AUTO CORRELATION AND CROSS CORRELATION FUNCTIONS

- **Auto Correlation**
- In auto correlation a signal is compared to a time delayed version of itself. This results in the Auto Correlation Function or ACF.
- Consider the function v(t), (which in general may be random or deterministic).

- The ACF, R(τ) , is given by

$$R(\tau) = \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} v(t)v(t-\tau)dt$$

- Of particular interest is the ACF when $\tau = 0$, and v(t) represents a voltage signal:

$$R(0) = \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} v(t)^2 \, dt$$

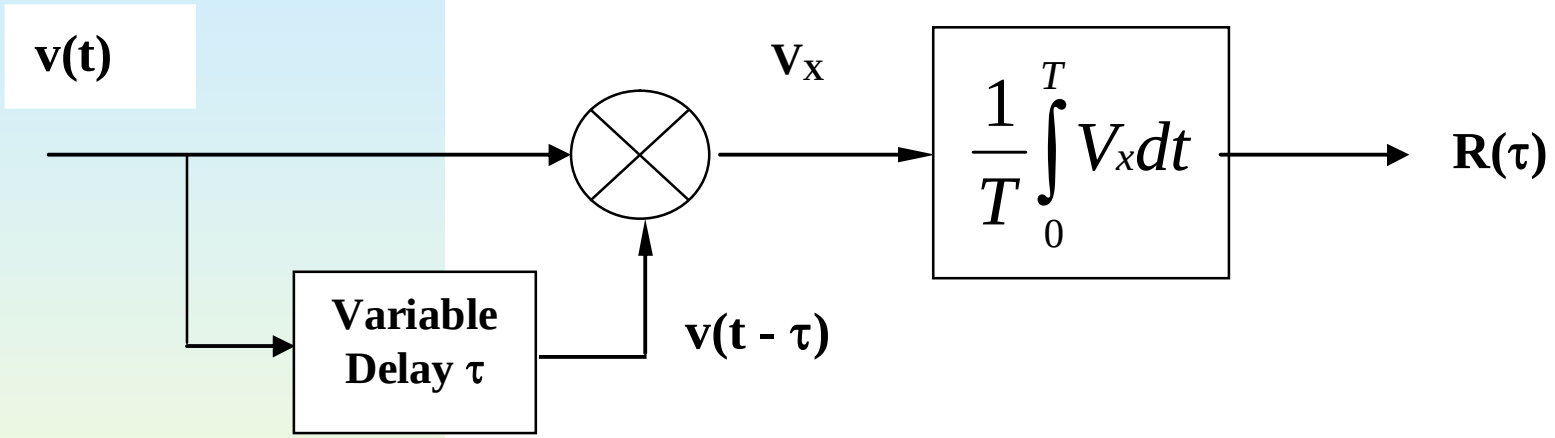
- R(0) represents the mean square value or normalised average power in the signal v(t)

- **Cross Correlation**
- In cross correlation, two ‘separate’ signals are compared, eg the functions v1(t) and v2(t) previously discussed.

- The CCF is

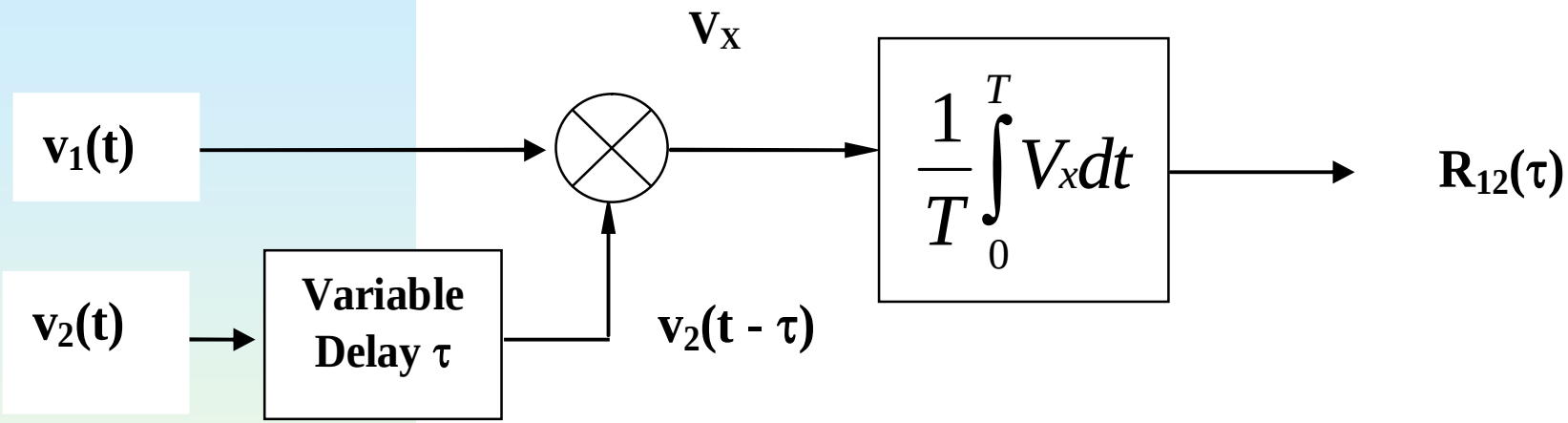
$$R_{12}(\tau) = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} v_1(t)v_2(t-\tau)dt$$

- Diagrams for ACF and CCF
- Auto Correlation Function, ACF



- Note, if the input is v1(t) the output is R11(τ)
- if the input is v2(t) the output is R22(τ)

Cross Correlation Function, CCF



- **CORRELATION COEFFICIENT**
- The correlation coefficient, ρ, is the normalised correlation function.

- For cross correlation (ie the comparison of two separate signals), the correlation coefficient is given by:

$$\rho = \frac{R_{12}(\tau)}{\sqrt{R_{11}(0).R_{22}(0)}}$$

- Note that $R_{11}(0)$ and $R_{22}(0)$ are the mean square values of the functions $v_1(t)$ and $v_2(t)$ respectively.

- For auto correlation (ie the comparison of a signal with a time delayed version of itself), the correlation coefficient is given by:

$$\rho = \frac{R(\tau)}{\sqrt{R(0).R(0)}} = \frac{R(\tau)}{R(0)}$$

- For signals with a zero mean value, ρ is in the range $-1 < \rho < +1$

- If $\rho = +1$ then the are equal (Positive correlation).
- If $\rho = 0$, then there is no correlation, the signals are considered to be orthogonal.
- If $\rho = -1$, then the signals are equal and opposite (negative correlation)

EXAMPLES OF CORRELATION – CONTINUOUS TIME FUNCTIONS