is a voltmeter. Current in the coil induces both vanes to become magnetized and repulsion between the similarly magnetized vanes produces a proportional rotation.

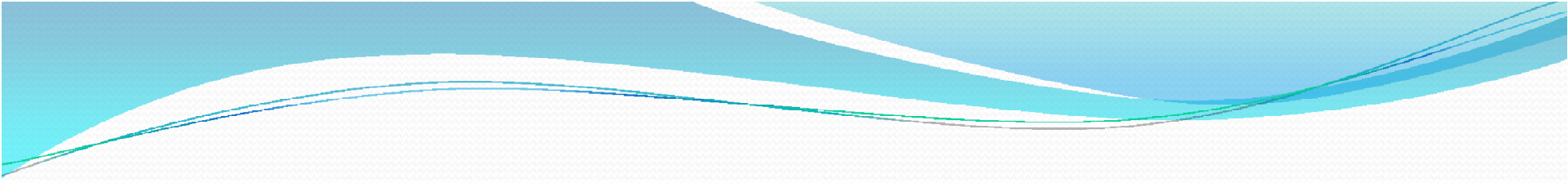


The deflecting torque is proportional to the square of the current in the coil, making the instrument reading is a true 'RMS' quantity. Rotation is opposed by a hairspring that produces the restoring torque. Only the fixed coil carries load current, and it is constructed so as to withstand high transient current. Moving iron instruments having scales that are nonlinear and somewhat crowded in the lower range of calibration. Another type of instrument that is usually classed with the attractive types of instrument is shown in figure.



This instrument consists of a few soft iron discs (B) that are fixed to the spindle, pivoted in jeweled bearings. The spindle also carries a pointer, a balance weight, a controlling weight and a damping piston, which moves in a curved fixed cylinder. The special shape of the moving-iron discs is for obtaining a scale of suitable form.

Remark: Moving-iron vanes instruments may be used for DC current and voltage measurements and they are subject to minor frequency errors only. The instruments may be effectively shielded from the influence of external magnetic fields by enclosing the working parts, except the pointer, in a laminated iron cylinder with laminated iron end covers.



Torque Expressions: Torque expression may be obtained in terms of the inductance of the instrument. Suppose the initial current is I , the instrument inductance L and the deflection θ . Then let I change to $I + dI$, dI being a small change of current; as a result let θ changes to $(\theta + d\theta)$, and L to $(L + dL)$. In order to get an incremental change in current dI there must be an increase in the applied voltage across the coil.

$$\text{Applied voltage } v = \frac{d(LI)}{dt} = I \frac{dL}{dt} + L \frac{dI}{dt} \quad (42.6)$$

The electric energy supplied to the coil in dt is

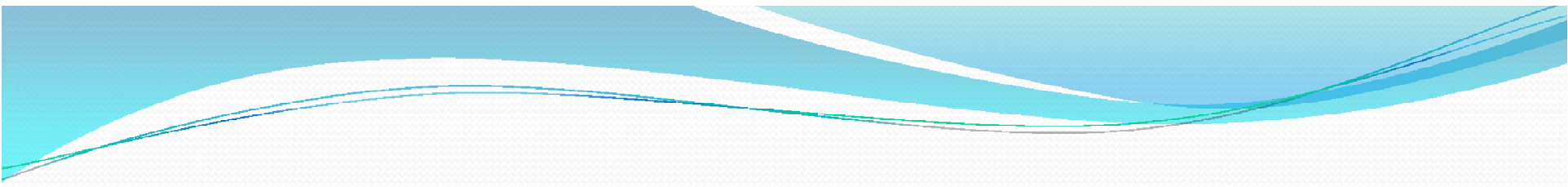
$$vI dt = I^2 dL + IL dI \quad (42.7)$$

$$\begin{aligned} \text{Increase in energy stored in the magnetic field} &= \frac{1}{2}(I + dI)^2 (L + dL) - \frac{1}{2}I^2 L \\ &\simeq IL dI + \frac{1}{2}I^2 dL \end{aligned}$$

(neglecting second and higher terms in small quantities)

If T is the value of the control torque corresponding to deflection θ , the extra energy stored in the control due to the change $d\theta$ is $T d\theta$. Then, the stored increase in stored

$$\text{energy} \simeq IL dI + \frac{1}{2}I^2 dL + T d\theta$$



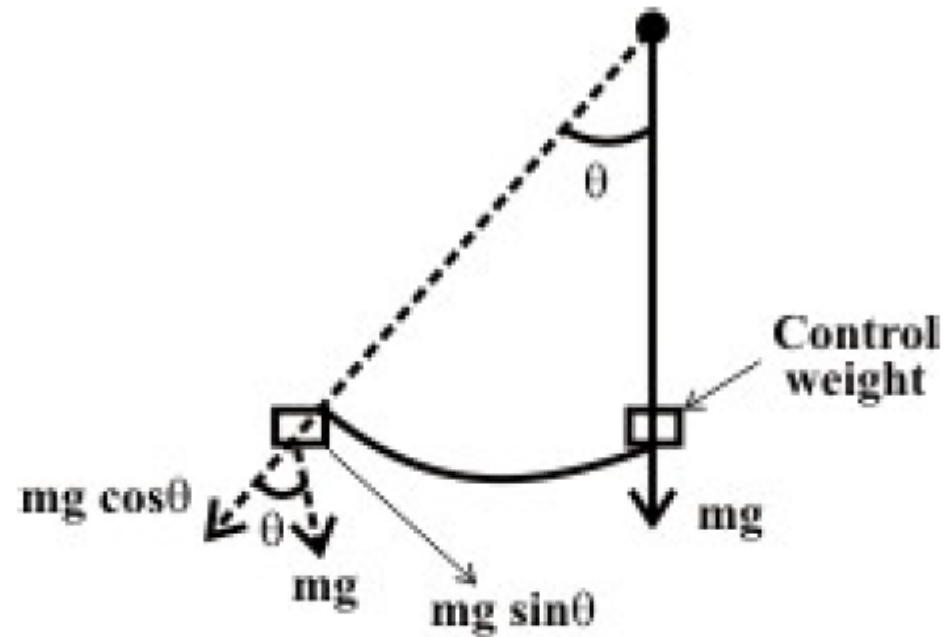
From principle of the conservation of energy, one can write the following expression
Electric energy drawn from the supply = increase in stored energy + mechanical work done

$$I^2 dL + IL dI = IL DI + \frac{1}{2} I^2 dL + T d\theta$$

$$T (\text{torque}) = \frac{1}{2} I^2 \frac{dL}{d\theta} \quad (\text{Nm}) \quad (42.9)$$

Controlling torque:

- i Spring control: $T_s = K_s \theta$ where K_s is the spring constant.
- ii Gravity control: $T_g = K_g \sin \theta$. Where $K_g = mgl$



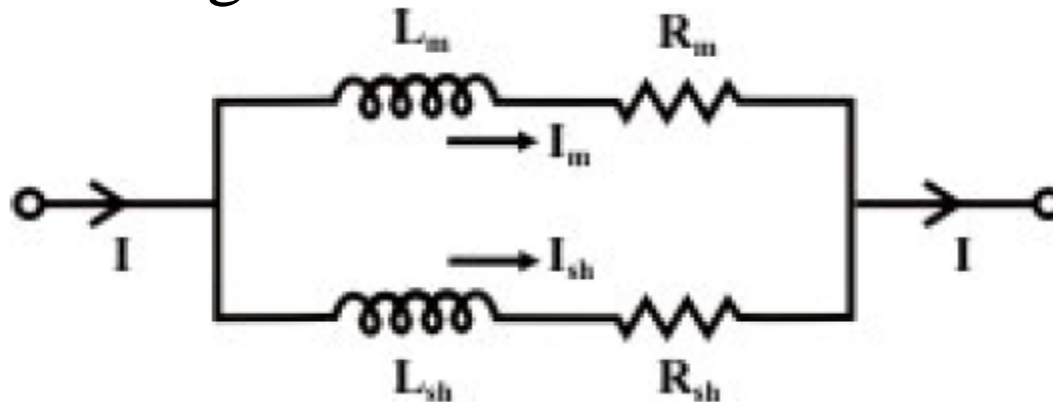
At equilibrium i.e. for steady deflection, Deflecting torque = Controlling torque. If the instrument is gravity controlled

$$T_D = T_C$$

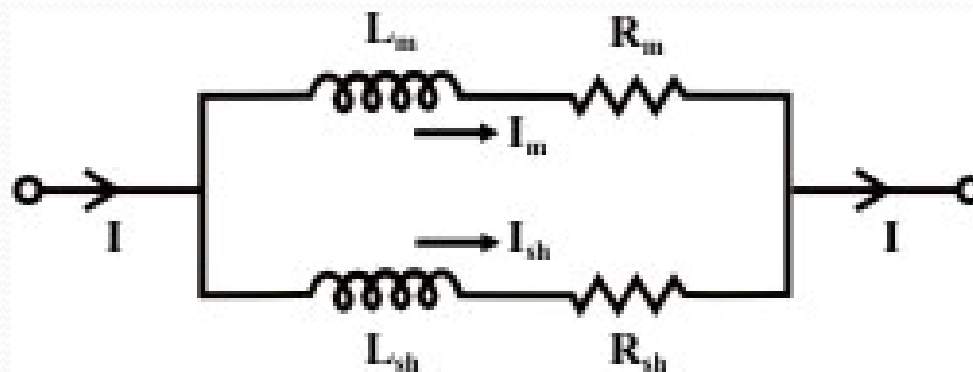
$$KI = K_g \sin \theta \Rightarrow \theta = \sin^{-1} \left(\frac{K}{K_g} I \right) \quad (42.10)$$

Extension of Ranges

For a given moving-iron instrument the ampere-turns necessary to produce full-scale deflection are constant. One can alter the range of ammeters by providing a shunt coil with the moving coil.



- Shunts and Multipliers for MI instruments
- **For moving-iron ammeters:** For the circuit shown in R and L are respectively the resistance and inductance of the coil and sh R and L the corresponding values for shunt.
- The ratio of currents in two parallel branches is shown in the figure.



$$\frac{I_{sh}}{I_m} = \frac{\sqrt{R_m^2 + (\omega L_m)^2}}{\sqrt{R_{sh}^2 + (\omega L_{sh})^2}} = \frac{R_m \sqrt{1 + \left(\frac{\omega L_m}{R_m}\right)^2}}{R_{sh} \sqrt{1 + \left(\frac{\omega L_{sh}}{R_{sh}}\right)^2}}$$

The above ratio will be independent of frequency ω provided that the time constants of the two parallel branches are same i.e $\frac{L_m}{R_m} = \frac{L_{sh}}{R_{sh}}$

In other words, $\frac{I_{sh}}{I_m} = \frac{R_m}{R_{sh}}$ if $\frac{L_{sh}}{R_{sh}} = \frac{L_m}{R_m}$

Now,

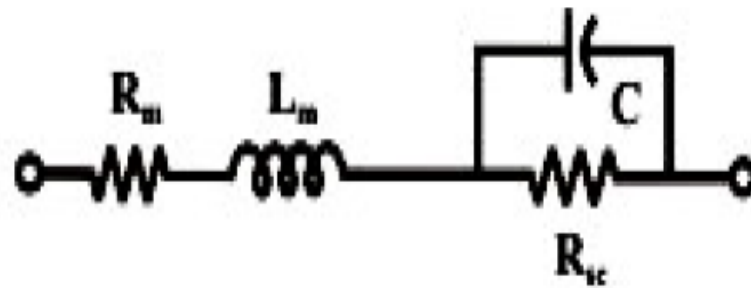
$$I = I_{sh} + I_m = I_m \frac{R_m}{R_{sh}} + I_m = I_m \left(1 + \frac{R_m}{R_{sh}}\right)$$

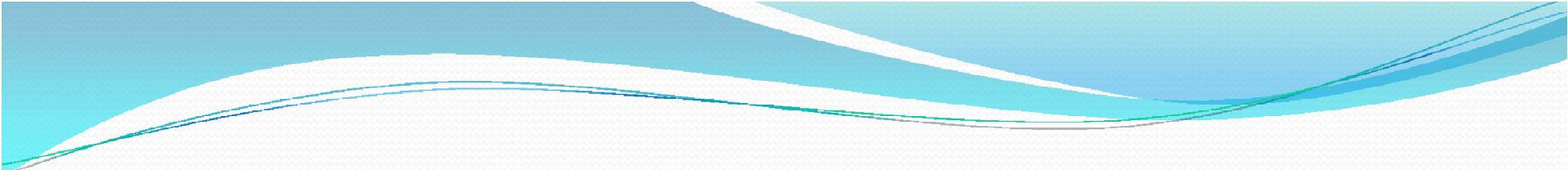
Multipliers for the shunt = $\left(1 + \frac{R_m}{R_{sh}}\right)$. It is difficult to design a shunt with the appropriate inductance, and shunts are rarely incorporated in moving iron ammeters. Thus the multiple ranges can effectively be obtained by winding the instrument coil in sections which may be connected in series, parallel or series-parallel combination which in turn changing the total ampere-turns in the magnetizing coil.

For moving-iron voltmeters: Voltmeter range may be altered connecting a resistance in series with the coil. Hence the same coil winding specification may be employed for a number of ranges. Let us consider a high resistance R_{se} is connected in series with the moving coil and it is shown below.



of the instrument with change of frequency introduces error in signal measurements. In order to compensate the frequency error, the multiplier may be easily shunted by the capacitor.



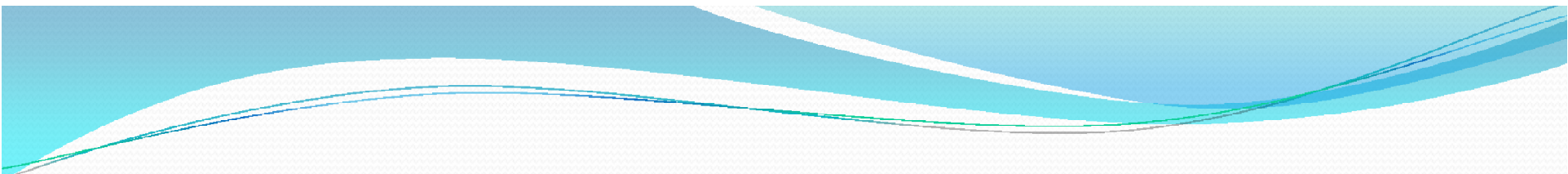


Advantages:

- The instruments are suitable for use in a.c and d.c circuits.
- The instruments are robust, owing to the simple construction of the moving parts.
- The stationary parts of the instruments are also simple.
- Instrument is low cost compared to moving coil instrument.
- Torque/weight ration is high, thus less frictional error.

Errors:

- i. Errors due to temperature variation.
- ii. Errors due to friction is quite small as torque-weight ratio is high in moving-iron instruments.
- iii. Stray fields cause relatively low values of magnetizing force produced by the coil. Efficient magnetic screening is essential to reduce this effect.
- iv. Error due to variation of frequency causes change of reactance of the coil and also changes the eddy currents induced in neighboring metal.
- v. Deflecting torque is not exactly proportional to the square of the current due to non-linear characteristics of iron material.

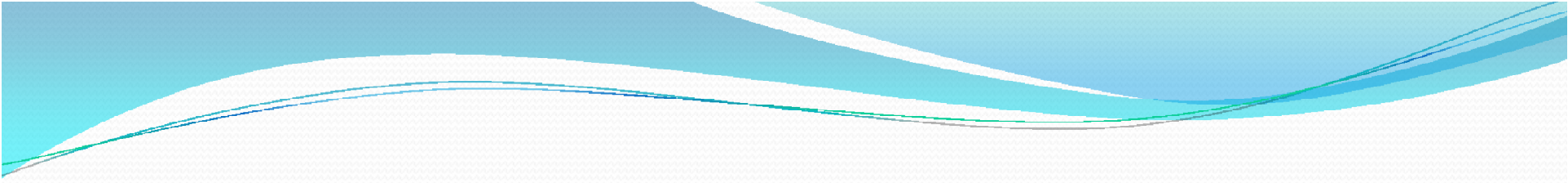


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 - $I_a R_a = I_s R_s$
 - The milliammeter is converted into a voltmeter by connecting a high value resistance (called a **multiplier**) in series with it as shown in Figure 6.3(b).
 - From Figure 10.4(b), $V = V_a + V_M = I_a R_a + I R_M$

Extension of Ranges for Moving Iron voltmeter and Ammeter

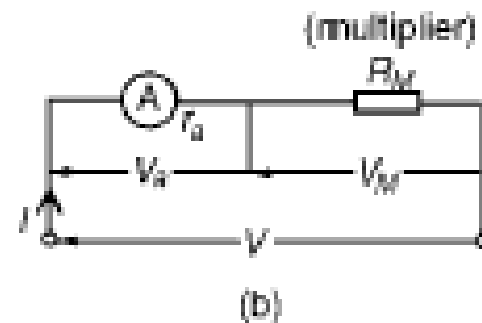
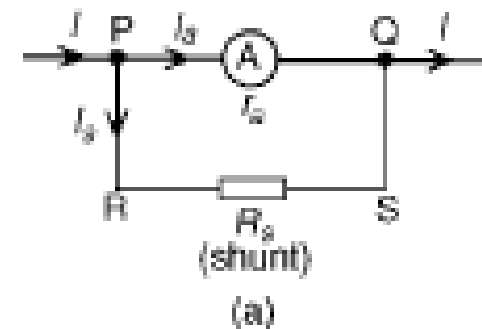
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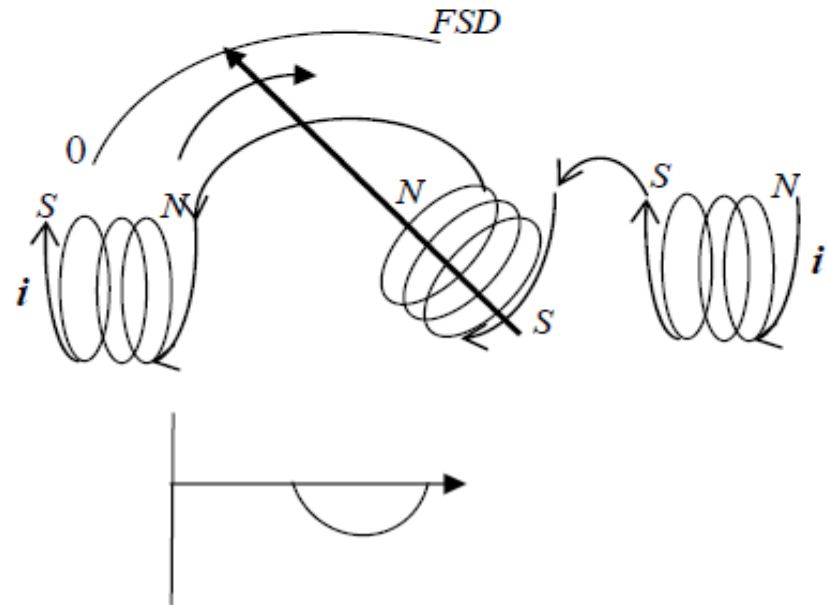
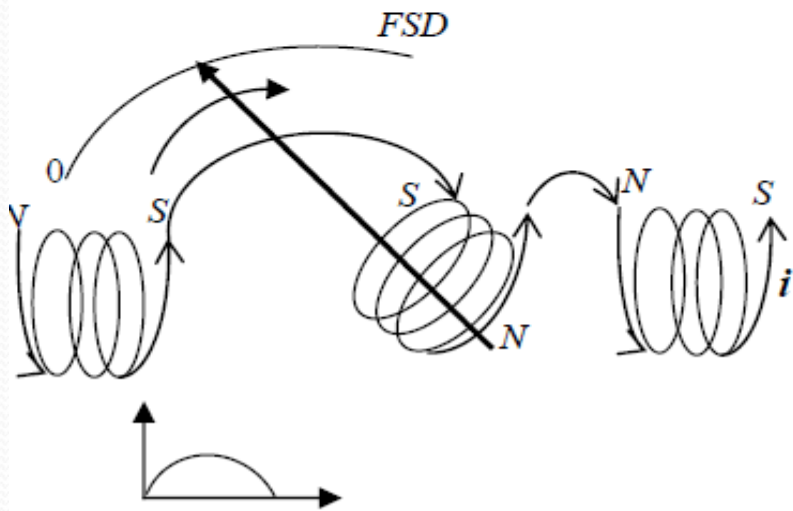
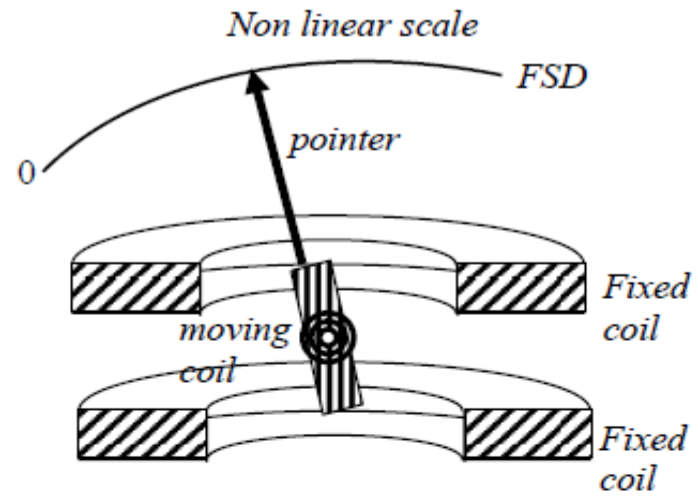
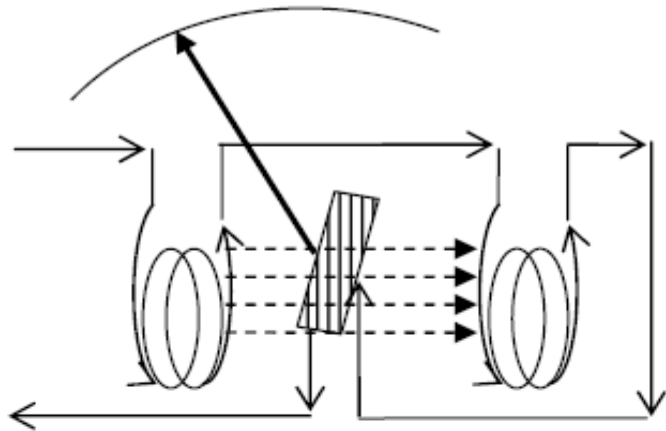
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 $= I r_a + I R_M$





Dynamometer:

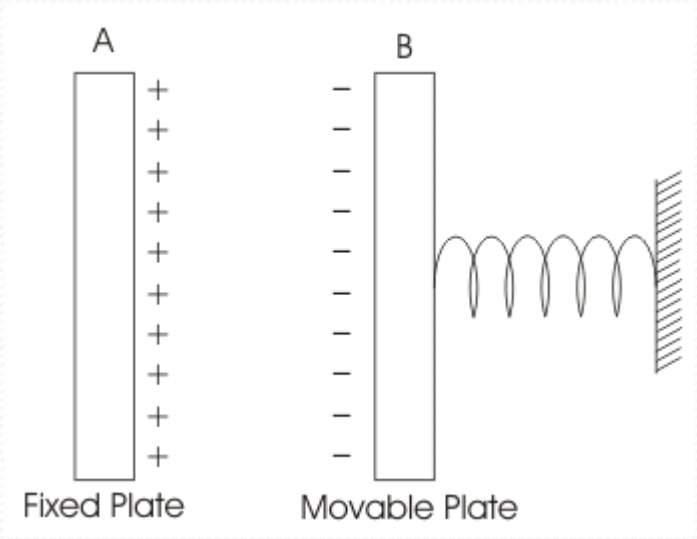
This instrument is suitable for the measurement of direct and alternating current, voltage and power. The deflecting torque in dynamometer is relies by the interaction of magnetic field produced by a pair of fixed air cored coils and a third air cored coil capable of angular movement and suspended within the fixed coil.





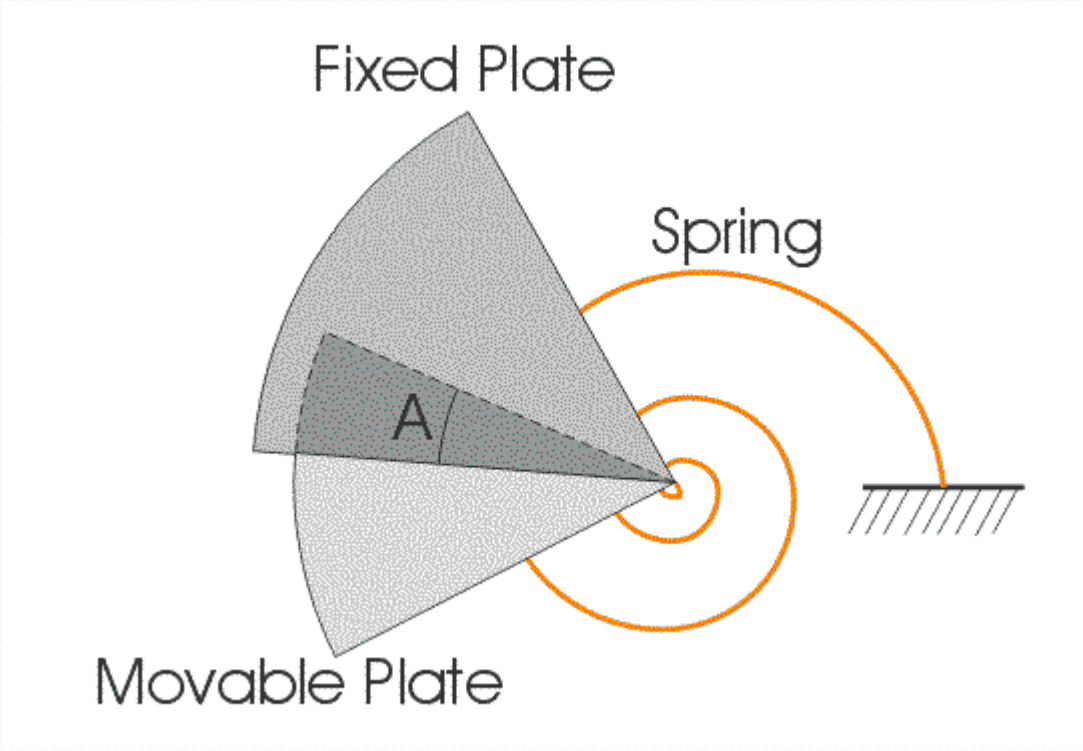
Electro static Instrument

As the name suggests the **electrostatic type instrument** use static electrical field to produce the deflecting torque. These types of instrument are generally used for the measurement of high voltages but in some cases they can be used in measuring the lower voltages and powers of a given circuit. Now there are two possible ways in which the electrostatic force can act. The two possible conditions are written below,



Construction of Electrostatic Type Instruments

- (a) When one of the plates is fixed and other plate is free to move, plates are oppositely charged in order to have attractive force between them. Now due this attractive force movable plate will move towards the stationary or fixed plate till the moving plate stored maximum electrostatic energy.
- (b) In other arrangement there may be force of attraction or repulsion or both, due to some rotary of plate.



Force & Torque Equation of Electrostatic Type Instrument

- Now let us derive **force equation for the linear electrostatic type instruments**. Let us consider two plates as shown in the diagram given below.
- Linear Electrostatic Instrument

Plate A is positively charged and plate B is negatively charged. As mentioned above as per the possible condition (a) we have linear motion between the plates. The plate A is fixed and plate B is free to move. Let us assume there exists some force F between the two plates at equilibrium when electrostatic force becomes equal to spring force. At this point, the electrostatic energy stored in the plates is



Torque Equation

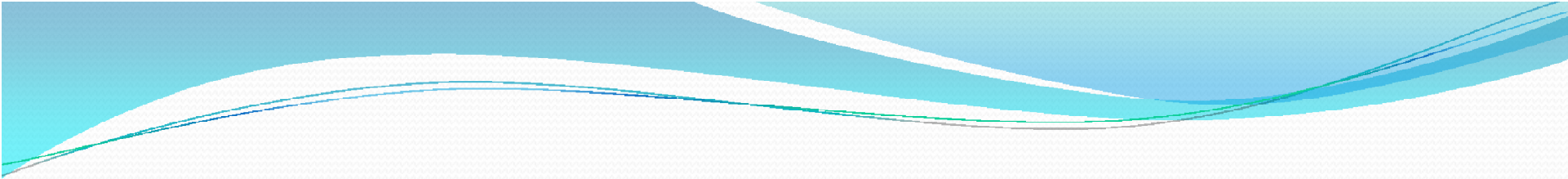
Now suppose we increase the applied voltage by an amount dV , due to this the plate B moves towards the plate A by a distance dx . The work done against the spring force due to displacement of the plate B be $F \cdot dx$. The applied voltage is related to current as

From this value of [electric current](#) the input energy can be calculated as

From this we can calculate the change in the stored energy and that comes out to be

By neglecting the higher order terms that appears in the expression. Now applying the principle of energy conservation we have input energy to the system = increase in the stored energy of the system + mechanical work done by the system. From this we can write,

From the above equation the force can be calculated as

- 
- Now let us derive force and torque equation for the rotary **electrostatic type instruments**. Diagram is shown below,
Rotary Type Electrostatic Instruments

- In order to find out the expression for deflecting torque in case of rotary type electrostatic instruments, just replace the in the equation (1) F by T_d and dx by dA . Now rewriting the modified equation we have deflecting torque is equals to

Now at steady state we have controlling torque is given by the expression $T_c = K \cdot A$. The deflection A can be written as

From this expression we conclude that the deflection of the pointer is directly proportional to the square of the voltage to be measured hence the scale will be non uniform. Let us now discuss about Quadrant electrometer. This instrument is generally used in measuring the voltage ranging from 100V to 20 kilo volts. Again the deflecting torque obtained in the Quadrant electrometer is directly proportional to the square of the applied voltage; one advantage of this is that this instrument can used to measure both the ac and dc voltages. One advantage of using the electrostatic type instruments as voltmeters is that we can extend the range of voltage to be measured. Now there are two ways of extending the range of this instrument. We will discuss them one by one.

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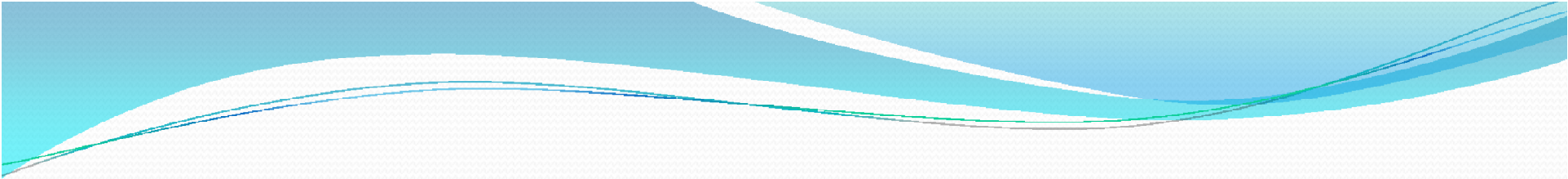
Extension of Ranges

(b) By using capacitor multiplier technique: We can increase the range of voltage to be measured by placing a series of capacitors as shown in the given circuit.

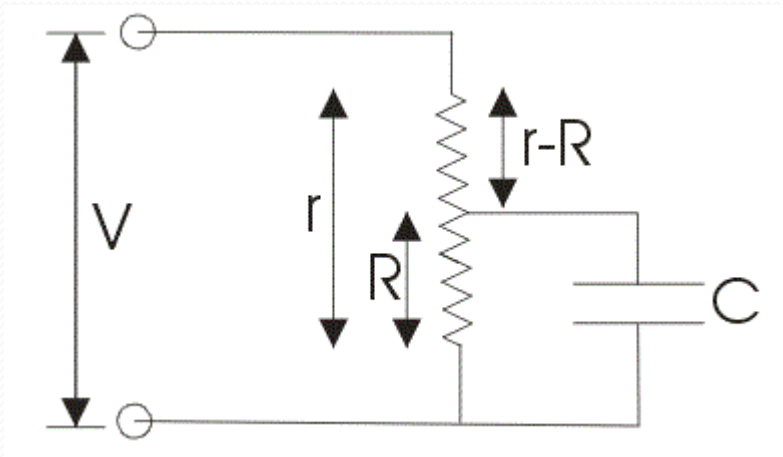
Let us derive the expression for multiplying factor for the circuit diagram 1. Let us mark the capacitance of the voltmeter be C_1 and series capacitor be C_2 as shown in the given circuit diagram. Now the series combination of these capacitor be equal to

Which is the total capacitance of the circuit. Now the impedance of the voltmeter is equal to $Z_1 = 1/j\omega C_1$ and thus total impedance will be equal to

Now the multiplying factor can be defined as the ratio of Z/Z_1 which is equal to $1 + C_2 / C_1$. Similarly the multiplying factor can also be calculated. Hence by this way we can increase the range of voltage to be measure.



From this expression we conclude that the deflection of the pointer is directly proportional to the square of the voltage to be measured hence the scale will be non uniform. Let us now discuss about Quadrant electrometer. This instrument is generally used in measuring the voltage ranging from 100V to 20 kilo volts. Again the deflecting torque obtained in the Quadrant electrometer is directly proportional to the square of the applied voltage; one advantage of this is that this instrument can be used to measure both the ac and dc voltages. One advantage of using the electrostatic type instruments as voltmeters is that we can extend the range of voltage to be measured. Now there are two ways of extending the range of this instrument. We will discuss them one by one.



Advantages of Electrostatic Type Instruments

Now let us look at some **advantages of electrostatic type instruments**.

(a) The first and the most important advantage is that we can measure both ac and dc voltage and the reason is very obvious the deflecting torque is directly proportional to the square of the voltage.

(b) Power consumption is quite low in these types of instruments as the electric current drawn by these instruments is quite low.

(c) We can measure high value of voltage.

Disadvantages of Electrostatic Type Instruments

Instead of various advantages, electrostatic instruments possess few **disadvantages** and these are written below.

- (a) These are quite costly as compared to other instruments and also these have large size.
- (b) The scale is not uniform.
- (c) The various operating forces involved are small in magnitude.

HOT-WIRE INSTRUMENTS

This instrument depends for its action on the heating and stretching of a wire due to the passing of current through it. The heating with a d.c. current is at the rate $I^2 R$, where R is the resistance of the internal hot wire. But with a.c. the heating is also at the rate $I^2 R$ provided that I is the rms current - indeed, it has been shown that rms current is defined as that a.c. current which produces the same heating as a d.c. current of the same numerical value.

Consequently a hot-wire instrument to which a.c. is applied will correctly indicate the rms value of the applied quantity.

