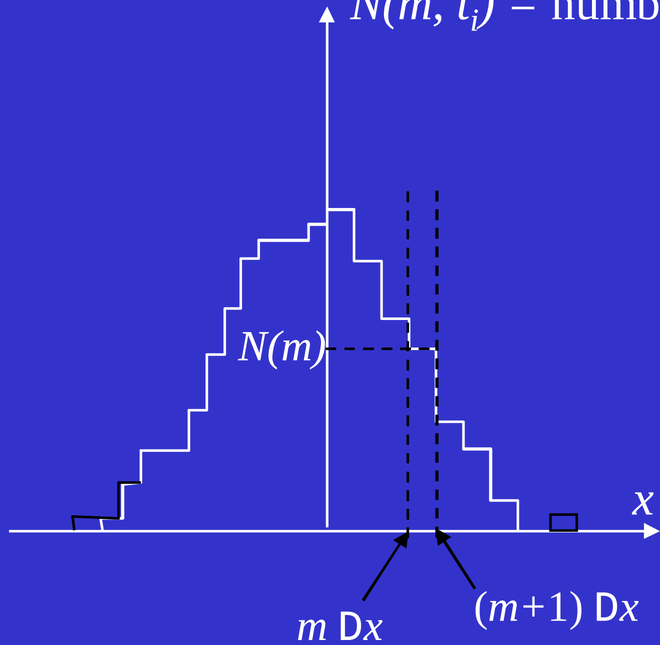
Probability density function (PDF)

- The characteristics of a random process or a random variable can be interpreted from the histogram

$N(m, t_i)$ = number of events: "$x_i = x +$ Dx"

Precision of measurment



$$\Rightarrow \quad \text{Prob}\left[m\Delta x \le x_i < (\mathrm{m}+1)\Delta x\right] = \lim_{N_{mes}\to\infty} \frac{N(m,t_i)}{N_{mes}}$$

$$et \quad \text{Prob}\left[m\Delta x \le x_i < p\Delta x\right] = \lim_{N_{mes}\to\infty} \frac{\sum_{k=m}^{p-1} N(k,t_i)}{N_{mes}}$$

N_{mes} = total number of measurments

PDF properties

- Id Δx=dx (trop petit) so, the histogram becomes continuous. In this case we can write:

$$\Rightarrow \quad \text{Prob}\left[x_1 \le x_i < x_2\right] = \int_{x_1}^{x_2} f\left(x, t_i\right) dx$$

$$\text{where} \quad f\left(x, t_i\right) = \lim_{N_{mes} \to +\infty} \frac{n\left(x, t_i\right)}{N_{mes}}$$

$$\text{Prob}\left[x = x_1\right] = 0$$

Theorem The DPF $f(x)$ satisfies the following basic properties:

(a) Non-negativity:

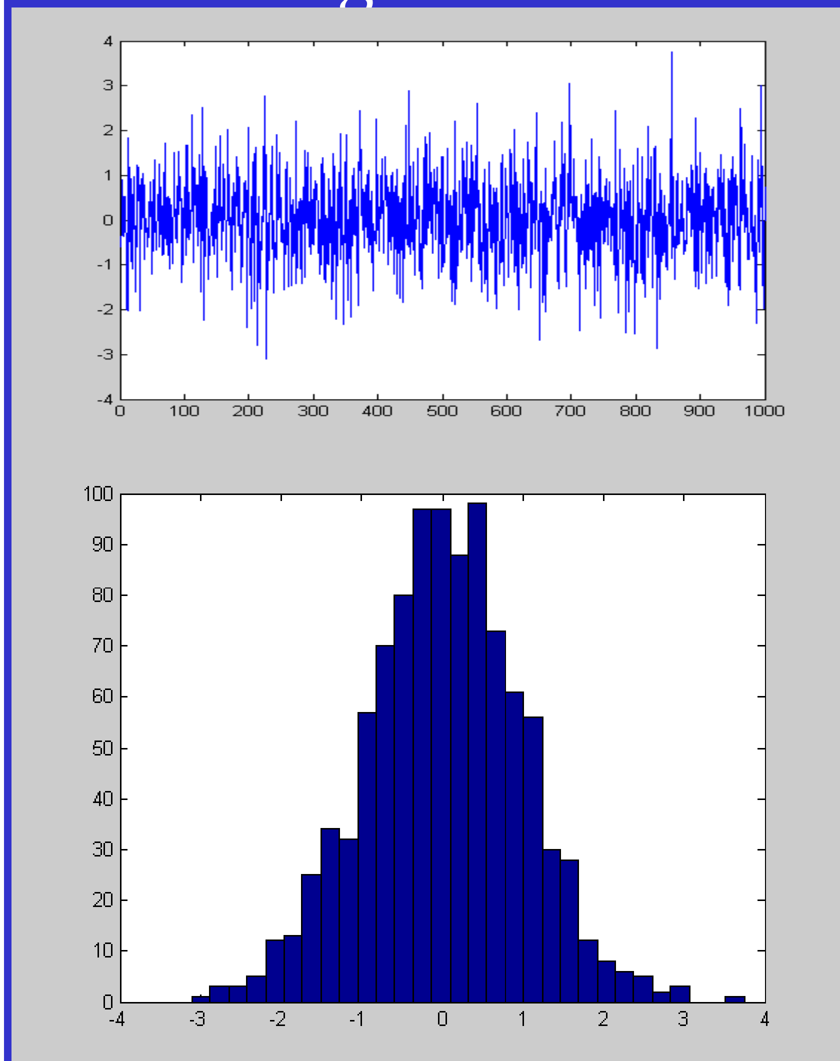
$$f(x) \ge 0, \quad x \in \mathbb{R}$$

(b) Normalization condition:

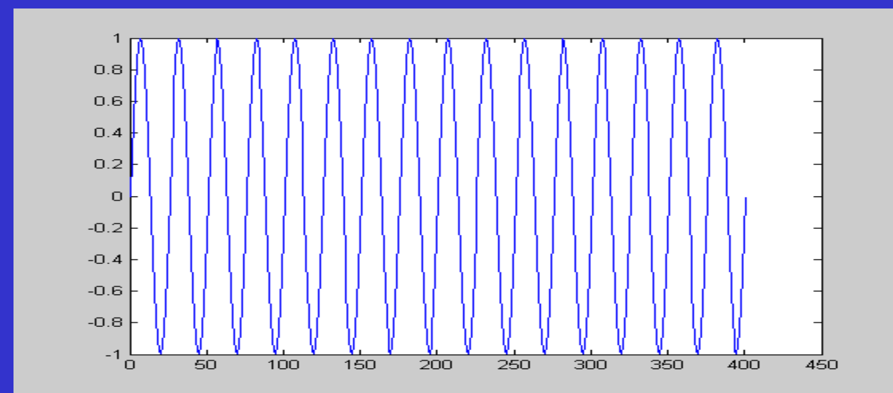
$$\int_{-\infty}^{\infty} f(x) dx = 1$$

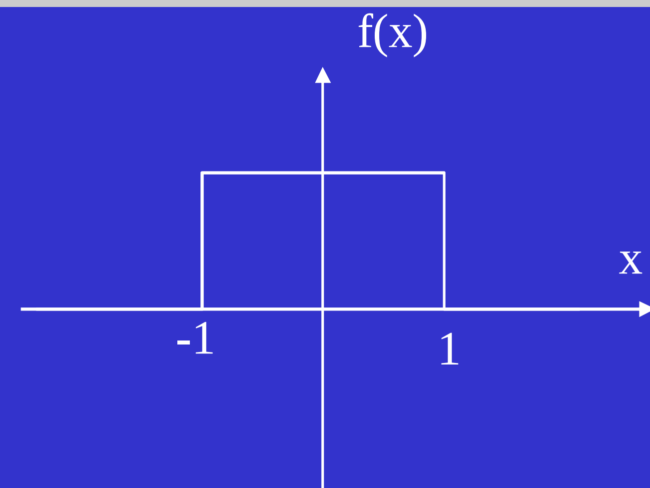
Histogram or PDF

Random signal



Sine wave :





Uniform PDF

Cumulative density function

Theorem Let X be a continuous RV with PDF $f(x)$ and CDF $F(x)$:

(a) The CDF $F(x)$ is continuous on $\mathbb{R}$ and for for every $x \in \mathbb{R}$:

$$F(x) = \int_{-\infty}^{x} f(t)\, dt \qquad \text{all } x \in \mathbb{R}$$

$F(x) = P(X \leq x)$ is equal to the area under the graph of $f(t)$ from $t = -\infty$ to $t = x$:

(b) If f is continuous at x:

$$F'(x) = \frac{dF(x)}{dx} = f(x)$$

Theore Let X be a continuous RV with PDF $f(x)$. For any real numbers $a \leq b$, we have:

(a) $P(a \leq X \leq b) = \int_{a}^{b} f(x)dx$)

(b) $P(X = a) = 0$

(c) $P(a \leq X \leq b) = P(a < X \leq b)$

$= P(s \leq X < b) = P(a < X < b)$

examples

Example: Let X be a continuous RV with PDF

$$f(x) = \begin{cases} c, & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

where c is a constant.

(a) Determine the constant c and sketch $f(x)$.

(b) Determine and sketch $F(x)$.

(c) Compute $P(-1/2 \leq X \leq 3/4)$.

Example: Let X be a continuous RV with PDF

$$f(x) = \begin{cases} ce^{-x}, & 0 \leq x \\ 0, & x < 0. \end{cases}$$

where c is a constant.

Expectation, variance

Every function of a random variable is a random variable. If we know the probability distribution of a RV, we can deduce the expectation value of the function of a random variable:

$$E\{g_i\} = \int_{-\infty}^{+\infty} g(x) f(x, t_i) dx$$

Statistical parameters :

Average value : $\mu_x(t_i) = E\{\hat{x}_i\} = \int_{-\infty}^{+\infty} x \cdot f(x, t_i) dx$

Mean quadratic value: $m_{x^2}(t_i) = E\{\hat{x}_i^2\} = \int_{-\infty}^{+\infty} x^2 \cdot f(x, t_i) dx$

Variance : $\sigma_x^2(t_i) = E\{\hat{x}_i - \mu_x(t_i)^2\} = m_{x^2} - \mu_x(t_i)^2$

Standard deviation : $\sigma_x(t_i) = \sqrt{m_x^2 - \mu_x^2}$

Moments of higher order

- The definition of the moment of order r is:

$$moment = E\left\{\hat{x}_i^r\right\} = \int_{-\infty}^{+\infty} x^r f\left(x, t_i\right) dx$$

- The definition of the characteristic function is:

$$Characteristic\ \ function = \varphi_X(u)$$

$$\varphi_X(u) = E\left\{e^{jux}\right\} = \int_{-\infty}^{+\infty} e^{jux} f\left(x\right) dx$$

We can demonstrate: $$E\left\{\hat{x}^n\right\} = \frac{1}{j^n}\varphi^{(n)}(0)$$