# Directivity or Directive Gain

# Directivity

- **Definition1:**

***The ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.***

$$D = U_{\text{given direc}} / U_{\text{av}}$$

**Definition2:**

The avg U is obtained by power radiated by 4π,(rad int of isotropic antenna)

***The ratio of its radiation intensity (U) in a given direction over that of an isotropic source.***

$$D = U/U_0$$

# Antenna Directivity

Total power radiated $P_{rad} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \varphi) \sin \vartheta d\vartheta d\varphi$

Average radiation intensity $U_{\text{avg}} = \dfrac{P_{rad}}{4\pi}$

$$D = \frac{U}{U_{avg}} = \frac{U}{P_{rad} / 4\pi}$$

- ***NOTE:*** ***D*** **Has no units**

# Antenna Directivity

- **Definition3:**

  If the direction is not specified then we calculate directivity in the direction of maxima

***The ratio of radiation intensity in max. direction to the radiation intensity of isotropic source. ($D_0$)***

$$D_{max} = D_0 = U_{max}/U_0 = 4\pi\, U_{max}/P_{rad}$$

D=directivity

Do=

U=radiation intensity

Umax=

Uo=

Prad=

P

# Properties of D

- D=1 for isotropic source
- D>1 for non isotropic

  (max. directivity is greater than 1, Umax>Uo)
- D= $\boldsymbol{4\pi / \Omega_A}$

smaller the beam area..........

# Antenna Radiation Efficiency

**Conduction and dielectric losses of an antenna ( $I^2R$ losses) = $e_{cd}$ efficiency.**

Let **$R_{cd}$** represent the actual losses due to conduction and dielectric heating. Then the efficiency is given as

$$K / e_{cd} = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{cd} + R_{rad}}$$

**Practically losses are there, so** $R_{cd} + R_{rad} > R_{cd}$

**Practical antenna: K<1**

**Ideal antenna: K=1**

# Overall Antenna Efficiency

**The overall antenna efficiency is a coefficient that accounts for all the different losses present in an antenna system.**

$$e = e_p \overbrace{e_r e_c e_d}^{e_t} = e_p \cdot e_r e_{cd}$$

$$e_p = polarization \quad mismatches$$

$$e_r = reflection \quad efficiency\,(impedance\, mismatch)$$

$$e_c = conduction \quad losses$$

$$e_d = dielectric \quad losses$$

$$e_{cd} = conductor\,\&\, dielectric \quad losses$$

**If antenna is perfectly matched, er=0**

**$e_t$ = e cd.**

# Reflection Efficiency

**The reflection efficiency through a reflection coefficient (G) at the input (or feed) to the antenna.**

$$e_r = 1 - |\Gamma|^2$$

$$\Gamma = \frac{R_{input} - R_{generator}}{R_{input} + R_{generator}}$$

$$R_{input} = antenna \quad input \quad impedance(\Omega)$$

$$R_{output} = generator \quad output \quad impedance(\Omega)$$

# Antenna Gain

- Directivity= $D = \frac{4\pi U}{P_{rad}/}$
- Gain= $D = \frac{4\pi U}{P_{in}/}$
- We know K=Prad/Pin
- G= $\frac{K\,4\pi U}{P_{rad}/}$
- So

G=KD

# Gain

- It can be measured by **comparing radiation intensity of the antenna under test (AUT) with a reference antenna.**
- Ref antenna =dipole, horn (whose gain can be calculated)

- G= U(AUT)/U(ref)

If ref antenna is isotropic

G=U(AUT)/Pin/4π

**G= 4π U (AUT)/Pin**

# Antenna Gain

- The directivity and gain are measures of the ability of an antenna to concentrate power in a particular direction.
- Directivity – power radiated by antenna ($P_0$)
- Gain – power delivered to antenna ($P_T$)

$$G = KD$$

$$K = \frac{P_{rad}}{P_{in}}$$

- K: radiation efficiency (50% - 75%)
- G has no units
  - Usually relates to the peak directivity of the main radiation lobe
  - Often expressed in dB
  - Known as "Absolute Gain" or "Isotropic Gain"

# Antennas – Gain

*Gain*

The power gain, G, of an antenna is very much like its directive gain, but also takes into account efficiency

$$G(\theta,\phi) = eD(\theta,\phi)$$

The maximum power gain

$$G_{\max} = eD_{\max}$$

The maximum power gain is often expressed in dB.

$$G_{\max}(dB) = 10\log_{10}(G_{\max})$$

*Example*

D8.3: Suppose an antenna has D = 4, $R_{rad}$ = 40 $\Omega$ and $R_{diss}$ = 10 $\Omega$. Find antenna efficiency and maximum power gain. (Ans: e = 0.80, $G_{max}$ = 3.2).

Antenna efficiency

$$e = \frac{R_{rad}}{R_{rad} + R_{diss}} = \frac{40}{10 + 40} = 0.8 \text{ (or) } 80\%$$

Maximum power gain

$$G_{\max} = eD_{\max} = (4)(0.8) = 3.2$$

Maximum power gain in dB

$$G_{\max}(dB) = 10\log_{10}(G_{\max}) = 10\log_{10}(3.2) = 5.05$$

# PFD vs. Antenna Gain

$$S(\vartheta,\varphi)=\frac{\Phi(\vartheta,\varphi)\Delta\vartheta\Delta\varphi}{(r\Delta\vartheta)(r\Delta\varphi)}=\frac{\Phi(\vartheta,\varphi)}{r^2}$$

$$=G(\vartheta,\varphi)\frac{P_0}{4\pi r^2}$$

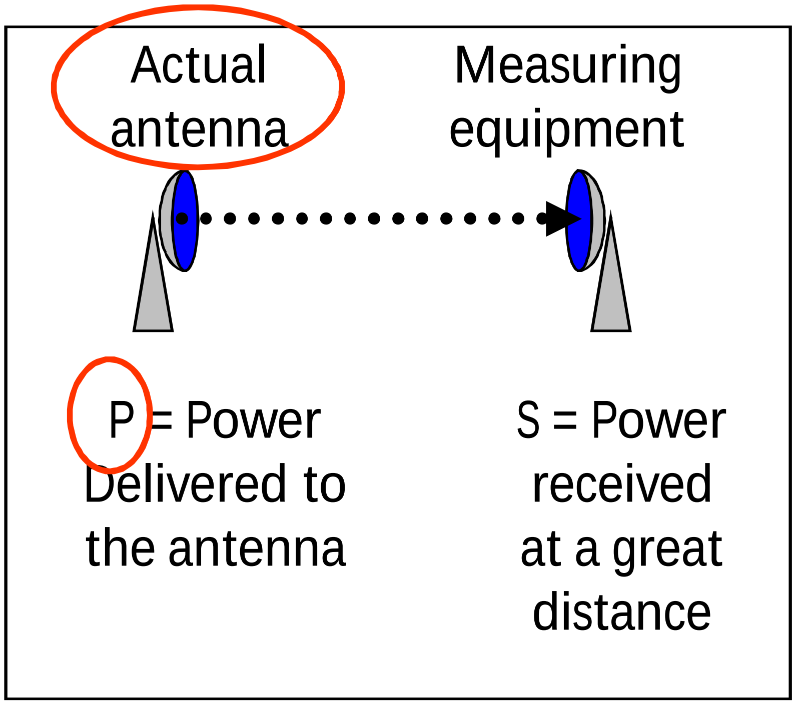
$$=G(\vartheta,\varphi)S_0$$

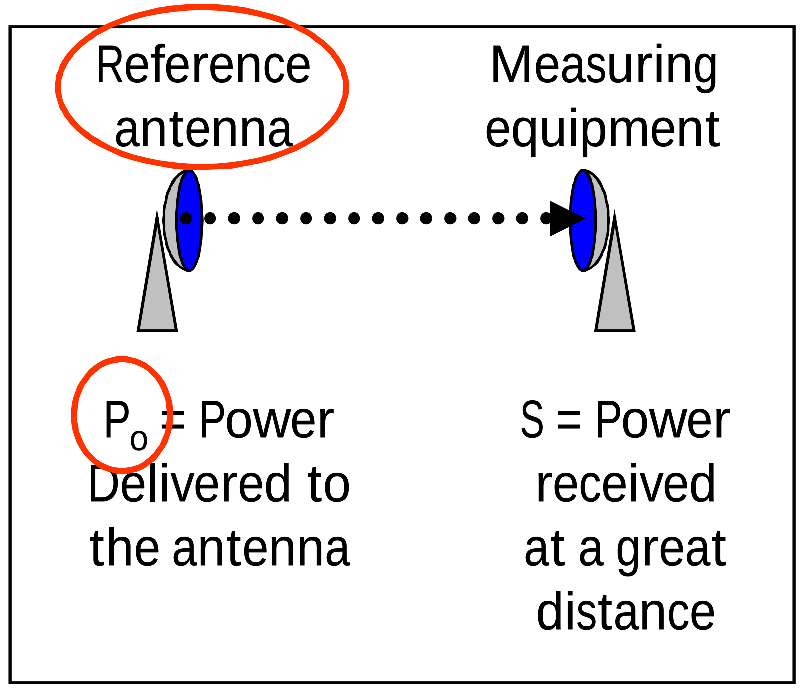
$S_0$ = PFD produced by a loss-less isotropic radiator

# Other Definitions of Gain

- For practical purposes, the antenna gain is defined as the ratio (usually in dB), of the power required at the input of a loss-free reference antenna to the power supplied to the input of the given antenna to produce, in a given direction, the same field strength or the same power flux-density at the same distance.
- When not specified otherwise, the gain refers to the direction of maximum radiation.
- The gain may be considered for a specified polarization. [RR 154]

# Antenna Gain




Antenna Gain (in the specific direction) = $P / P_o$

# Reference Antennas

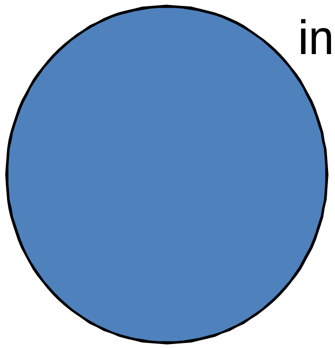
- Isotropic radiator
  - isolated in space ($G_i$, absolute gain, or isotropic gain)
- Half-wave dipole
  - isolated in space, whose equatorial plane of symmetry contains the given direction ($G_d$)
- Short vertical antenna
  - (much shorter than $\lambda/4$), close to, and normal to a perfectly conducting plane which contains the given direction ($G_v$)

# Reference Antennas (1)

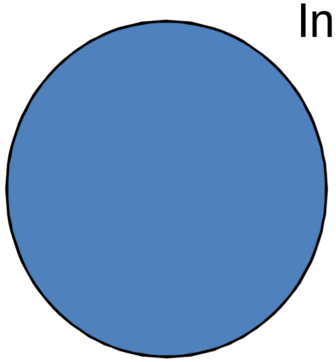
Isotropic antenna

- Sends (receives) energy equally in (from) all directions
- Gain = 1 (= 0 dB)
- When supplied by P, produces at distance r power flux density = $P/(4\pi r^2)$
- Theoretical concept, cannot be physically realized

Radiation pattern
in vertical plane



Radiation pattern
In horizontal plane

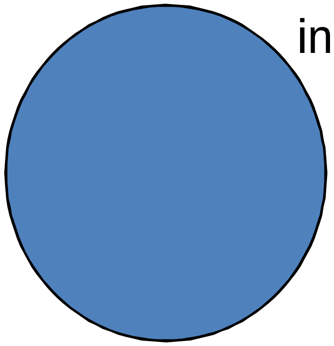


# Reference Antennas (2)

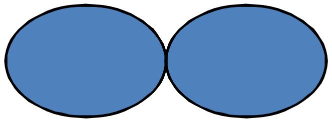
Half-Wave Dipole

- Linear antenna, realizable
- Gain = 1.64 (= 2,15 dB) in the direction of maximum radiation
- Figure-eight-shaped radiation pattern in the dipole plane, omnidirectional (circular) in the orthogonal plan

Radiation pattern
in vertical plane



Radiation pattern
In horizontal plane



# Effective aperture and aperture efficiency

Receiving antenna extracts power from incident wave

$$P_{rec} = S_{in} \cdot A_e$$

Aperture and beam area are linked:

$$A_e = \frac{\lambda^2}{\Omega_A}$$

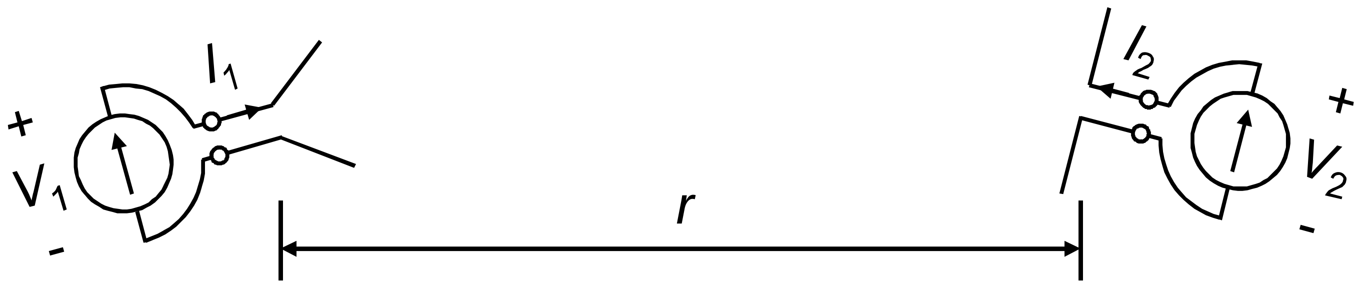
For some antennas, there is a clear physical aperture and an aperture efficiency can be defined

$$\varepsilon_{ap} = \frac{A_e}{A_p}$$

# Reciprocity

- Transmission and reception antennas can be used interchangeably
- Medium must be linear, passive and isotropic
- Caveat: Antennas are usually optimised for reception or transmission not both !

# Receiving Antennas and Reciprocity



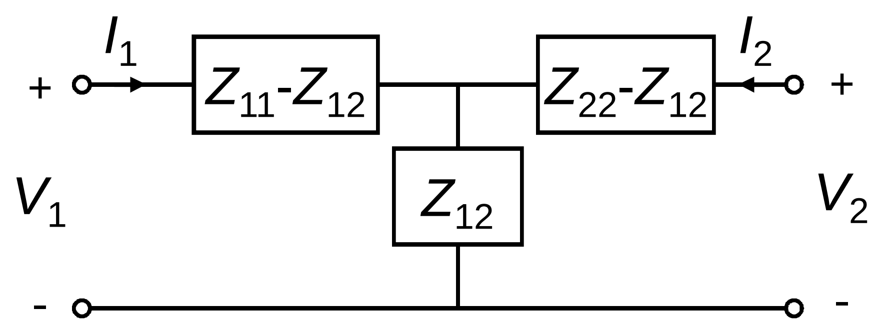

For a linear two-port

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Reciprocity

$$Z_{12} = Z_{21}$$

Equivalent Circuit

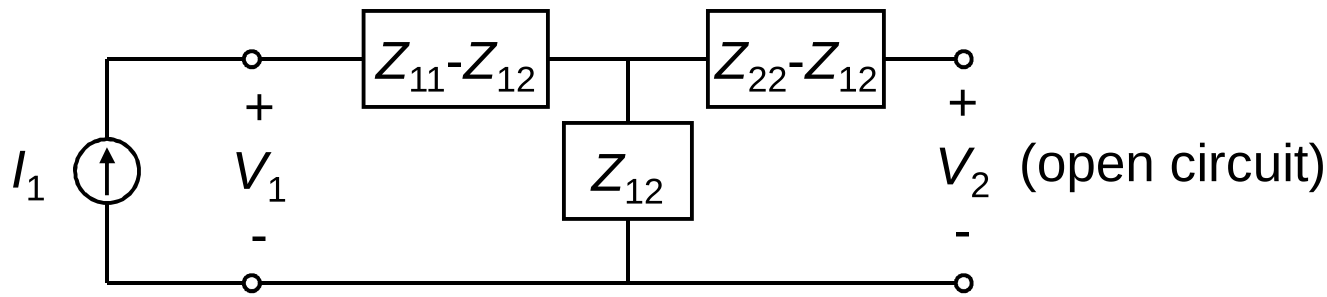



If $I_2 = 0$, $V_2 = Z_{12}I_1 \sim 1/r$

For r large,

$$|Z_{12}| << |Z_{11}|, |Z_{22}|$$

# Circuit Relation for Radiation into Free Space




$V_1 = Z_{11} I_1$

$V_2 = V_{OC} = Z_{12} I_1$

Transmitted power

$$P_T = (1/2)\mathrm{Re}\left(V_1 I_1^*\right) = (1/2)\mathrm{Re}\left(Z_{11}|I_1|^2\right) = (1/2) R_{r1} |I_1|^2$$

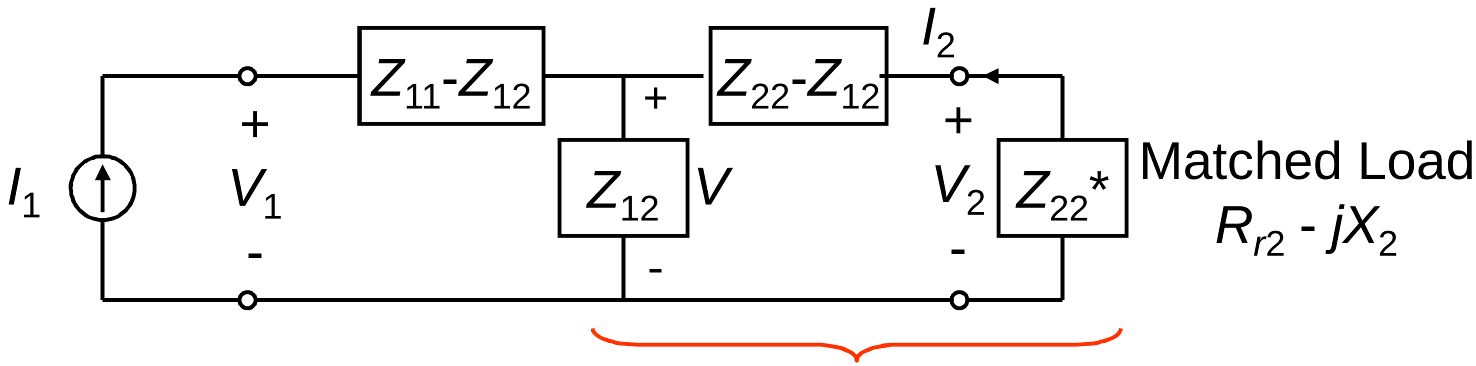
where $R_{r1}$ = radiation resistance of antenna 1

Therefore : $Z_{11} = R_{r1} + jX_1$

Similarily : $Z_{22} = R_{r2} + jX_2$

where $R_{r2}$ = radiation resistance of antenna 2

# Received Power and Path Loss Ratio




Current $I_1$ divides between branches $I_2 = -I_1 \dfrac{Z_{12}}{Z_{12}+\left(Z_{22}-Z_{12}+Z_{22}^*\right)} = -I_1 \dfrac{Z_{12}}{2R_{r2}}$

Received Power for Matched Load $P_R = \dfrac{1}{2}\left|I_2\right|^2 R_{r2} = \dfrac{1}{2}\left|\dfrac{I_1 Z_{12}}{2R_{r2}}\right|^2 = \left|I_1\right|^2 \dfrac{\left|Z_{12}\right|^2}{8R_{r2}}$

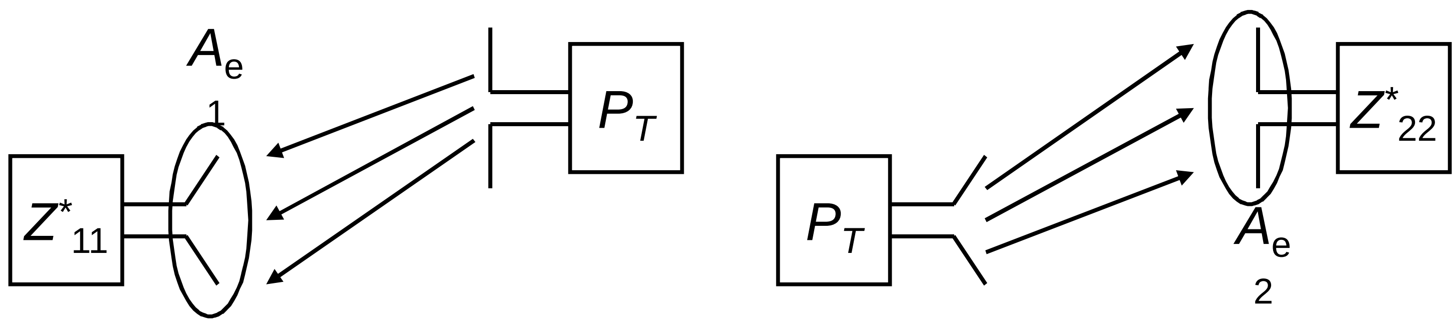
Path Gain $PG \equiv \dfrac{P_R}{P_T} = \dfrac{\left|I_1\right|^2\left|Z_{12}\right|^2 / 8R_{r2}}{\left|I_1\right|^2 R_{r1} / 2} = \dfrac{\left|Z_{12}\right|^2}{4R_{r1}R_{r2}}$

Final expression for $PG$ is the same if antenna 2 radiates and antenna 1 receives.

# Effective Area of Receiving Antenna

Effective Area = $A_e$

$$P_R = \underline{P} \cdot \underline{g}_r A_e = P_T \frac{g(\theta,\phi)}{4\pi r^2} A_e$$




$$PG = \frac{P_R}{P_T} = \frac{g_2 A_{e1}}{4\pi r^2} \quad \text{and by reciprocity} \quad PG = \frac{P_R}{P_T} = \frac{g_1 A_{e2}}{4\pi r^2}$$

Therefore $g_2 A_{e1} = g_1 A_{e2}$ or $\dfrac{A_{e1}}{g_1} = \dfrac{A_{e2}}{g_2} =$ same for all antennas