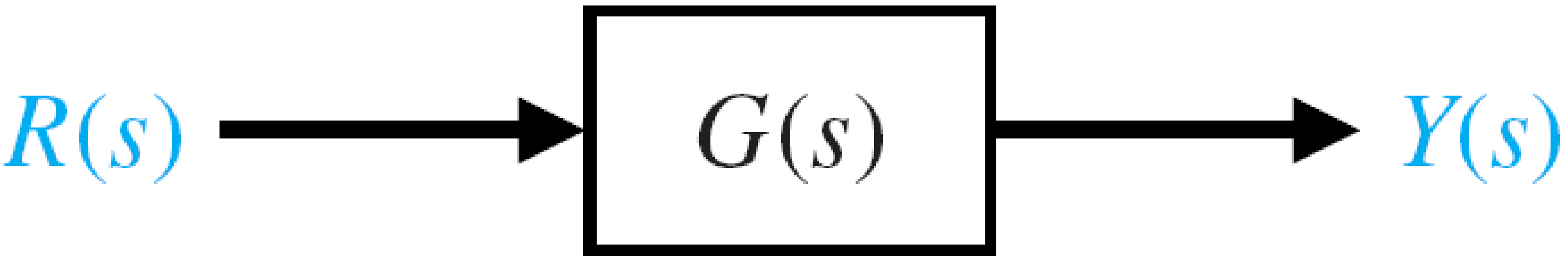
Steady-state response

If the steady-state response of the output does not agree with the steady-state of the input exactly, the system is said to have a steady-state error.

It is a measure of system accuracy when a specific type of input is applied to a control system.



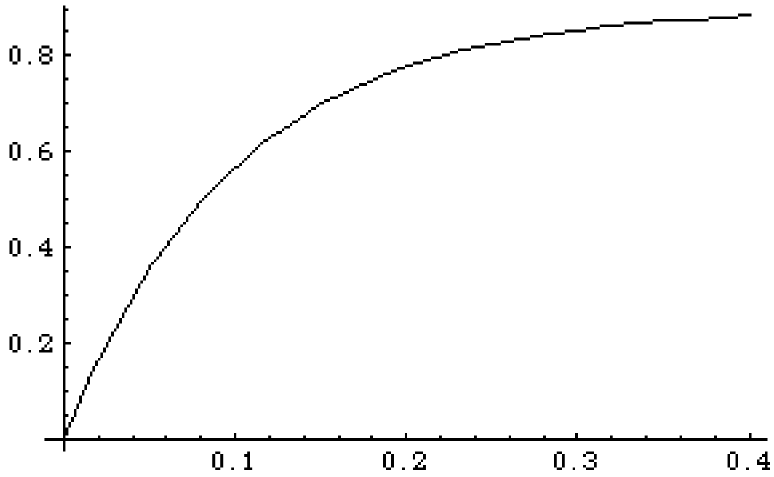
$$Y(s) = R(s)\,G(s)$$

Steady-state error

$T(s) = 9/(s + 10)$

$Y(s) = 9/s(s+10)$

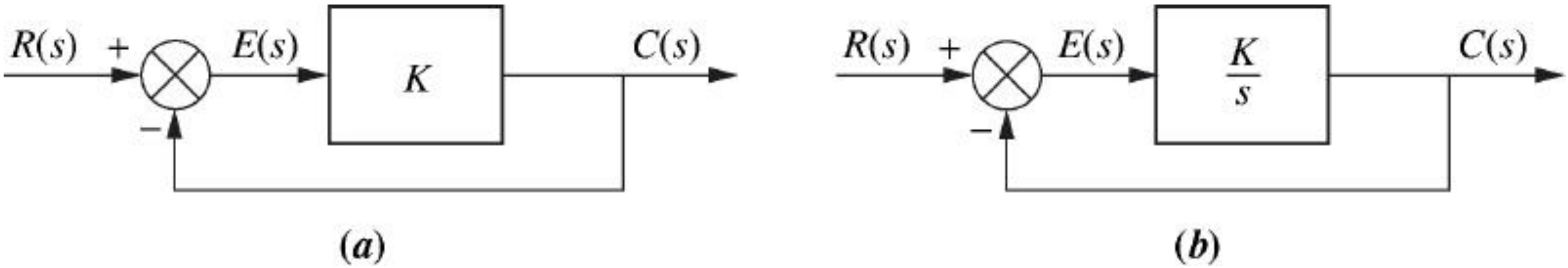
$y(t) = 0.9(1- e^{-10t})$



$y(\infty) = 0.9$

$E(s) = R(s) - Y(s)$

$e_{ss} = \lim_{s \to 0} s\,E(s) = 0.1$



$$E(s) = \frac{R(s)}{1+K}$$

$$e(t) = \frac{1}{1+K}$$

$$E(s) = \frac{R(s)}{1+\frac{K}{s}}$$

$$e(t) = e^{-K\,t}$$

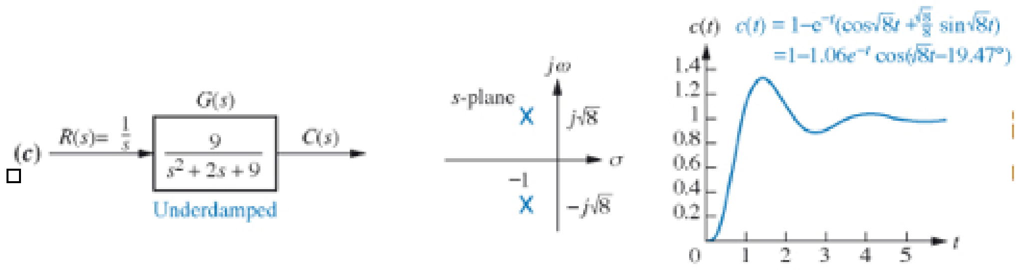
InverseLaplaceTransform$\left[\left\{\frac{1}{s+a}, \frac{a}{s+a}\right\}, s, t\right]$

$\left\{E^{-(a\ t)}, \frac{a}{E^{a\ t}}\right\}$

InverseLaplaceTransform$\left[\left\{\frac{1}{s} * \frac{1}{s+a}, \frac{1}{s} * \frac{a}{s+a}\right\}, s, t\right]$

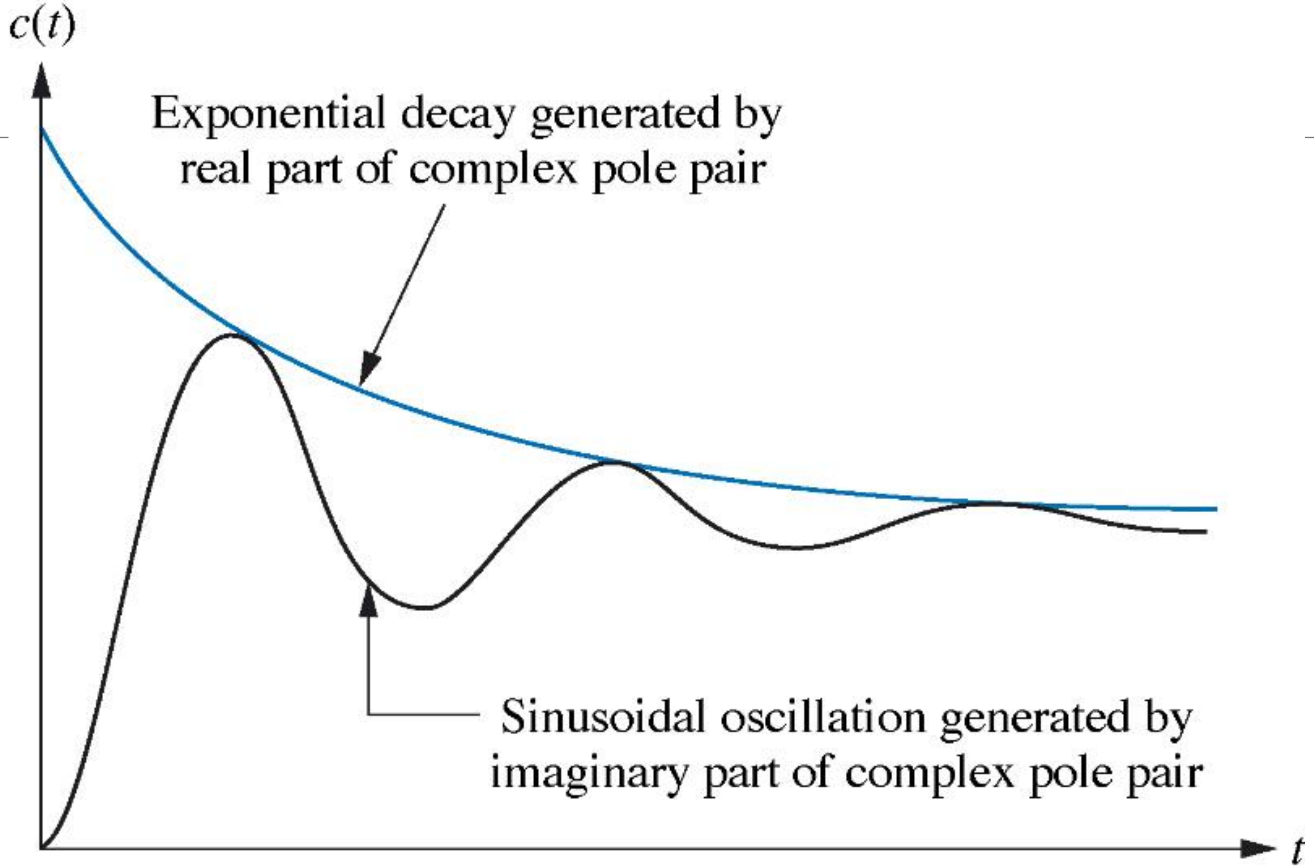
$\left\{\frac{1 - E^{-(a\ t)}}{a}, 1 - E^{-(a\ t)}\right\}$

Underdamped



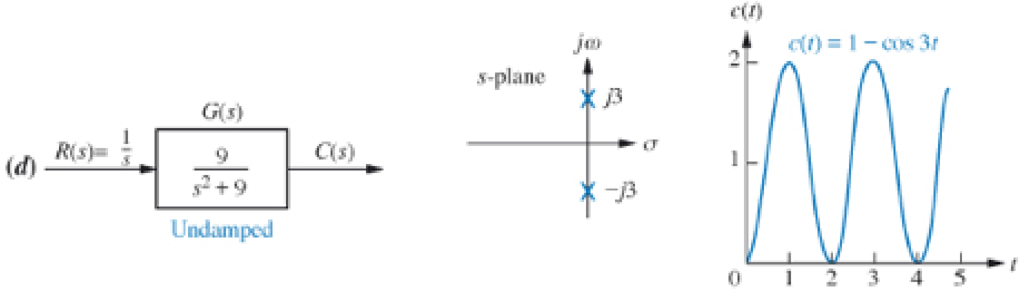
$$C(s)=\frac{9}{s(s^2+2s+9)}=\frac{9}{s(s+1-j\sqrt{8})(s+1+j\sqrt{8})}$$

$$c(t)=1-e^{-t}(\cos\sqrt{8}t+\frac{\sqrt{8}}{8}\sin\sqrt{8}t)$$



$$c(t) = 1 - e^{-t}\left(\cos\sqrt{8}t + \frac{\sqrt{8}}{8}\sin\sqrt{8}t\right)$$

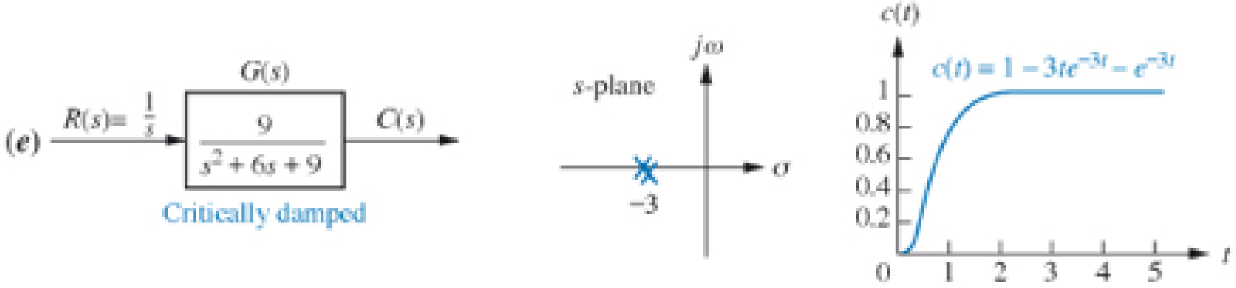
Undamped



$$C(s) = \frac{9}{s(s^2+9)}$$

$$c(t) = 1 - \cos 3t$$

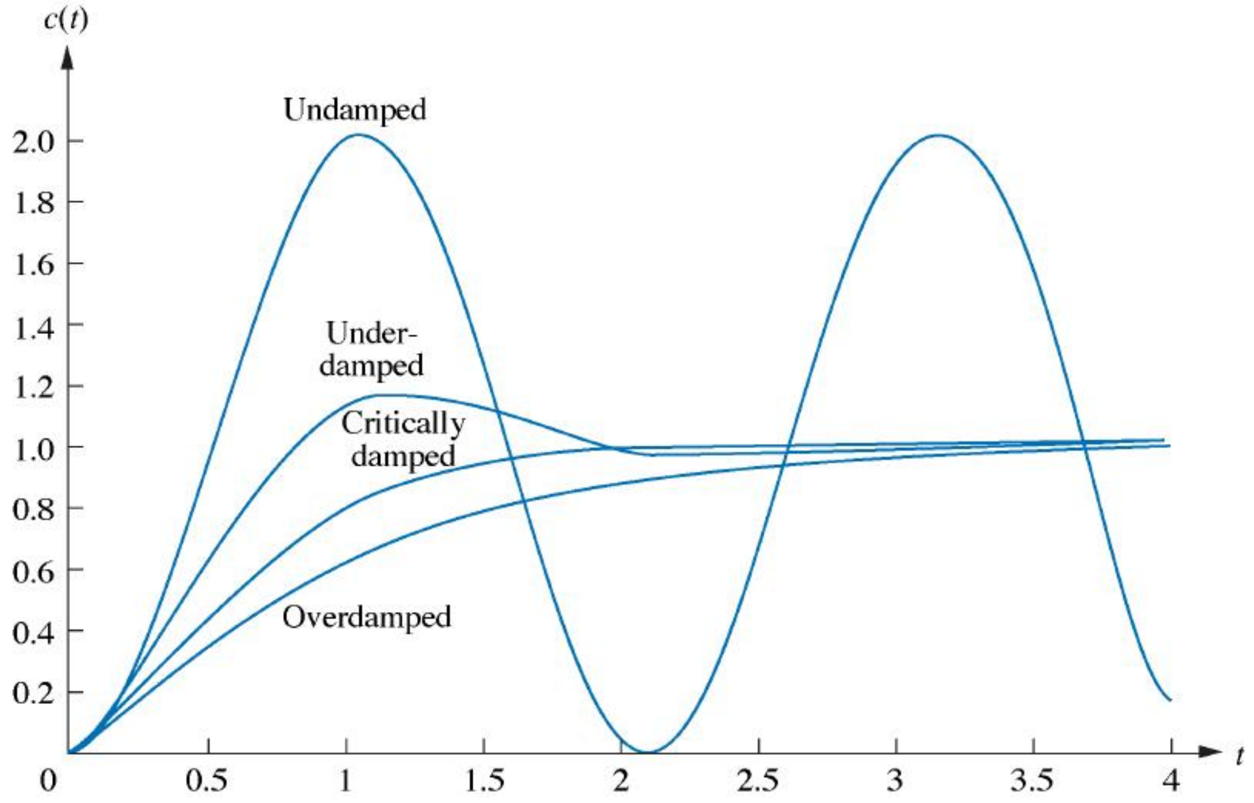
Critically damped



$$C(s) = \frac{9}{s(s^2 + 6s + 9)} = \frac{9}{s(s+3)^2}$$

$$c(t) = 1 - 3te^{-3t} - e^{-3t}$$

Step response for second order system damping cases



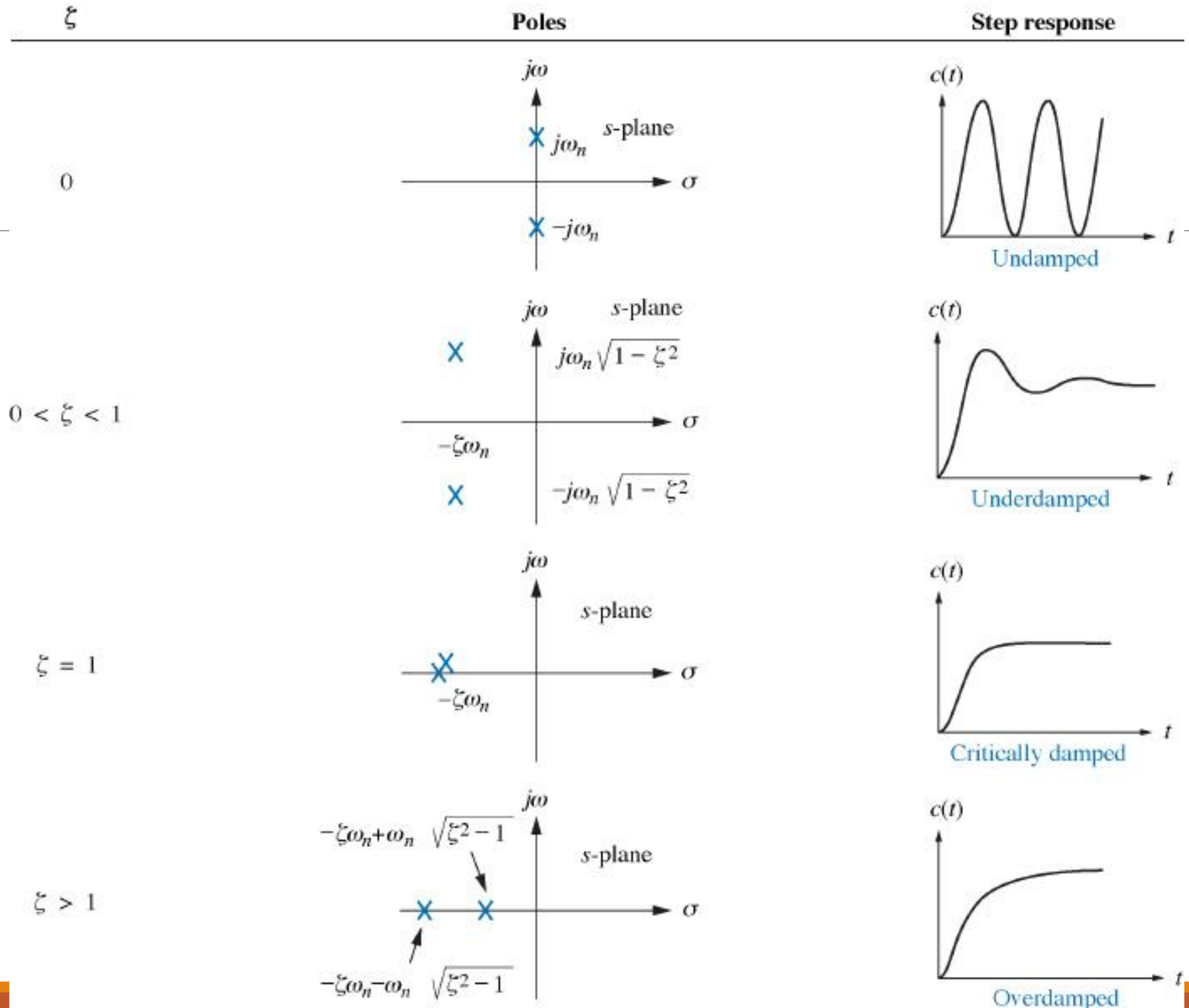

ζ
Poles
Step response
0
jω
s-plane
jω_n
σ
−jω_n
c(t)
t
Undamped
0 < ζ < 1
jω_n √(1 − ζ²)
−ζω_n
−jω_n √(1 − ζ²)
Underdamped
ζ = 1
Critically damped
ζ > 1
−ζω_n+ω_n √(ζ² − 1)
−ζω_n−ω_n √(ζ² − 1)
Overdamped

fig_04_11

Summary

Overdamped

Poles: Two real at $-\sigma_1, -\sigma_2$

Underdamped

Poles: Two complex at $-\sigma_d + j\omega_d, -\sigma_d - j\omega_d$

Undamped

Poles: Two imaginary at $+j\omega_1, -j\omega_1$

Critically damped

Poles: Two real at $-\sigma_1,$