SCALAR ELECTRIC &
VECTOR MAGNETIC POTENTIAL
POTENTIAL

&

RETARDED POTENTIALS

Scalar Electric Potential

Coulomb's Law of Electro-Static Force

3.2. COULOMB'S LAW OF ELECTROSTATIC FORCE

Conclusions drawn by Charles Augustin De Coulombs in 1785 on the basis of expe known as **Coulomb's Law or inverse square law** which gives the force existing between tw Coulomb's Law states that **"the force (F) between two charges (Q_1 and Q_2) varies product of the charges and inversely as the square of the distance between them".**

Mathematically $$F \propto \frac{Q_1 Q_2}{r^2} = k \frac{Q_1 Q_2}{r^2} \text{ Newton}$$

where F = Force experienced, in Newton ; Q_1, Q_2 = charges, in coulombs.

r = distance between two charges Q_1 and Q_2, in metres.

k = Proportionality Constant.

$k = 1/4\pi\varepsilon$ in International system of units (SI) or rationalized M.K.S. System of

where ε is the permittivity or dielectric constant of medium in which the two charg situated and is related as

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

Here ε_0 = Permittivity of free space = 8.854×10^{-12} Farad/metre

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$$\simeq \frac{1}{36\pi \times 10^9} \text{ F/m}$$

and ε_r = Relative permittivity of the medium w.r.t. free space.

= 1 for free space or air

$\therefore$ $\varepsilon = \varepsilon_0$ .... for space or air

Hence Eqn. (3.1) can be written as

$$\boxed{F = \frac{1}{4\pi\varepsilon} \cdot \frac{Q_1 Q_2}{r^2}} \text{ Newton (in medium)}$$

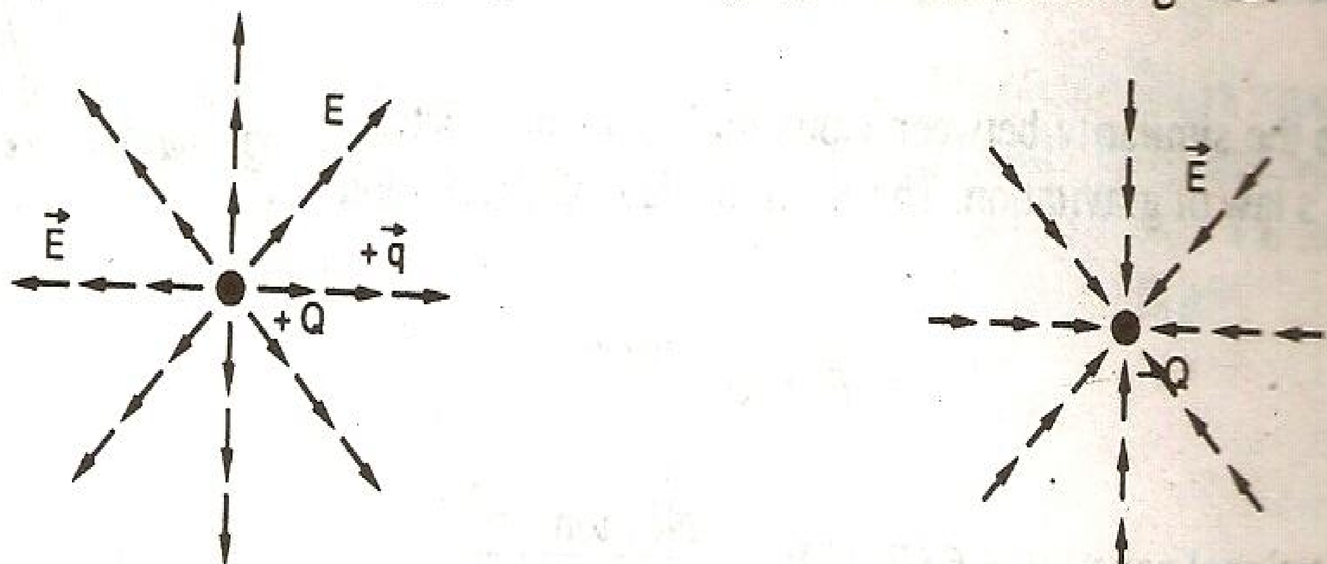
and

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2} \text{ Newtons (in air/vacuum)}$$

if we assume that medium between the two charges is vacuum or air. In equation (3.4) constant (4 in the denominator of Coulomb's Law so that the same would not appear in forth coming Maxwell's etc. This simplifies the relations in electromagnetic theory. The unit system with introduction of

3.3. ELECTRIC FIELD INTENSITY

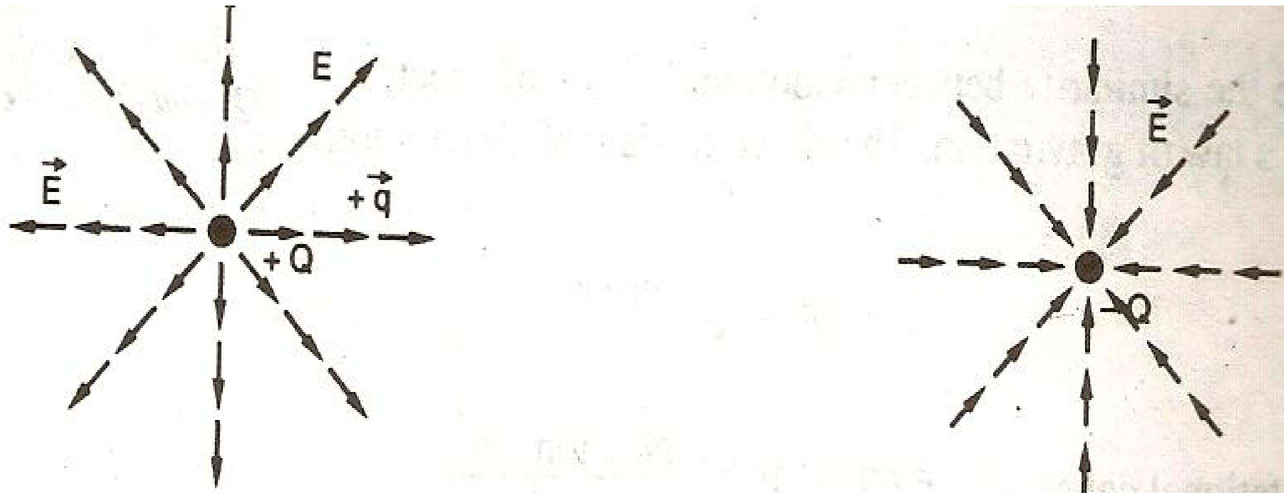
Electric field intensity or simply electric intensity or electric field is denoted by E. If a sm
probe) charge q is placed at any point near a second fix charge (Q), the probe charge q experienc
The magnitude and the direction of this will depend upon the location of the probe charge (q) w.r.t. fix
Q. About the charge Q, there is said to be an electric field of strength E and the magnitude of E at
is measured as *force per unit charge* at that point. The direction of E is the direction of force on th
test charge along the outward radial from the positive charge Q as illustrated in Fig. 3.2.



(a) charge with positive numerical value. *(b) charge with negative numerical value.*

Fig. 3.2. Fixed change Q with vectors showing magnitude and direction of associated electric field.

Thus, the electric intensity E may be defined as "*The force per unit charge exerted on a test (*
charge in the field". It is sometimes also called as "Elect



(a) charge with positive numerical value. *(b) charge with negative numerical value.*

Fig. 3.2. Fixed change Q with vectors showing magnitude and direction of associated electric field.

Thus, the electric intensity **E** may be defined as "*The force per unit charge exerted on a test charge in the field*" . It is sometimes also called as "*Electric field strength*" and its unit is volt / metre be found by applying Coulomb's Law, Eq. 3.5. The magnitnde of the force on the test charge given by

$$F = \frac{Q \cdot q}{4 \pi \varepsilon r^2}$$

and the magnitude of the electric field intensity **E** due to fixed charge Q at test charge q is

$$E = \frac{F}{q} = \frac{Q \cdot q}{q \cdot 4 \pi \varepsilon r^2} \quad \text{or} \quad E = \frac{Q}{4 \pi \varepsilon r^2}$$

Thus from Eqn. 3.6 and 3.7, it is clear that the force on the test charge q is dependent upon the

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Thus from Eqn. 3.6 and 3.7, it is clear that the force on the test charge q is dependent upon the of the probe charge but Electric field intensity is not. Therefore, if the charge on the test charge is al approach zero, then the force per unit charge remains constant *i.e.* electric field due to fixed cha considered to exist immaterial whether test charge q is there to detect its presence or not

The direction and magnitude of electric field about a point charge ($q = 1$ for point charge) indicated by writing Eq. 3.7 in vector form *e.g.*

$$\boxed{\mathbf{E} = \frac{Q}{4 \pi \varepsilon r^2} \mathbf{a}_r}$$

where $\mathbf{a}_r$ = unit vector along the outward radial from the charge Q.

If the test charge q is made small enough, so that it may be regarded as of infinitesimal size, ultimate value of the electric field intensity at a point becomes the force ΔF on a positive test cha divided by the charge Δq with the limit taken as the charge approaches zero *i.e.*

TRIC FIELD DUE TO SEVERAL POINT CHARGES [Bang. Univ. BE (Su

a test charge q is situated at a point (say P) in the field of a single charge Q, it experi Coulomb's Law as

$$F = \frac{Q\,q}{4\,\pi\,\varepsilon\,r^2}\,\boldsymbol{a}_r \quad \text{Newtons}$$

ield intensity $\boldsymbol{E}$ is given by Eqn. 3.8 (a)

$$E = \frac{Q}{4\,\pi\,\varepsilon\,r^2}\,\boldsymbol{a}_r \ \text{v/m}$$

e several charges present, each charge will exert a force on the test charge at P, the of which is given by Eq. 3.7 (a). The resultant or total force on q is the vector sum of into account both the direction and the magnitude of the force. *Hence the electric i the vector sum of electric intensities due to each charge acting alone*. This is ca e superposition.

$Q_1, Q_2, O_3 \ldots Q_n$ be the charge located at a distance $r_1, r_2, r_3 \ldots r_n$ from the point P ntensity at point P is the given by

$$= \frac{Q_1}{4\,\pi\,\varepsilon\,r_1^2}\,\boldsymbol{a}_{r_1} + \frac{Q_2}{4\,\pi\,\varepsilon\,r_2^2}\,\boldsymbol{a}_{r_2} + \frac{Q_3}{4\,\pi\,\varepsilon\,r_3^2}\,\boldsymbol{a}_{r_3} + \ldots \frac{Q_n}{4\,\pi\,\varepsilon\,r_n^2}\,\boldsymbol{a}_{r_n}$$

$$= \frac{1}{4\pi\varepsilon}\left[\frac{Q_1}{r_1^2}\,\boldsymbol{a}_{r_1} + \frac{Q_2}{r_2^2}\,\boldsymbol{a}_{r_2} + \frac{Q_3}{r_3^3}\,\boldsymbol{a}_{r_3} + \ldots \frac{Q_n}{r_n^2}\,\boldsymbol{a}_{r_n}\right]$$

3.5. (SCALAR) ELECTRIC POTENTIAL (I.A

An electric field is a field of force. If a body being acted upon by a force is moved from o to another, work will be done on or by the body. If some point is taken as reference or zero point the force can be described by the work that must be done in moving the body from reference point upto a in the field.

A reference point that is usually used is a point at infinity. For example, if a small body havin Q and a second body with a small test charge q is moved from infinity along a radius line to a poi distance R from the charge Q, then work done (W) on the system in moving the test charge q against F is given by

$$W = -\int_{\infty}^{R} F.\, dr$$

By Coulomb's Law

$$F = \frac{Qq}{4\pi\varepsilon r^2}$$

$$\therefore \quad W = -\int_{\infty}^{R} \frac{Qq}{4\pi\varepsilon r^2}\, dr = -\frac{Qq}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_{\infty}^{R} = \frac{Qq}{4\pi\varepsilon R}$$

If the test charge is unit charge (*i.e.* $q = 1$), then work done on the test charge per unit potential V, at the point P, due to charge Q, If the test charge is unit charge (i.e. $q = 1$), then work done on the test charge per unit potential V, at the point P, due to charge Q,

$$\therefore \quad \boxed{V = \frac{Q \times 1}{4 \pi \varepsilon R} = \frac{Q}{4 \pi \varepsilon R}}$$

where $\quad V =$ Electric potential at a point P due to charge Q.

Since Electric potential has magnitude without any direction, electric potential is a scalar quantity and is usually called as the '*scalar potential*'. Hence, electric potential at a point is *defined as the work done on the test charge per unit charge in moving a charge from infinity to the point*. The unit of electric potential is volt or joules per coulomb [$\because$ 1 volt = 1 Joule/coulomb].

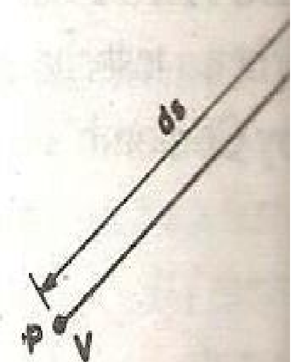

Fig. 3.3.

In case there are two points which are separated by an small distance ds, then the work done by an external force in moving an unit positive charge from one point to the another is

$$dW = V - (V + dV) = \mathbf{E}\, ds$$

or $$dv = -\mathbf{E}\, ds$$

But V is a function of x, y, z and hence 3.14 is written as

$$\delta V \qquad \delta V \qquad \delta V$$

$$dW = V - (V + dV) = \mathbf{E}\, d\mathbf{s}$$

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But V is a function of x, y, z and hence 3.14 is written as

$$\frac{\delta V}{\delta x} dx + \frac{\delta V}{\delta y} dy + \frac{\delta V}{\delta z} dz = -\mathbf{E}\, d\mathbf{s}$$

or $$\left(\frac{\delta V}{\delta x} \mathbf{a}_x + \frac{\delta V}{\delta y} \mathbf{a}_y + \frac{\delta V}{\delta z} \mathbf{a}_z\right)(\mathbf{a}_x\, dx + \mathbf{a}_y\, dy + \mathbf{a}_z\, dz) = -\mathbf{E}\, d\mathbf{s}$$

or $$\nabla V . d\mathbf{s} = -\mathbf{E}\, d\mathbf{s}.$$

or $$\boxed{\mathbf{E} = -\nabla V = -\text{Grad } V}$$

Hence electric field intensity at any point is the negative of the potential gradient at that point. the direction of the electric field is the direction in which the *gradient* is greatest on Eqn. 3.15. Grad gradient of V and may also be represented with ∇ the del operator or Nabla as ∇V.

3.6. ELECTRIC CHARGE DENSITY (ρ) AND CONTINUOUS DISTRIBUTION OF CHA

The electric charge density (ρ) is the ratio of total charge Q in a volume V, to volume V i.

$$\rho = \frac{Q}{V} \quad \text{..}$$

e dimensions of charge per unit volume and its unit in SI unit is the couiomo per cubi
$^{-3}$.

ectric charge is continuously distributed throughout a region, then charge density at an
ΔQ in a small volume element Δv divided by the volume, with the limit of this rati
hrinks to zero around the point P or symbolically,

$$\rho = \lim_{\nabla V \to 0} \frac{\Delta Q}{\Delta v} \quad \text{...}$$

med here that the electric charge is continuously distributed but in fact it is not and is ass
rticles *e.g.* electrons or atoms which are *separated* by finite atomic distances. The
above is also sometimes called as *volume charge density* (ρ_v).

y, when the charge is distributed continuously over a surface, then the *surface charge*
is defined as the charge per unit area and its unit is coulomb per square metre. Hen

$$\rho_s = \lim_{\Delta S \to 0} \frac{\Delta Q}{\Delta s} \quad \text{...}$$

med that charge is continuously distributed over a surface.

, when the charge is continuously distributed along a length instead of a surface or v
charge density (ρ_L) is used. This is defined as **the charge per unit length and its**
etre. Hence

$$\rho_L = \lim_{\Delta L \to 0} \frac{\Delta Q}{\Delta L} \quad \text{...}$$

The negative sign indicates that the charge is attractive towards plate. Similarly there are problems which can be solved by image method.

3.29. POISSON'S EQUATION AND LAPLACE'S EQUATION

Besides divergence operator, there is another Laplacian (Laplah-ci-an) operator. Eqn. 3.71 i between the flux density ***D*** and the charge density ρ that exist in the region.

Thus $$\nabla \cdot D = \rho$$

But $$D = \varepsilon E$$

∴ $$\nabla \cdot (\varepsilon E) = \rho$$

If the region is homogeneous and isotropic, the dielectric const or permittivity ε wil quantity, and hence.

$$\varepsilon \nabla \cdot E = \rho$$

But *E*

∴ $$-\varepsilon \nabla \cdot (\nabla V) = \rho$$

or $$\boxed{\nabla^2 V = -\frac{\rho}{\varepsilon}}$$

This Eqn. is known as **Poisson's equation and is useful in vacuum tubes and gaseous problems particularly.**

The divergence of a gradient (the double operator) is written as ∇^2 (del square) and is ca *Laplacian operator*.

In free space when there is no charge (*i.e.* ρ = 0), above eqn. becomes

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$$\boxed{\nabla^2 V = 0}$$

This eqn. is known as *Laplace's equation*.

Expanding equation 3.174 in rectangular co-ordinate, we get,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Further when $\rho = 0$, then eqn. 3.74.

$$\nabla \cdot D = 0$$

or

$$\nabla \cdot \varepsilon E = 0$$

or

$$\nabla \cdot E = 0$$

Laplace's eqn. is of great importance in electromagnetic theory. Eqn. 3.174 is special case eqn. for charge free regions but eqns. 3.175 and 3.176 are the alternative forms.

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3.30. CAPACITOR

A capacitor (also called as condenser formerly) is an electric device having two conductor: by an insulator or dielectric medium. The capacitance of a capacitor is defined as **the ratio of the one of its conductors to the potential difference between them.** Symbolically the capacitance of is given by

$$\boxed{C = \frac{Q}{V} \text{ Coulombs/volt or Farad}}$$

If $V = 1$ volt, $Q = 1$ coulomb. Then $C = 1$ Farad

Hence, capacitance of a capacitor is one Farad, if charge stored is one coulomb with difference of one volt. In practice lower value *i.e.* microfarad (*i.e.* 10^{-6} Farad) and micro-micr 10^{-12} Farad) or Pico-Farad is used as Farad is a larger capacitance.

Vector Magnetic Potential

4.13. BIOT-SAVART'S LAW (AMIE,

This deals with the magnetic field of current carrying element. The magnetic flux density by a current element ($I\,dl$) at any point in space or in any medium where the magnetic field is current element is governed by Biot-Savart's Law.

Let the aligning torque on an arbitrarily small perfectly mounted magnetic needle be use the field B produced by an incremental current carrying element of Δl, shown in Fig. 4.1 measurement, it is found that the incremental B is a function of I, Δl, r and θ and is given by

$$\Delta B = K \frac{I \Delta l \sin \theta}{r^2}$$

where K is proportionality constant and $= \frac{\mu}{4\pi}$ *i.e.*

$$K = \frac{\mu}{4\pi}$$

where μ is the permeability and its unit is that of inductance divided by length *i.e.* Henry/metre. permeability is given by

$$\mu = \mu_0 \mu_r$$

where μ_0 = Permeability in vacuum $= 4\pi \times 10^{-7}$ H/m

μ_r = Relative permeability w.r.t. vacuum or free space.

Putting Eqn. 4.45 into Eqn. 4.44 and writing infinitesimals instead of incrementals we fundamental relation as

$$dB = \frac{\mu}{4\pi} \frac{Idl \sin \theta}{r^2}$$

The direction of dB is perpendicular to the page inward at the point P.

$$dB = \frac{\mu}{4\pi} \frac{Idl \sin\theta}{r^2}$$

The direction of dB is perpendicular to the page inward at the point P.

In order to find the value of $\mathbf{B}$ at a point P due to a current I in a long straight or curved placed in the plane of page as illustrated in Fig. 4.16 it is assumed that the conductor is made of ele segments of infinitesimal length dl, all connected in series. The total flux density $\mathbf{B}$ at the point P is sum of the contributions from all these elements and is expressed by the integral of Eqn. 4.47. Hence

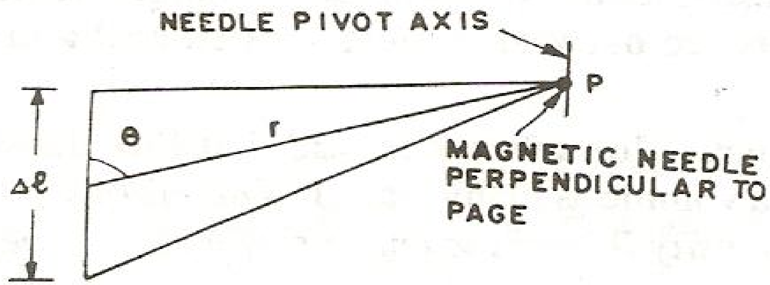

Fig. 4.15. Measurement of B produced by short current-carrying element Δ l as a function of radius r, angle θ, current I and length Δ l.

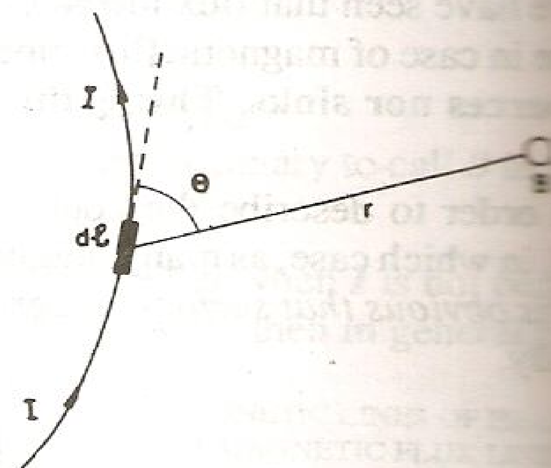

Fig. 4.16. Calculation of flux density B at a point P due to current I in a long conductor.

$$B = \int dB = \frac{\mu I}{4\pi} \int \frac{\sin\theta}{r^2}\, dl$$

or

$$B = \frac{\mu I}{4\pi} \int \frac{\sin\theta\, dl}{r^2}$$

= Flux density at P, in T.

= Permeability of the medium.

= Current in conductor, A.

= Length of current element, in m.

= Distance from element dl to P, in metre

= Angle measured clockwise from positive direction of current along dl to the direction of radiu vector r extending from dl to P.

ntegration in Eqn. 4.48 is done over the entire length of the conductor. Eqns. 4.47 and 4.48 a of the Biot-Savart law.

NETIC FIELD OF A LINEAR CONDUCTOR OF INFINITE LENGTH

eometry of the infinite linear conductor and the field produced by it, a distance r from it, 4.17.

...ation of B (magnetic flux density) near a long straight conductor.

current (I) is in the direction shown then the magnetic flux density $\boldsymbol{B}$ at a distance r fr

ard in to the page shown. From Fig. 4.17, APC,

$$\frac{R}{r} = \sin\theta \quad \text{or} \quad R = r\sin\theta \qquad \dots .4.49$$

rom the Fig. 4.18 which is redrawn from the Fig. 4.17 in the amplified form.

$$\frac{\text{Arc}}{\text{radius}} = \text{Angle}$$

$$\frac{DE}{r} = d\theta$$

$$DE = dl\sin\theta$$

$$\frac{dl\sin\theta}{r} = d\theta$$

$$dl\sin\theta = r\,d\theta \qquad \dots 4.49$$

by the Biot-Savart law eqn. 4.48, $\boldsymbol{B}$ at point P is given by

$$B = \frac{\mu I}{4\pi}\int_0^{\pi}\frac{\sin\theta\,dl}{r^2} \qquad \dots (4$$

$$B = \frac{\mu I}{4\pi}\int_0^{\pi}\frac{r\,d\theta}{r^2} \qquad \text{[By Eqn. 4.49}$$

$$B = \frac{\mu I}{4\pi}\int_0^{\pi}\frac{d\theta}{r} \qquad \dots (4$$

The integration is taken between $\theta = 0$ to π for the entire length of infinite conductor. On inte after putting the value of Eqn. 4.49 (a), we get

$$B = \frac{\mu I}{4\pi}\int_0^{\pi} \frac{\sin\theta}{R}\, d\theta = \frac{\mu I}{4\pi R}\Big[-\cos\theta\Big]_0^{\pi} = \frac{-\mu I}{4\pi R}[-1-1] = +\frac{\mu}{4\pi R}[2]$$

$$\boxed{B = \frac{\mu I}{2\pi R} \text{ webre/m}^2}$$

where B = Magnetic flux density.

μ = Permeability of the medium H/m.

I = Current in the conductor, in A.

R = Perpendicular distance in metre.

If the conductor is of finite length and making angles θ_1 and θ_2 at point P from their ends the Eqn. 4.48, we will have

$$B = \frac{\mu I}{4\pi}\int_{\theta_1}^{\theta_2} \frac{\sin\theta\, dI}{r^2} = \frac{\mu I}{4\pi R}\Big[-\cos\theta\Big]_{\theta_1}^{\theta_2}$$

[∵ limit varies from θ_1 to θ_2 instead of

$$\boxed{B = \frac{\mu I}{4\pi R}[\cos\theta_1 - \cos\theta_2]} \quad \ldots$$

This is the expression of B for a finite length of linear conductor.

4.25. MAGNETIC VECTOR POTENTIAL (P.U., B.E. EMT June/Dec. 1982, AMIE EM

The electric potential depends upon the charges which establishes the field. It is *scalar func* the *field expressed in terms* of gradient of the potential function, is not generally useful for disc magnetic field. Hence it is desirable to set up a magnetic potential the space derivative of which wo **B** or **H**, as electric field was space derivative of *V*.

Now in magnetic case, source for producing a magnetic field is *current element* whereas i electric field it is *charge*. Since the charge (having magnitude only) is a scalar quantity and so electrostatic electric potential but in the magnetic case, the *current element* is having direction and m both. Hence the potential in case of magnetic field must be *a vector potential*, the direction of whic related to the direction of current element, the source of magnetic field. Let us denote this magnet potential or usually called *simply vector potential by a vector* **A**, then it is possible to obtain **B** or **H** derivative of **A**, as *E* was obtained as space derivative of *V*.

Further since space operation of a vector quantity may be the divergence and the curl. But di of a vector is a scalar where as curl of a vector is a vector and hence curl operation is the only space d operation which can be accepted. Therefore, the vectors **H** or **B** may be derivable from a suitable r vector potential *A* through the relations.

$$\mathbf{H} = \nabla \times \mathbf{A}$$

or

$$\mathbf{B} = \nabla \times \mathbf{A}$$

out of these two alternatives, the latter is more widely used. Thus

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Now, in order to define *A*, of course, for homogeneous, isotopic media, the relation betwee vector potential A through the relations.

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out of these two alternatives, the latter is more widely used. Thus

$$B = \nabla \times A$$

Now, in order to define *A*, of course, for homogeneous, isotopic media, the relation betwee element (*I dl*) source and the magnetic vector potential *A*, must be of the type.

$$dA = k\left(\frac{Idl}{r}\right)$$

where k = constant, yet to be determined

But seeing the Biot-Savart law the definition of **B** – the definition of *A* for current element *I* guessed as

$$dA = \frac{\mu}{4\pi}\left(\frac{Idl}{r}\right) \qquad \left(\because k\right.$$

Hence the magnetic vector potential due to current flow in a entire circuit is obtained by in the vector potentials caused due to all current elements that comprise the circuit. Thus

$$\int dA = \int \frac{\mu}{4\pi}\left(\frac{Idl}{r}\right)$$

or

$$\boxed{A = \int \frac{\mu}{4\pi}\left(\frac{Idl}{r}\right)}$$

$$B = \nabla \times A$$

Now, in order to define A, of course, for homogeneous, isotopic media, the relation betwee element ($I\,dl$) source and the magnetic vector potential A, must be of the type.

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$$\int dA = \int \frac{\mu}{4\pi}\left(\frac{Idl}{r}\right)$$

or

$$\boxed{A = \int \frac{\mu}{4\pi}\left(\frac{Id\boldsymbol{l}}{r}\right)}$$

where the integration is over the complete circuit in which the current I flows. As far as direction c element is concerned, either I or dl may be made a vector quantity i.e.

$$\boxed{A = \int \frac{\mu}{4\pi}\cdot\frac{Idl}{r} = \int \frac{\mu}{4\pi}\cdot\frac{Id\boldsymbol{l}}{r}}$$

If the expression is generalized, when the current flow throughout a volume with current c then we have

$$\boxed{Id\boldsymbol{l} = \boldsymbol{J}dv}$$

...urrent element it is rather customary, although not necessary, to use $I d\boldsymbol{l}$ i...
...de of filamentary current is constant. Hence on introducing Eqn. 4.86
...A as

$$\boxed{\boldsymbol{A} = \int_v \frac{\mu \boldsymbol{J} \, dv}{4 \pi r}}$$

...he expression for the vector potential can also be used in differential fo...
...des of Eqn. 4.84.

$$\nabla \times \boldsymbol{B} = \nabla \times \nabla \times \boldsymbol{A} \qquad \because \boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \cdot ...$$

$$... (\mu \boldsymbol{H}) = \nabla \times (\nabla \times \boldsymbol{A}) = (\nabla \cdot \boldsymbol{A}) \nabla - (\nabla \cdot \nabla) \boldsymbol{A}$$

$$... (\nabla \times \boldsymbol{H}) = (\nabla \cdot \boldsymbol{A}) \nabla - \nabla^2 \boldsymbol{A}$$

$$\mu \boldsymbol{J} = (\nabla \cdot \boldsymbol{A}) \nabla - \nabla^2 \boldsymbol{A} \qquad \because \nabla \times \boldsymbol{H} = \boldsymbol{J}$$

...er to determine a vector uniquely, its divergence and its curl at all poi...
...qn. 4.84 leaves the $\nabla \cdot \boldsymbol{A}$ undertermined. Hence one can assume

$$\nabla \cdot \boldsymbol{A} = 0$$

$$\boxed{\nabla^2 \boldsymbol{A} = - \mu \boldsymbol{J}}$$

...s the differential eqn. for vector potential $\boldsymbol{A}$ and is similar to Poi...

$$\nabla^2 A = -\mu J$$

is the differential eqn. for vector potential A and is similar to Poi

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

μ = Absolute Permeability of the medium.

If Eqn. 4.88 is expanded, then

$$(\nabla^2 A_x) + a_y (\nabla^2 A_y) + a_z (\nabla^2 A_z) = -\mu (a_x J_x + a_y J_y +$$

$$\nabla^2 A_x = -J_x$$

$$\nabla^2 A_y = -J_y$$

$$\nabla^2 A_z = -J_z$$

shows that Eqns. 4.88 and 4.89, are having the same form as Poisson's Eqn.

$$\nabla^2 V = -\frac{\rho}{\varepsilon}$$

= Electrostatic potential and statisfies the Green's solution as

$$V = \frac{1}{4\pi\varepsilon} \int_V \frac{\rho \, dv}{r}$$

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ring Eqns. 4.88 and 3.173, we see

$$V \equiv \boldsymbol{A}$$

$$\rho \equiv \boldsymbol{J}$$

$$\mu \equiv \frac{1}{\varepsilon}$$

he vector potential **A** satisfy Poission Eqn. and therefore it must satisfy it
we can write the expression for **A** as

$$A_x = \frac{\mu}{4\pi} \int_v \frac{J_x\, dv}{r}$$

$$A_y = \frac{\mu}{4\pi} \int_v \frac{J_y\, dv}{r}$$

$$A_z = \frac{\mu}{4\pi} \int_v \frac{J_z\, dv}{r}$$

or combining all the three vectorially, we get

$$\boxed{\boldsymbol{A} = \int_v \frac{\mu}{4\pi} \left(\frac{\boldsymbol{J}}{r} \right) dv}$$

The unit of $\boldsymbol{A}$ is wb/m^2 or Tesla (T) because by putting $\boldsymbol{A}$ in the Eqn. 4.84 we get the $\boldsymbol{B}$.

According to eqn. 4.92, the vector potential $\boldsymbol{A}$ at a point due to a current distribution is e ratio $\boldsymbol{J}/r$ integrated over the volume occupied by the current distribution, where $\boldsymbol{J}$ is the curren each volume element dv and r is the distance from each volume element to the point P, where A being

If the current distribution is known, the vector potential A can be found. Knowing A at an flux density $\boldsymbol{B}$ at that can be calculated by taking curl of $\boldsymbol{A}$ since

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

In rectangular co-ordinate the curl of $\boldsymbol{A}$ is given by eqn. 2.96

$$\nabla \times \boldsymbol{A} = \boldsymbol{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \boldsymbol{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \boldsymbol{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

and $A = A_x \boldsymbol{a}_x + A_y \boldsymbol{a}_y + A_z \boldsymbol{a}_z$. Also in cylindrical co-ordinate $\nabla \times A$ is given by Eqn. 2.96

$$(\nabla \times A)_r = \left[\frac{1}{\,} \frac{\partial A_z}{\,} - \frac{\partial A_\varphi}{\,} \right]$$

$$B = \nabla \times A$$

In rectangular co-ordinate the curl of A is given by eqn. 2.96

$$\nabla \times A = a_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + a_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + a_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

and $A = A_x a_x + A_y a_y + A_z a_z$. Also in cylindrical co-ordinate $\nabla \times A$ is given by Eqn. 2.96

$$(\nabla \times A)_r = \left[\frac{1}{r} \frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z} \right]$$

$$(\nabla \times A)_\varphi = \left[\frac{\partial A_r}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z} \right]$$

$$(\nabla \times A)_z = \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\varphi) - \frac{\partial A_r}{\partial \varphi} \right]$$

The use of vector potential method for finding the magnetic field due to a given current convenience. On many problems of a more difficult nature the vector potential is indispensible. for this is the simplicity in evalutating the integral in Eqn. 4.92. Evaluation of Eqn. 4.92 is accomplished by evaluating separately in three rectangular coordinates.

The components B_x, B_y, B_z of magnetic field B are, therefore, can be written with Eqn. 2.96 in cartesian form as

$$B = \nabla \times A$$

$$(\nabla \times \boldsymbol{A})_r = \left[\frac{1}{r}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right]$$

$$(\nabla \times \boldsymbol{A})_\varphi = \left[\frac{\partial A_r}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right]$$

$$(\nabla \times \boldsymbol{A})_z = \frac{1}{r}\left[\frac{\partial}{\partial r}(r A_\varphi) - \frac{\partial A_r}{\partial \varphi}\right]$$

The use of vector potential method for finding the magnetic field due to a given curren convenience. On many problems of a more difficult nature the vector potential is indispensible. for this is the simplicity in evalutating the integral in Eqn. 4.92. Evaluation of Eqn. 4.92 is accomplished by evaluating separately in three rectangular coordinates.

The components B_x, B_y, B_z of magnetic field $\boldsymbol{B}$ are, therefore, can be written with Eqn. 2.96 in cartesian form as

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

or

$$B_x\, \boldsymbol{a}_x + B_y\, \boldsymbol{a}_y + B_z\, \boldsymbol{a}_z = (\nabla \times \boldsymbol{A})$$

or

$$B_x = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)$$

$$B_y = \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)$$

$$B_z = \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

RETARDED POTENTIALS

2.16 SHORT ELECTRIC DIPOLE (OR HERTZIAN DIPOLE)

A linear antenna can be regarded as a large number of very infinitesimally short cond
connected in series (end to end) and hence it is important first to consider the rad
properties of such short conductors. A short linear conductor is so short that c
may be assumed to be constant throughout its length as shown in Fig. 2.18. Thi
of short linear conductor is known as ***"Short dipole"*** or ***"Hertzian dipole"***, aft
German physicist Heinrich Hertz.

Definition. *Hertzian dipole is a hypothetical antenna and is defined as a isolated conductor carrying uniform alternating current.*

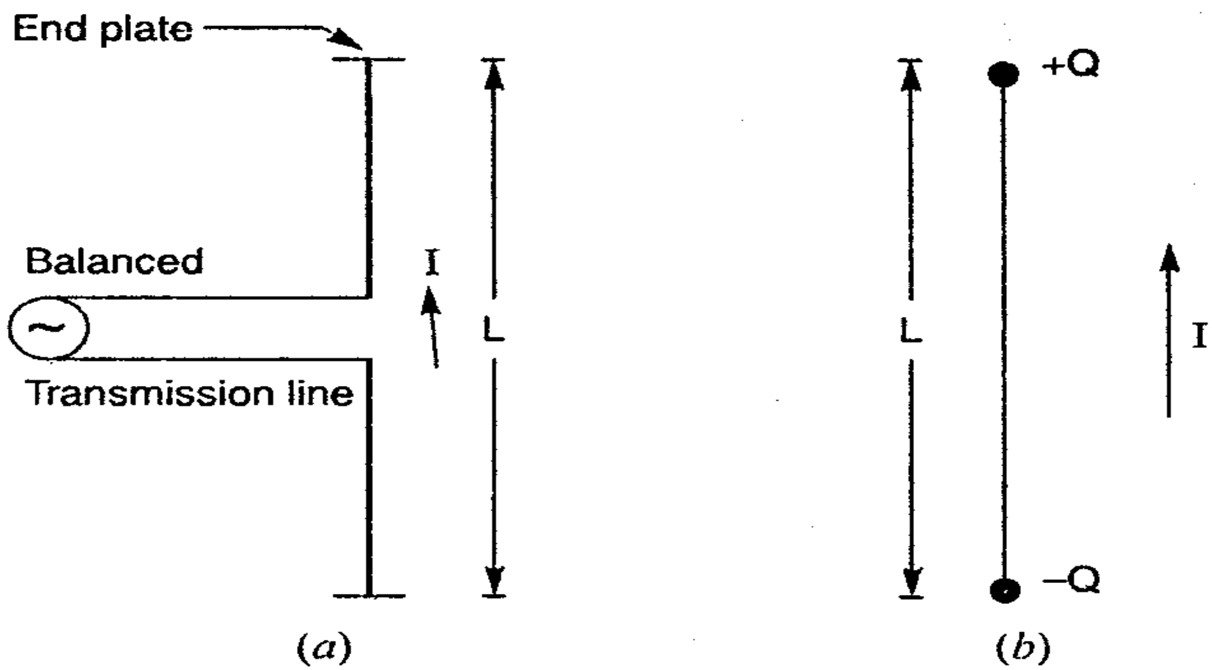

Fig. 2.18. A short dipole and its equivalent

A physical equivalent of short dipole is shown in Fig. 2.18(*b*) in which two ends of the dipole are represented by two spheres where charges are accumulated. If I be the current then it is related to charge as

$$I = \frac{dQ}{dt} \qquad \text{...(2.161)}$$

The electrically short dipole is theoretically the simplest and the most important structure. The term short dipole is commonly applied to any dipole no longer than 0.1λ. A short dipole that does not have a uniform current is known as ***Elemental dipole*** and is generally shorter than $\frac{1}{10}$ th λ. Elemental dipole are also known as *elementary dipole, elementary doublet* and *Hertzian dipole.*

When the length of the short dipole is vanishingly small, the term *infinitesimal dipole* is used. If dL be the infinitesimally small length and I be the current, then $\vec{I}\ \vec{dL}$ is called as ***current element.***

Since $I = I_0 \sin \omega t$ or $I_0 \cos \omega t$ ...(2.162*a*)

Current element $IdL = I_0\, dL \sin \omega t$ or $I_0\, dL \cos \omega t$ ...(2.162*b*)

$I\;dL$ is called as *current element*.

Since $I = I_0 \sin \omega t$ or $I_0 \cos \omega t$ ...(2.162*a*)

Current element $IdL = I_0\, dL \sin \omega t$ or $I_0\, dL \cos \omega t$ ...(2.162*b*)

Initially, a short dipole is in neutral condition. When a current (flow of electric charge) starts to flow in one direction, one half of the dipole acquires an excess charge and the other half a deficit, thereby causing a potential difference (voltage) between the two halvs of the dipole. When the current changes its direction this charge unbalance will first be neutralized and then changed.

Thus, the oscillating current will result in an oscillating voltage as well or vice-versa. If the current oscillation is sinusoidal, the voltage oscillation will also be sinusoidal and approximately 90° lagging the current in phase angle, i.e., a short dipole is capacitive in nature from current voltage relation point of view.

As electric charge oscillates in such short dipoles, they may also be called as *oscillating electric dipoles as against oscillating magnetic dipoles.*

2.17 RETARDED VECTOR POTENTIAL

If the expression for vector potential is integrated, it follows that potential due to various current elements are added up. Let the instantaneous current (I) in the elements be a sinusoidal function of time as

$$\boxed{\vec{I} = \vec{I_0} \sin \omega t} \quad \text{...(2.162)}$$

where I_0 = Maximum or peak current

I = Instantaneous current *i.e.*, current at any instant

and $\omega = 2\pi f$, the angular frequency.

The vector potential expression represents the superposition of potentials due to various current elements ($I\,dl$), at a distant point P at a distance of r. If these are simply added up, it means an *assumption is made that these field effects which are superimposed at time t, all started from the current elements of the same value of current and time,*

even though they have travelled different varying distances. In other words finite t
of propagation has been ignored which is not correct. This would have been co
provided the velocity of propagation would have been infinite which is actually

So, there is a necessity to introduce the concept of ***retardation*** or that the ef reaching a distant point P from a given element at an instant t is due to a current w which followed at an earlier time or that the current effective in producing a fiel earlier time. This time, of course, depends on the distance travelled from (dL) to P *other words finite time of propagation (or retardation time as used by Lorentz) mus taken to account.* Thus, the instantaneous current given by Eqn. (2.162) is modi now as

$$[I] = I_0 \sin \omega \left(t - \frac{r}{c} \right) \quad \text{...(2.16}$$

where r = distance travelled; c = velocity of propagation.

$[I]$ = Retarded current and the bracket is added to indicate that it is retard current

$\left(t - \frac{r}{c} \right)$ = Retarded time as phase of the wave at point P is retarded with resp to the phase of the current in the element by an angle ($\omega\, r/c$).

This equation (2.163) implies that the disturbance at time t at the distance r (po
P) from the element is caused by a retarded current $[I]$ that occured at an earlier ti
$(t - r/c)$. The time difference by an amount (r/c) is the interval needed by the disturba
to travel the distance r at the velocity at which electromagnetic wave travels i
velocity of light c.

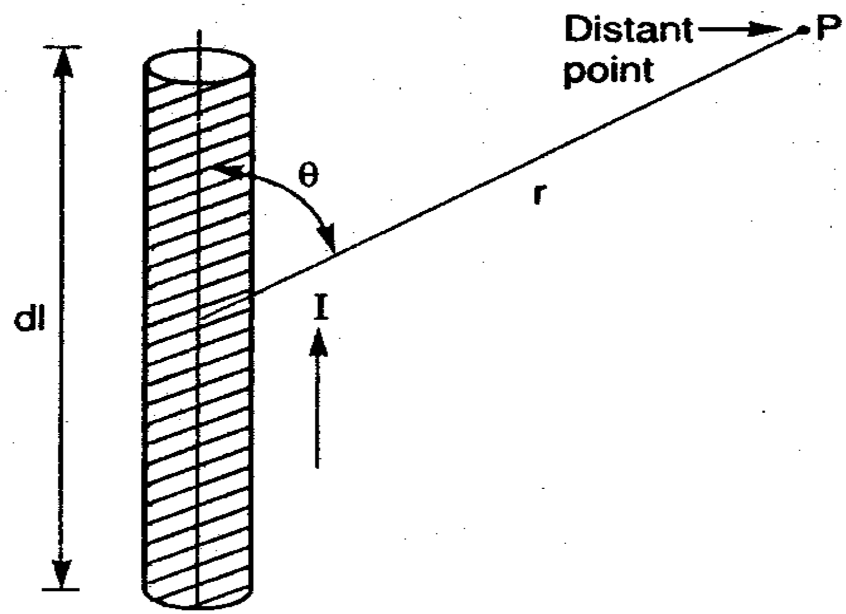

Fig. 2.19. A current carrying element

As $$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

$\therefore$ $$\sin \omega\left(t - \frac{r}{c}\right) = \sin (\omega t - \beta r) \qquad \text{...(2.16}$$

Thus, using Eqn. (2.164), equation for retarded current $[I]$, retarded current densi
$[\vec{J}]$ in exponential forms can respectively be written as

$$[\vec{I}] = \vec{I}_0 e^{j\omega\left(t-\frac{r}{c}\right)} = \vec{I}_0\, e^{j(\omega t - \beta r)} \text{ Amp.} \quad \text{...(2.165a)}$$

$$[\vec{J}] = \vec{J}_0 e^{j\omega\left(t-\frac{r}{c}\right)} = \vec{J}_0\, e^{j(\omega t - \beta r)} \text{ Amp/m}^2 \quad \text{...(2.165b)}$$

Accordingly the expressions for magnetic vector potential $\vec{A}$ when introduced with above eqns. we get **"Retarded vector potential"** which is applicable in time varying conditions where distances travelled are significant in terms of wavelength. Hence,

$$[\vec{A}] = \frac{\mu}{4\pi} \int_V \frac{[\vec{J}]}{\mathbf{r}} dv \qquad \because \vec{A} = \frac{\mu}{4\pi} \iiint \frac{\vec{J}}{r} dv$$

$$[\vec{A}] = \frac{\mu}{4\pi} \int_V \vec{J}_0\, e^{j\omega\left(t-\frac{r}{c}\right)} dv \quad \text{(exponential form)} \quad \text{...(2.166a)}$$

or

$$[\vec{A}] = \frac{\mu}{4\pi} \int_V \frac{\vec{J}\left(t-\frac{r}{c}\right)}{r} dv \quad (\textit{In general}) \quad \text{...(2.166b)}$$

For sinusoidal current element, the retarded vector potential is given by

$$[\vec{\mathbf{A}}] = \frac{\mu}{4\pi} \int \frac{\vec{\mathbf{J}}\left(t - \frac{r}{c}\right)}{r} ds \, . \, d\vec{l} \qquad \because dv = d\vec{s} \, . \, d\vec{l}$$

where ds is cross-section area and dl the length $I = \int \vec{\mathbf{J}} \, . \, d\vec{s}$

$$= \frac{\mu}{4\pi} \int \frac{\vec{\mathbf{I}}\left(t - \frac{r}{c}\right)}{r} d\vec{\mathbf{I}} = \frac{\mu}{4\pi} \int \frac{I_0 \sin \omega \left(t - \frac{r}{c}\right)}{r} d\vec{\mathbf{I}} \quad \text{...(2.167)}$$

Similarly, scalar potential into the form of retarded scalar potential is written as

$$[V] = \frac{1}{4\pi\varepsilon} \int \frac{[\rho]}{r} dv \quad \text{...(2.168a)}$$

or

$$[V] = \frac{1}{4\pi\varepsilon} \int_V \frac{\rho_0 \, e^{j\omega\left(t - \frac{r}{c}\right)}}{r} dv \quad \text{...(2.168b)}$$

where $[V]$ = Retarded scalar potential V in volts.

$[\rho] = \rho_0 \, e^{j\omega\left(t - \frac{r}{c}\right)}$ = Retarded charge density, cm^{-3}.