Correlation

CORRELATION

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INTRODUCTION

Correlation techniques are widely used in signal processing with many applications in telecommunications, radar, medical electronics, physics, astronomy, geophysics etc . . .. .

Correlation has many useful properties, giving for example the ability to:

- Detect a wanted signal in the presence of noise or other unwanted signals.
- Recognise patterns within analogue, discrete-time or digital signals.
- Allow the determination of time delays through various media, eg free space, various materials, solids, liquids, gases etc . . .

- Correlation is a comparison process.
- The correlation betweeen two functions is a measure of their similarity.
- The two ‘functions’ could be very varied. For example fingerprints: a fingerprint expert can measure the correlation between two sets of fingerprints.

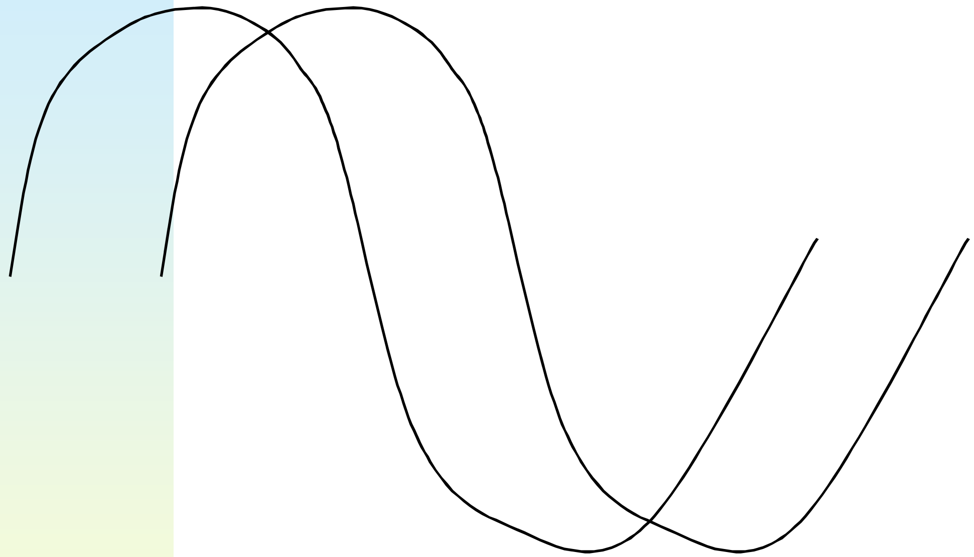
- This section will consider the correlation of signals expressed as functions of time. The signals could be continuous, discrete time or digital.

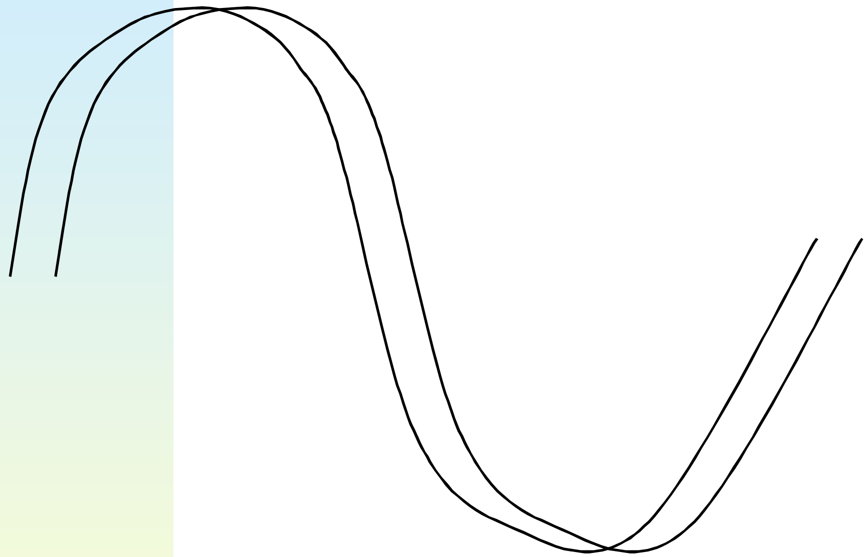
- When measuring the correlation between two functions, the result is often expressed as a correlation coefficient, ρ, with ρ in the range –1 to +1.

$\rho = -1$	$\rho = 0$	$\rho = +1$
Similar but opposite	No similarity	Exactly similar

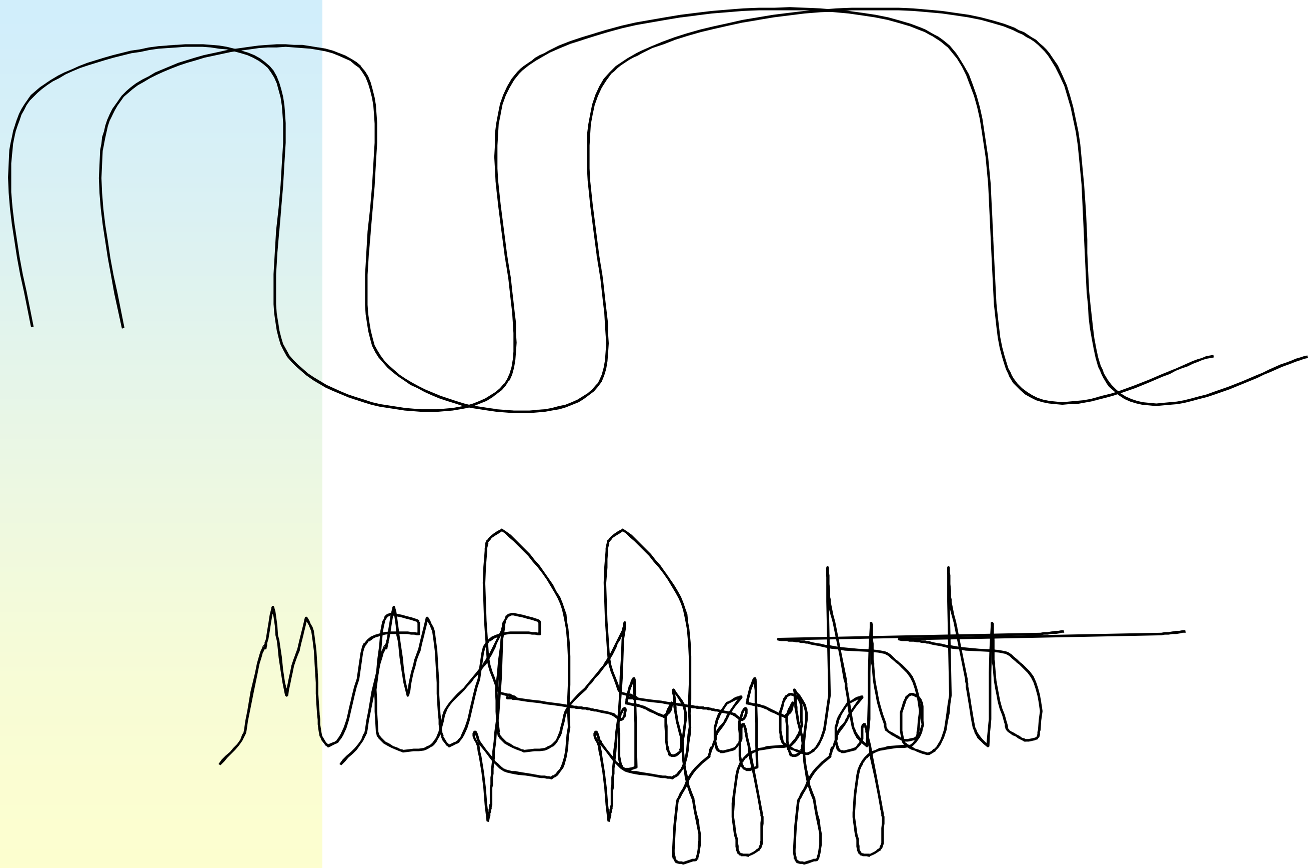
- **Correlation involves multiplying, 'sliding' and integrating**

- **Consider 2 functions**

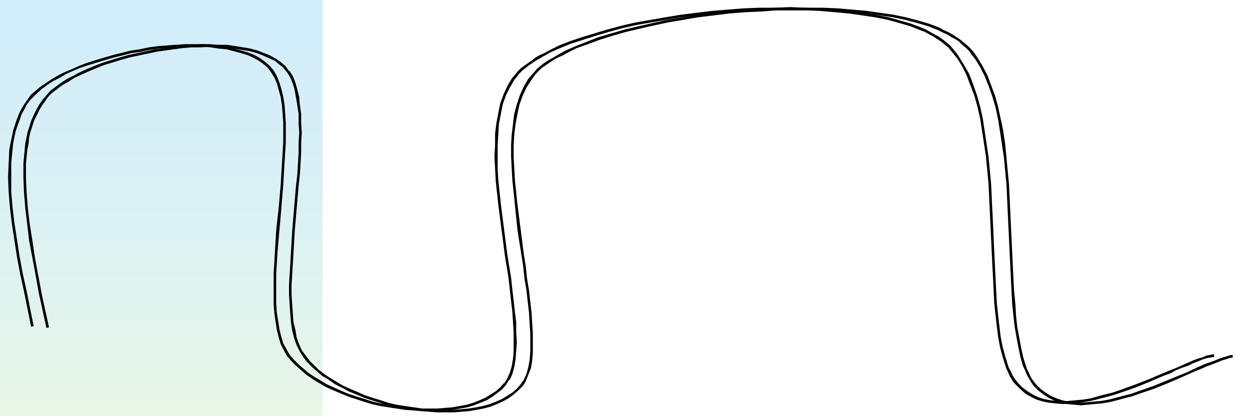


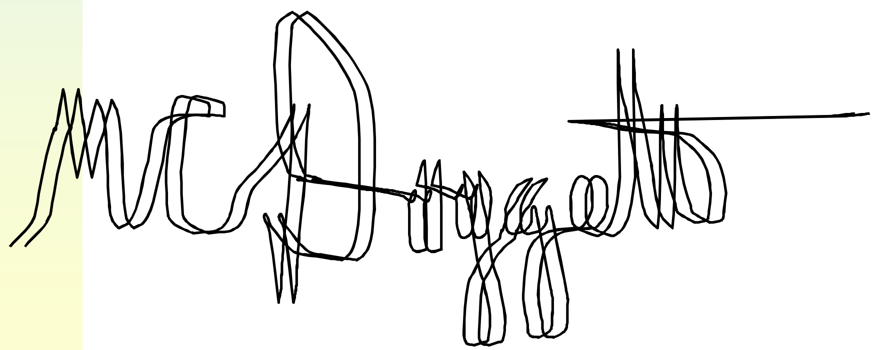


- **Consider 2 more functions**

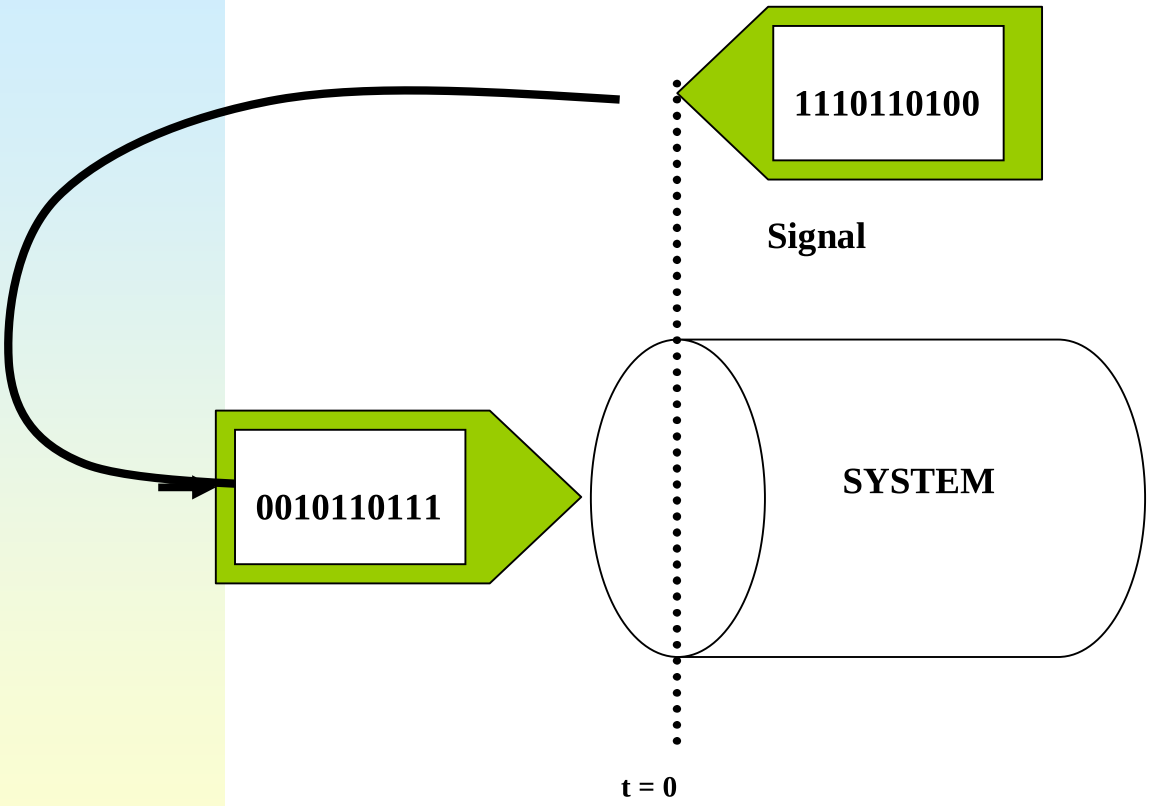


- **Consider 2 more functions**





CONVOLUTION



- **CORRELATION FUNCTION – CONTINUOUS TIME FUNCTIONS**
- Consider two continuous functions of time, $v_1(t)$ and $v_2(t)$. The functions may be random or deterministic.
- The correlation or similarity between these two functions measured over the interval T is given by:

$$R_{12}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} v_1(t) v_2(t-\tau)\, dt$$

- The functions may be deterministic or random.
- $R12(\tau)$ is the correlation function and is a measure of the similarity between the functions v1(t) and v2(t).
- The measure of correlation is a function of a new variable, τ, which represents a time delay or time shift between the two functions.

- Note that correlation is determined by multiplying one signal, $v1(t)$, by another signal shifted in time, $v2(t-\tau)$, and then finding the integral of the product,

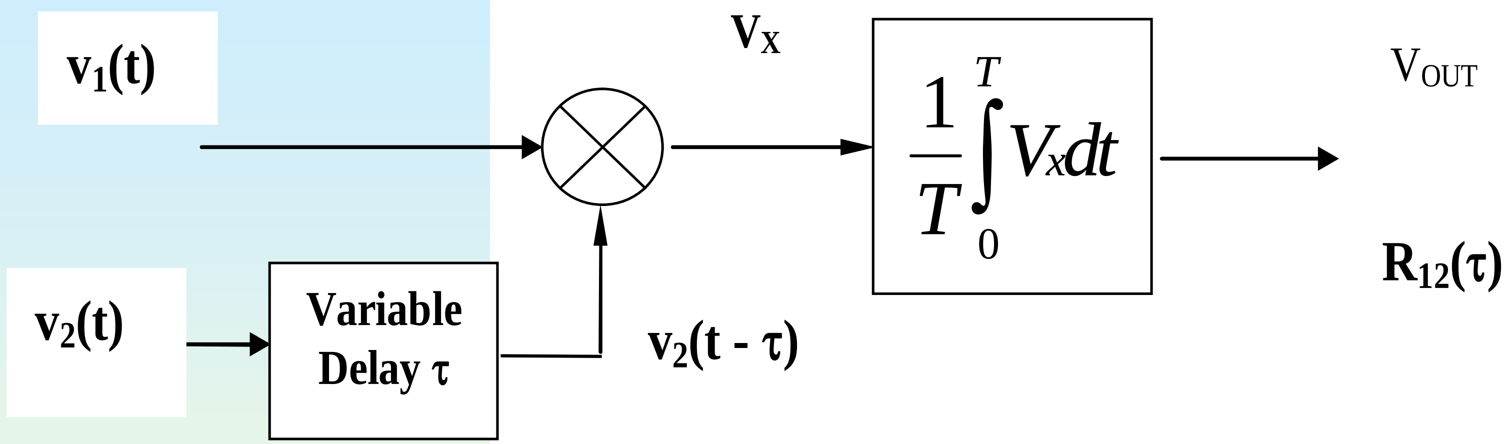
- Thus correlation involves multiplication, time shifting (or delay) and integration.

- The integral finds the average value of the product of the two functions, averaged over a long time (T $\rightarrow \infty$) for non-periodic functions.

- For periodic functions, with period T, the correlation function is given by:

$$R_{12}(\tau) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} v_1(t)v_2(t-\tau)dt$$

- The correlation process is illustrated below:



- As previously stated:

$$R_{12}(\tau) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} v_1(t)v_2(t-\tau)\,dt$$

- The output $R_{12}(\tau)$ is the correlation between the two functions as a function of the delay τ.
- The correlation at a particular value of τ would be solved by solving $R_{12}(\tau)$,