Chapter-4

FREQUENCY DOMAIN ANALYSIS , COMPENSATION & CONTROL COMPONENT

Frequency Response Methods and Stability

In previous chapters we examined the use of test signals such as a step and a ramp signal. In this chapter we consider the steady-state response of a system to a sinusoidal input test signal. We will see that the response of a linear constant coefficient system to a sinusoidal input signal is an output sinusoidal signal at the same frequency as the input. However, the magnitude and phase of the output signal differ from those of the input sinusoidal signal, and the amount of difference is a function of the input frequency. Thus we will be investigating the steady-state response of the system to a sinusoidal input as the frequency varies.

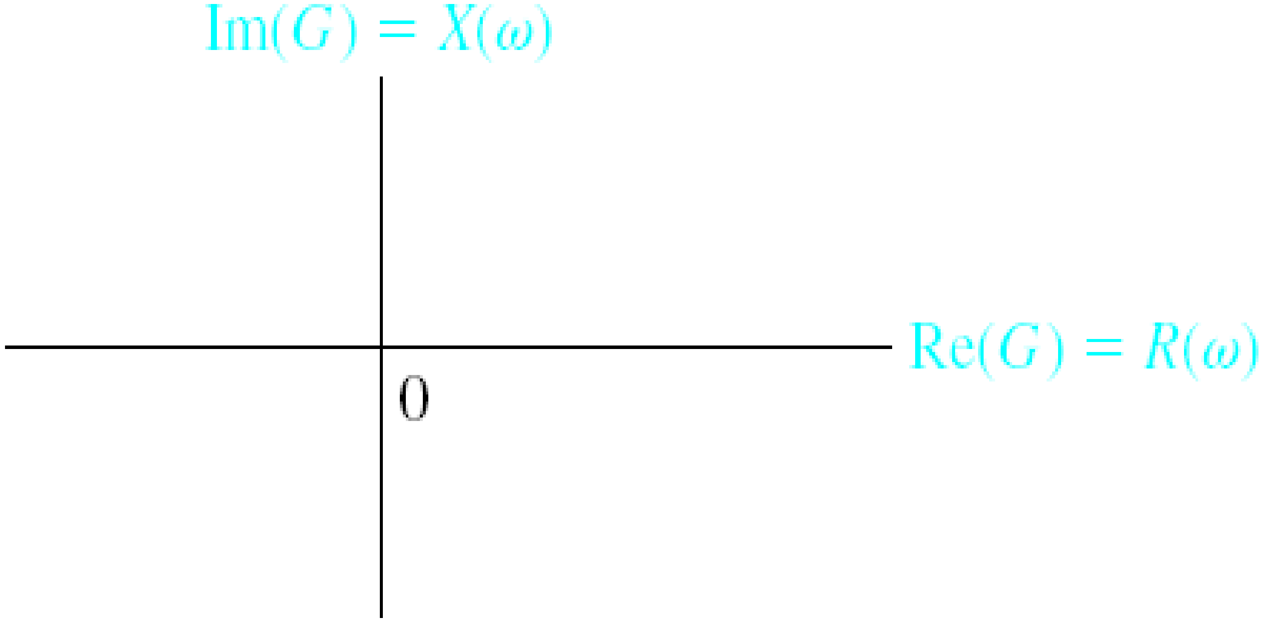
We will examine the transfer function G(s) when s =jw and develop methods for graphically displaying the complex number G(j)as w varies. The Bode plot is one of the most powerful graphical tools for analyzing and designing control systems, and we will cover that subject in this chapter. We will also consider polar plots and log magnitude and phase diagrams. We will develop several time-domain performance measures in terms of the frequency response of the system as well as introduce the concept of system bandwidth.

Introduction

The frequency response of a system is defined as the steady-state response of the system to a sinusoidal input signal. The sinusoid is a unique input signal, and the resulting output signal for a linear system, as well as signals throughout the system, is sinusoidal in the steady-state; it differs form the input waveform only in amplitude and phase.

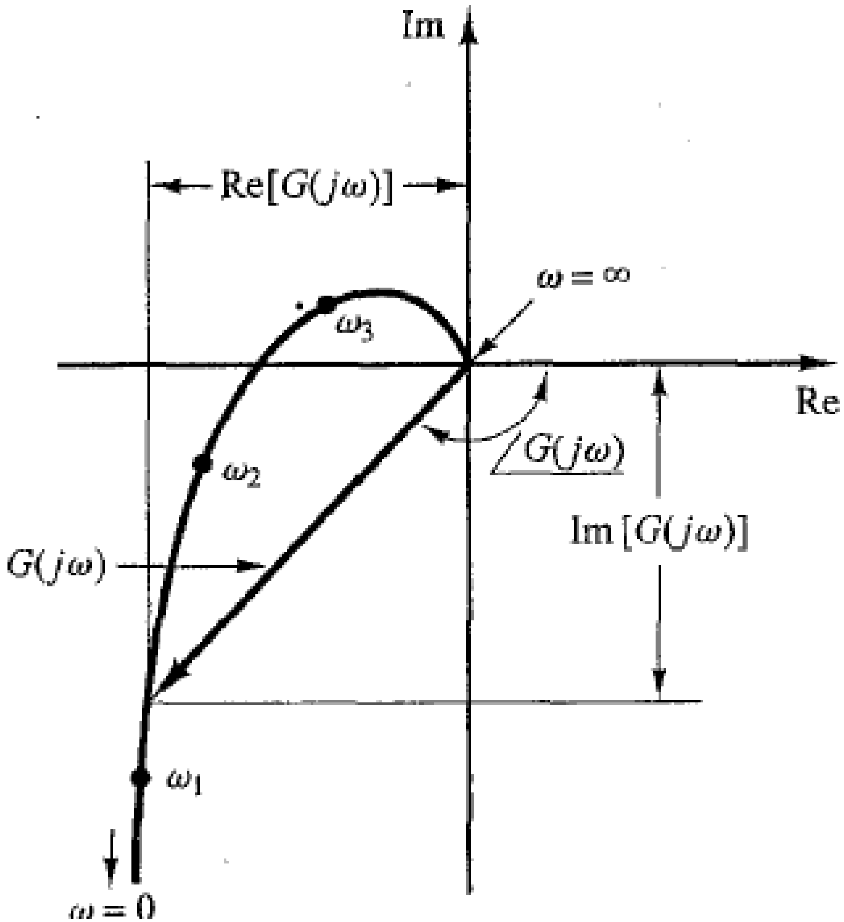
Frequency Response Plots

Polar Plots



DIRECT POLAR PLOTS

We can plot transfer function in polar plan



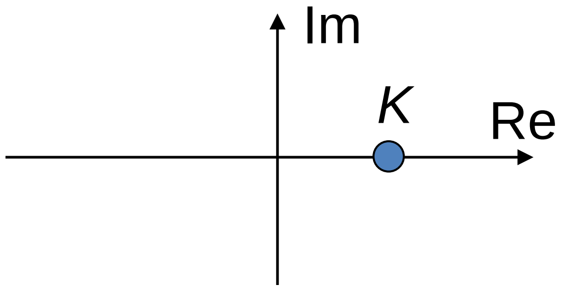
DIRECT POLAR PLOTS

1. **Proportional element**

Transfer function:

$$G(s) = \frac{C(s)}{R(s)} = K$$

$$G(j\omega) = K \rightarrow \begin{cases} |G(j\omega)| = K \\ \varphi(\omega) = \angle G(j\omega) = 0^o \end{cases}$$



Polar plot

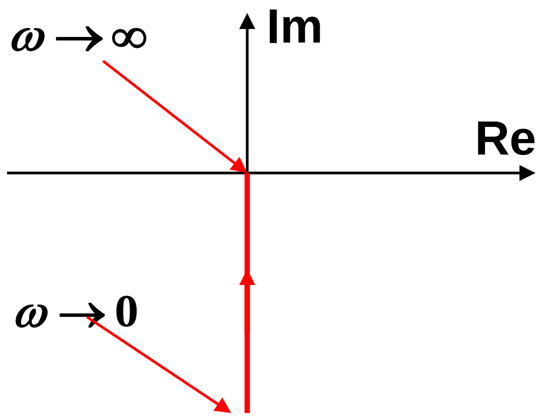
DIRECT POLAR PLOTS

2. Integrating element

Transfer function:

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s}$$

$$G(j\omega) = \frac{1}{j\omega} \rightarrow \begin{cases} |G(j\omega)| = \dfrac{1}{\omega} \\ \varphi(\omega) = \angle G(j\omega) = -90^{o} \end{cases}$$



Polar plot

DIRECT POLAR PLOTS

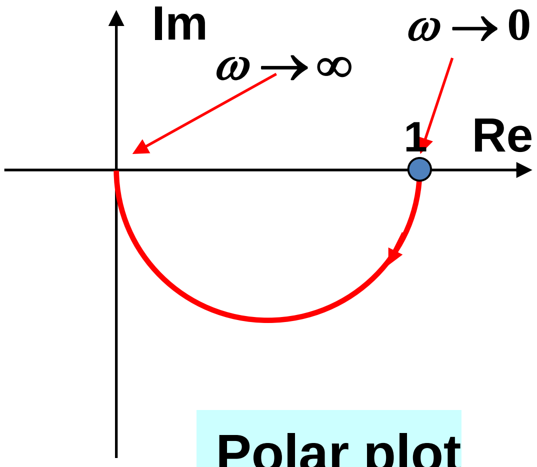
3. Inertial element

Transfer function:

$$G(s)=\frac{C(s)}{R(s)}=\frac{1}{Ts+1}\quad G(j\omega)=\frac{1}{j\omega T+1}$$

$$\left|G(j\omega)\right|=\frac{1}{\sqrt{1+(\omega T)^2}}$$

$$\varphi(\omega)=-tg^{-1}(\omega T)$$



Polar plot

DIRECT POLAR PLOTS

4. Oscillating element

Transfer function:

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{T^2 s^2 + 2\zeta T s + 1} \qquad 0 < \zeta < 1$$

$$G(j\omega) = \frac{1}{(1-\omega^2 T^2) + j2\zeta T\omega}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\left[1-(\omega^2 T^2)\right]^2 + (2\zeta T\omega)^2}} \qquad \varphi(\omega) = -tg^{-1}\left(\frac{2\zeta T\omega}{1-\omega^2 T^2}\right)$$

maximum value of $|G(j\omega)|$:

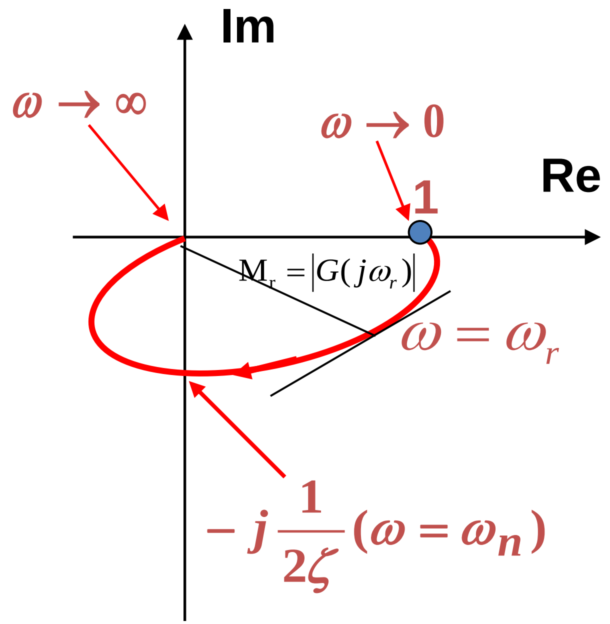
Make: $$\frac{d}{d\omega}(|G(j\omega)|) = 0 \Rightarrow \omega = \omega_r = \omega_n\sqrt{1-2\zeta^2} \quad \left(0 < \zeta < \frac{\sqrt{2}}{2}\right)$$

$\omega_r \rightarrow$ *resonant frequency*

$M_r \rightarrow$ *resonant peak*

$$\Rightarrow M_r = |G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

DIRECT POLAR PLOTS



Polar plot

$$1.\ \zeta \downarrow \Rightarrow \omega_r \uparrow (\to \omega_n) \Rightarrow M_r \uparrow$$

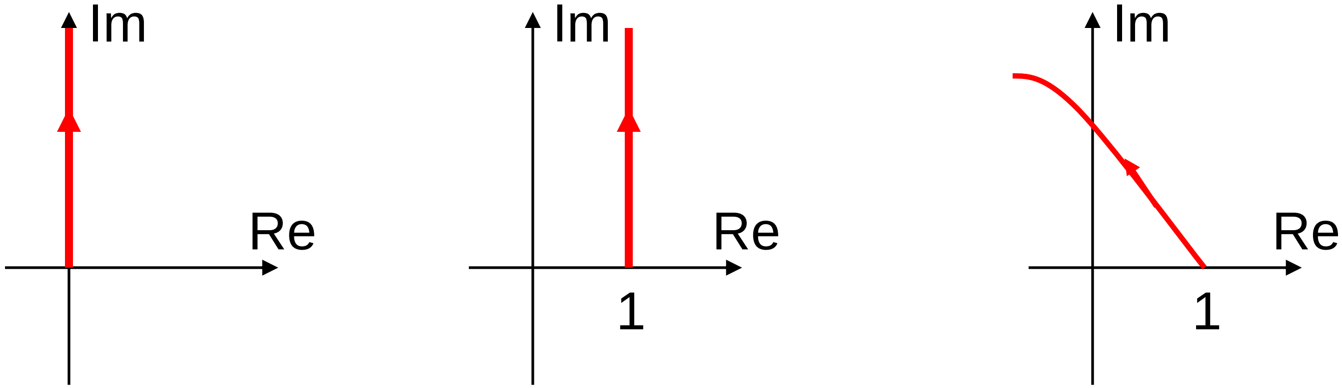
$$2.\ \begin{cases} \zeta = 0 \to \omega_r = \omega_n \Rightarrow \textit{unstable system} \\ \zeta = \sqrt{2}/2 \to \omega_r = 0\ \ \textit{No resonance},\ \text{Optimal Second-order System} \end{cases}$$

DIRECT POLAR PLOTS

5. Differentiating element

Transfer function:

$$G(s)=\begin{cases} s & \textit{differential} \\ Ts+1 & \textit{first-order differential} \\ Ts^2+2\zeta Ts+1 & \textit{second-order differential} \end{cases}$$



differential **1th-order differential** **2th-order differential**

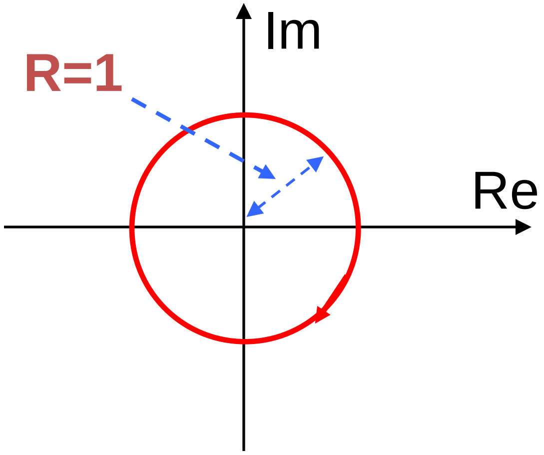
Polar plot

DIRECT POLAR PLOTS

6. Delay element

Transfer function: $$G(s) = \frac{C(s)}{R(s)} = e^{-\tau s}$$

$$G(j\omega) = e^{-j\omega\tau} \Rightarrow \begin{cases} |G(j\omega)| = 1 \\ \varphi(\omega) = \angle G(j\omega) = -\omega\tau \end{cases}$$



Polar plot

DIRECT POLAR PLOTS

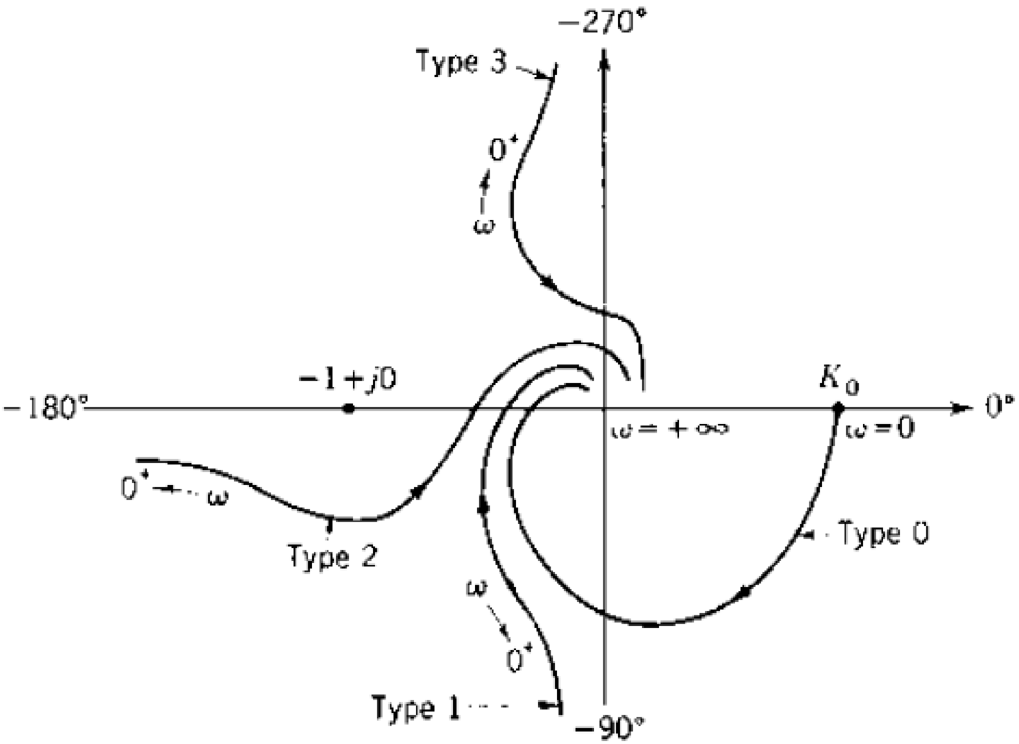
To obtain the direct polar plot of a system's forward transfer function, the following criteria are used to determine the key parts of the curve.

Step 1. The forward transfer function has the general form

$$\mathbf{G}(j\omega) = \frac{K_m(1 + j\omega T_a)(1 + j\omega T_b)\cdots(1 + j\omega T_w)}{(j\omega)^m(1 + j\omega T_1)(1 + j\omega T_2)\cdots(1 + j\omega T_u)}$$

For this transfer function the system type is equal to the value of m and determines the portion of the polar plot representing the $\lim_{\omega \to 0} \mathbf{G}(j\omega)$. The low-frequency polar plot characteristic (as $\omega \to 0$) of the different system types are summarized in Fig.

$$\mathbf{G}(j\omega) = \frac{K_m(1 + j\omega T_a)(1 + j\omega T_b)\cdots(1 + j\omega T_w)}{(j\omega)^m(1 + j\omega T_1)(1 + j\omega T_2)\cdots(1 + j\omega T_u)}$$

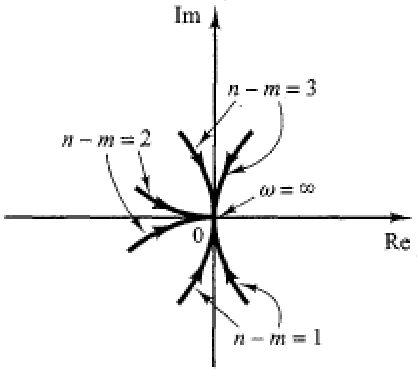


The angle at $\omega = 0$ is $m(-90^\circ)$.

The arrow on the polar plots indicates the direction of increasing frequency.

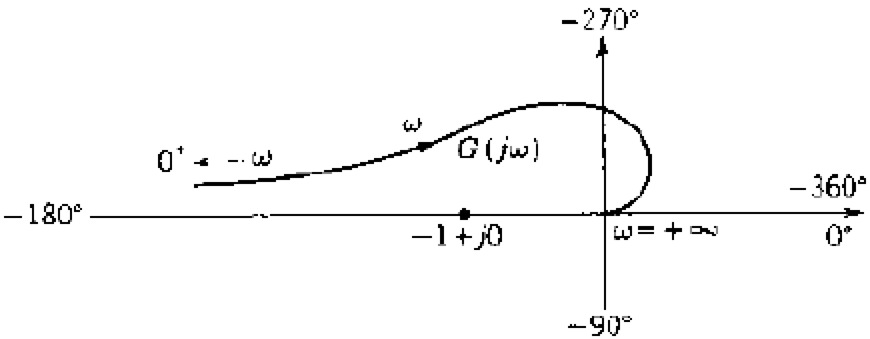
Step 2. The high-frequency end of the polar plot can be determined as follows:

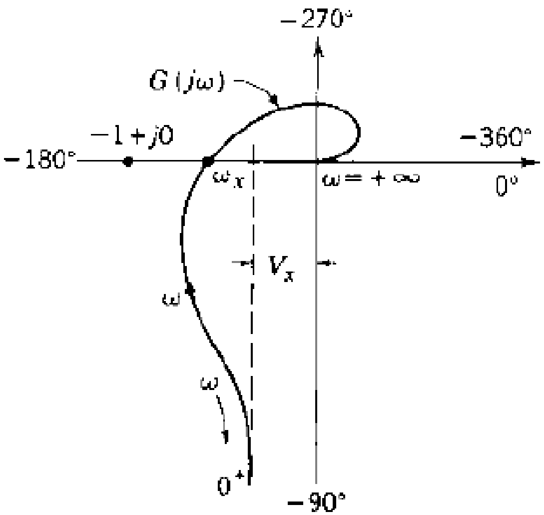
$$\lim_{\omega \to +\infty} G(j\omega) = 0 \underline{/(w - m - u)90^\circ}$$



$$G(j\omega) = \frac{b_o(j\omega)^m + \dots}{a_o(j\omega)^n + \dots}$$

Step 3. The asymptote that the low-frequency end approaches, for a Type 1 system, is determined by taking the limit as $\omega \to 0$ of the real part of the transfer function.

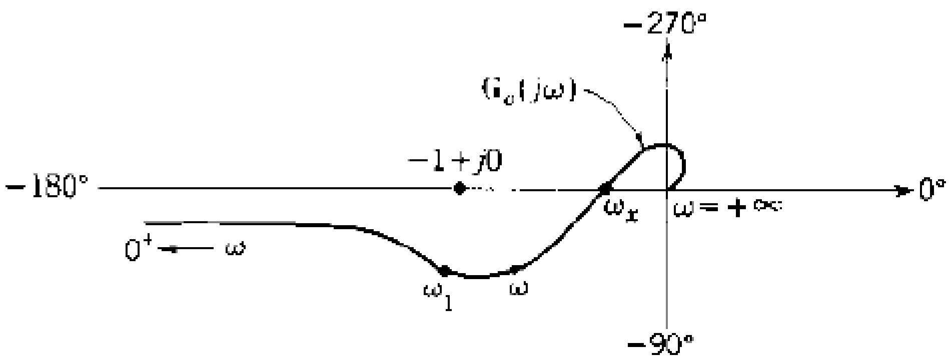


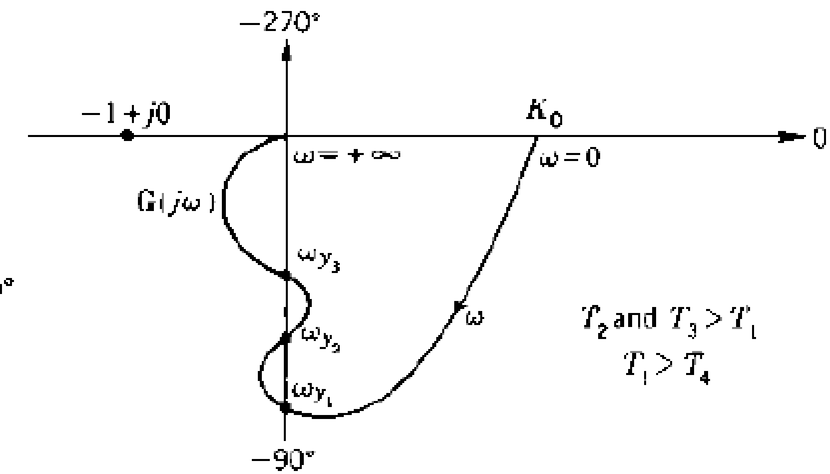


Step 4. The frequencies at the points of intersection of the polar plot with the negative real axis and the imaginary axis are determined, respectively, by setting

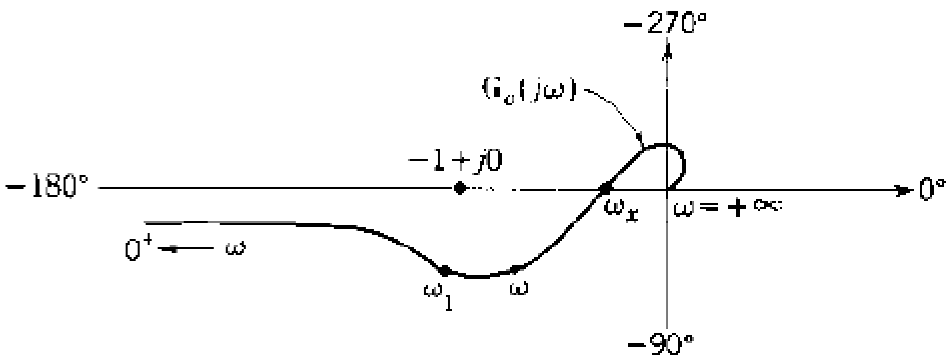
$$\mathrm{Im}\ G(j\omega)] = 0$$

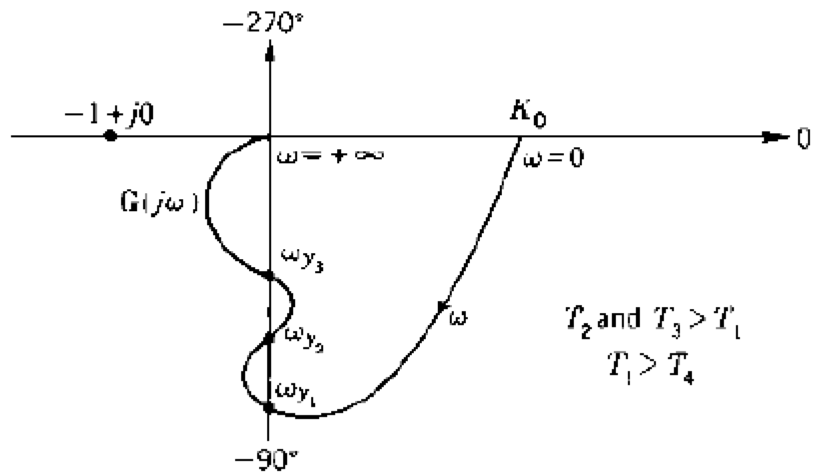
$$\mathrm{Re}\ G(j\omega)] = 0$$





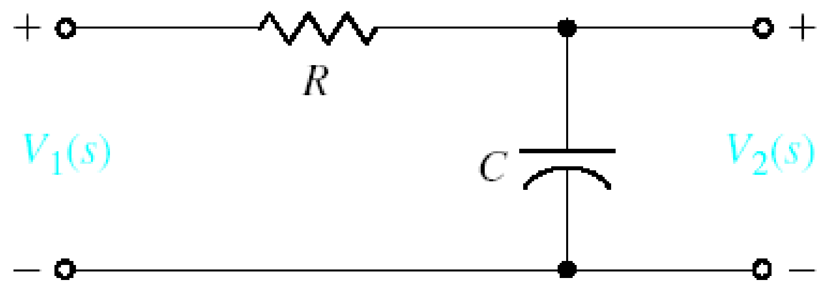
Step 5. If there are no frequency-dependent terms in the numerator of the transfer function, the curve is a smooth one in which the angle of G($j\omega$) continuously decreases as ω goes from 0 to ∞. With time constants in the numerator, and depending upon their values, the angle may not continuously vary in the same direction, thus creating "dents" in the polar plot.





Step 6. it is important to know the exact shape of the polar plot of G($j\omega$) in the vicinity of the $-1 + j0$ point and the crossing point on the negative real axis.

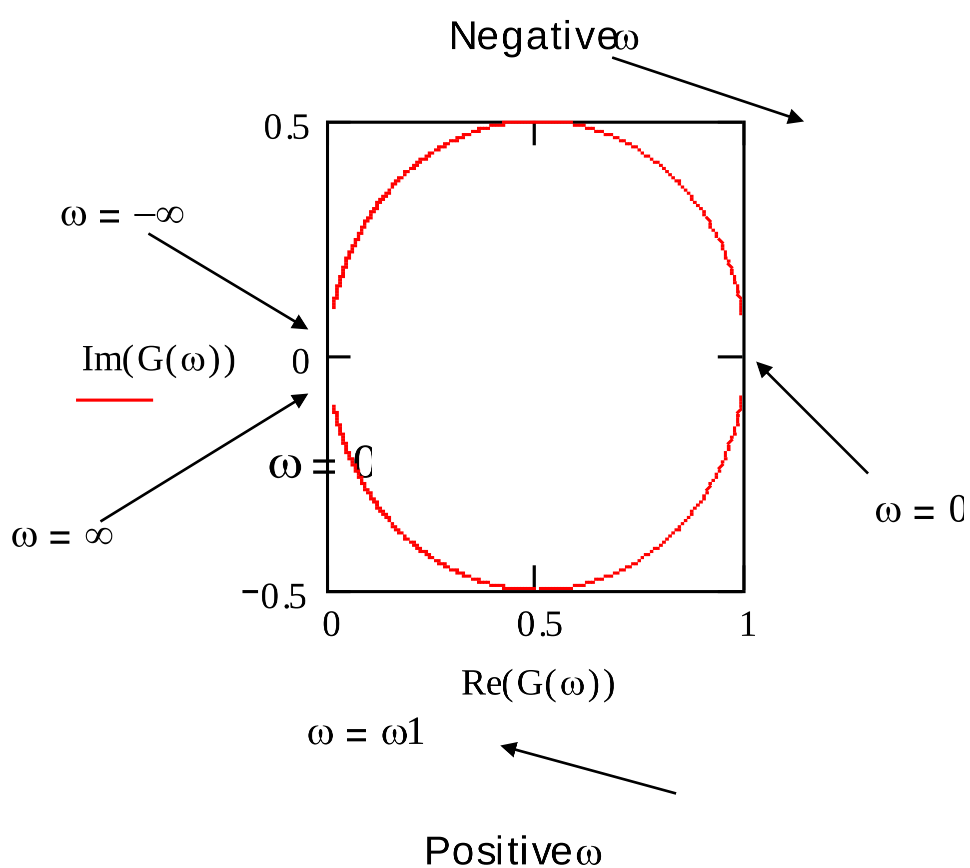
Frequency Response Plots



Polar Plots

$\omega := -1000, -999 .. 1000$ $\quad j := \sqrt{-1}$ $\quad R := 1$ $\quad C := 0.01$ $\quad \omega 1 := \dfrac{1}{R \cdot C}$

$$G(\omega) := \frac{1}{\left(j \cdot \dfrac{\omega}{\omega 1}\right) + 1}$$

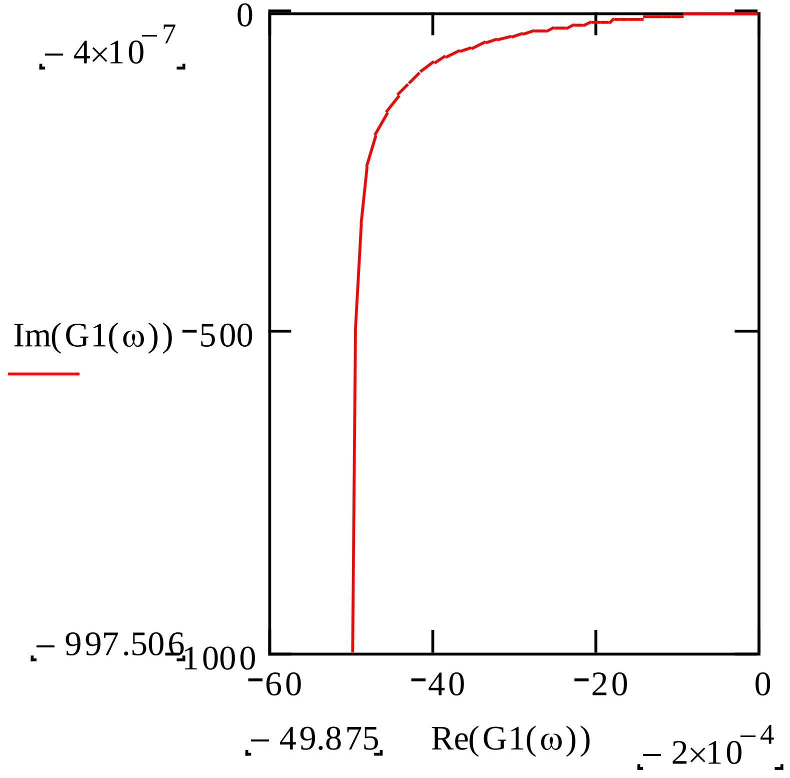


Frequency Response Plots

Polar Plots

$\omega := 0, .1 .. 1000 \qquad \tau := 0.5 \qquad K := 10($

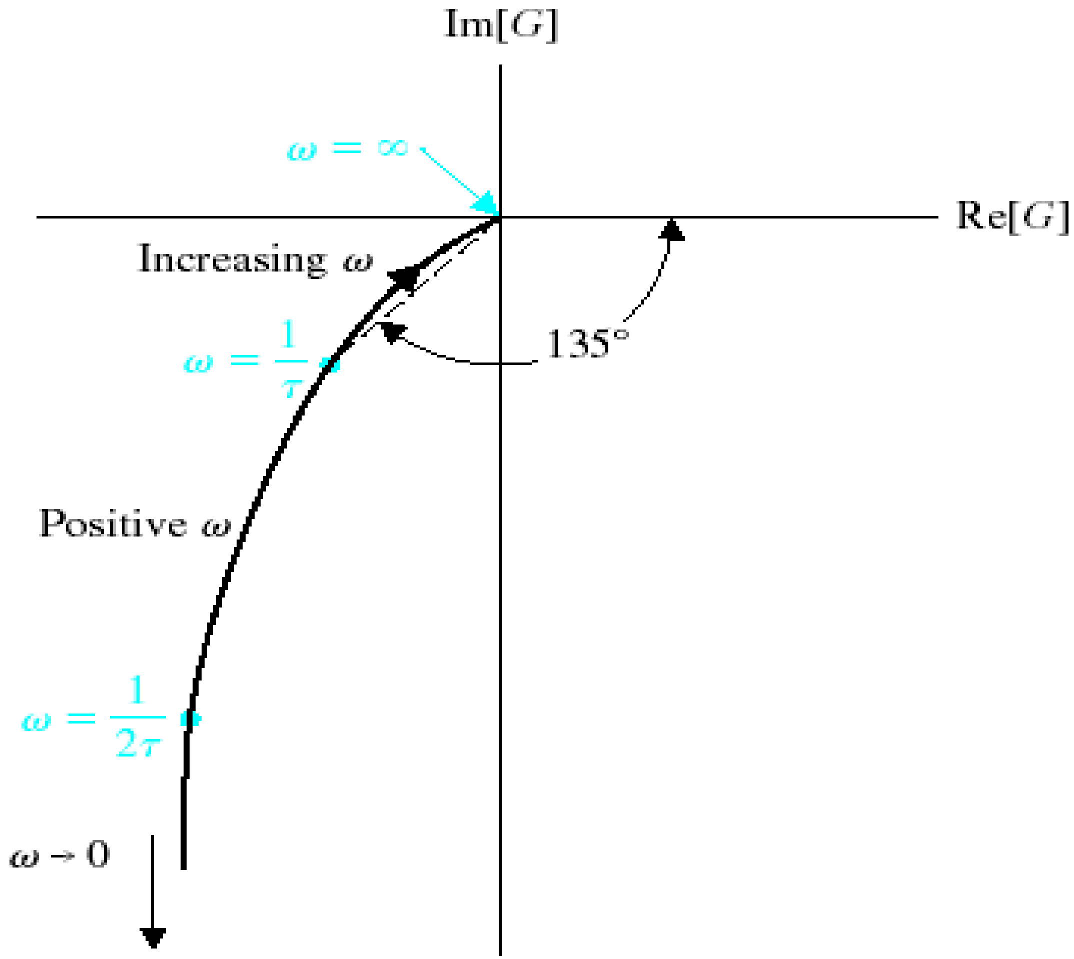
$$G1(\omega) := \frac{\frac{K}{\tau}}{j \cdot \omega \cdot \left(j \cdot \omega + \frac{1}{\tau} \right)}$$



Polar plot for $G(j\omega) = K/j\omega(j\omega\tau + 1)$. Note that $\omega = \infty$ at the origin.

Frequency Response Plots

Polar Plots



Polar plot for $G(j\omega) = K/j\omega(j\omega\tau + 1)$. Note that $\omega = \infty$ at the origin.