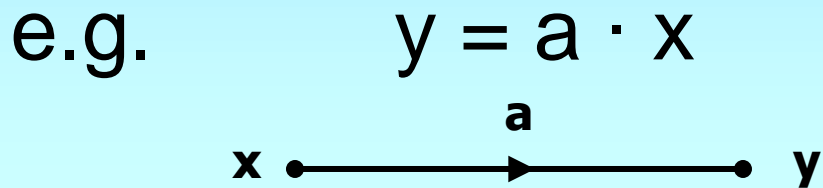
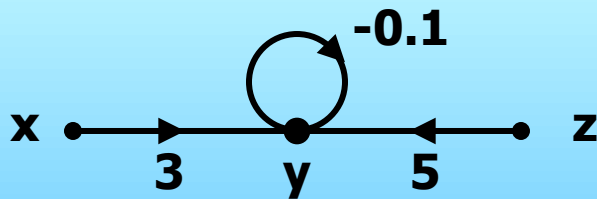


Signal Flow Graph

- nodes : variables
- branches : gains

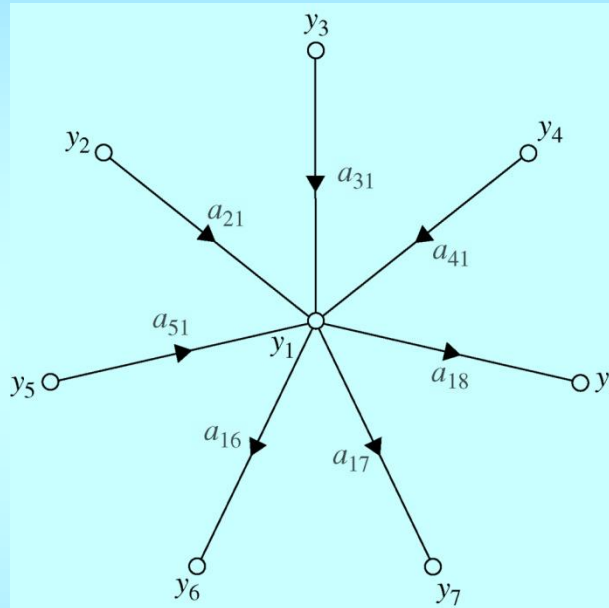


e.g. $y = 3x + 5z - 0.1y$



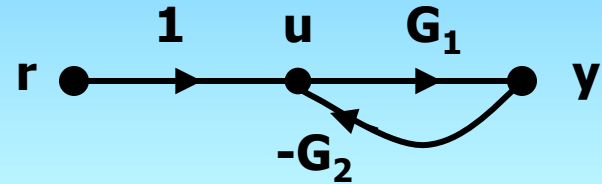
The value of a node is equal to the sum of all signal coming into the node.

The incoming signal needs to be weighted by the branch gains.



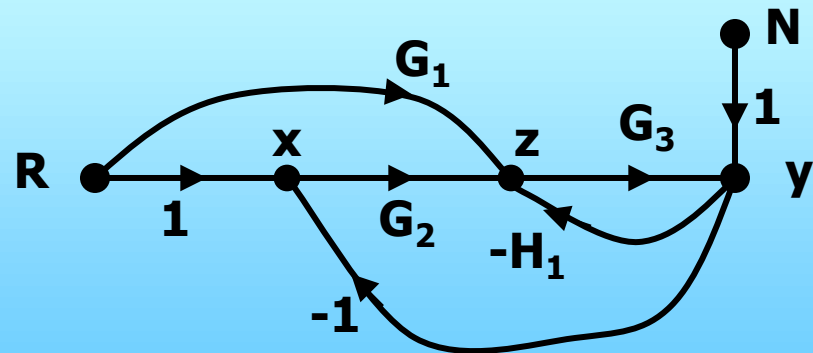
$$y_1 = a_{21}y_2 + a_{31}y_3 + a_{41}y_4 + a_{51}y_5$$

Note: y_6 , y_7 , y_8 are gained up versions of y_1 .

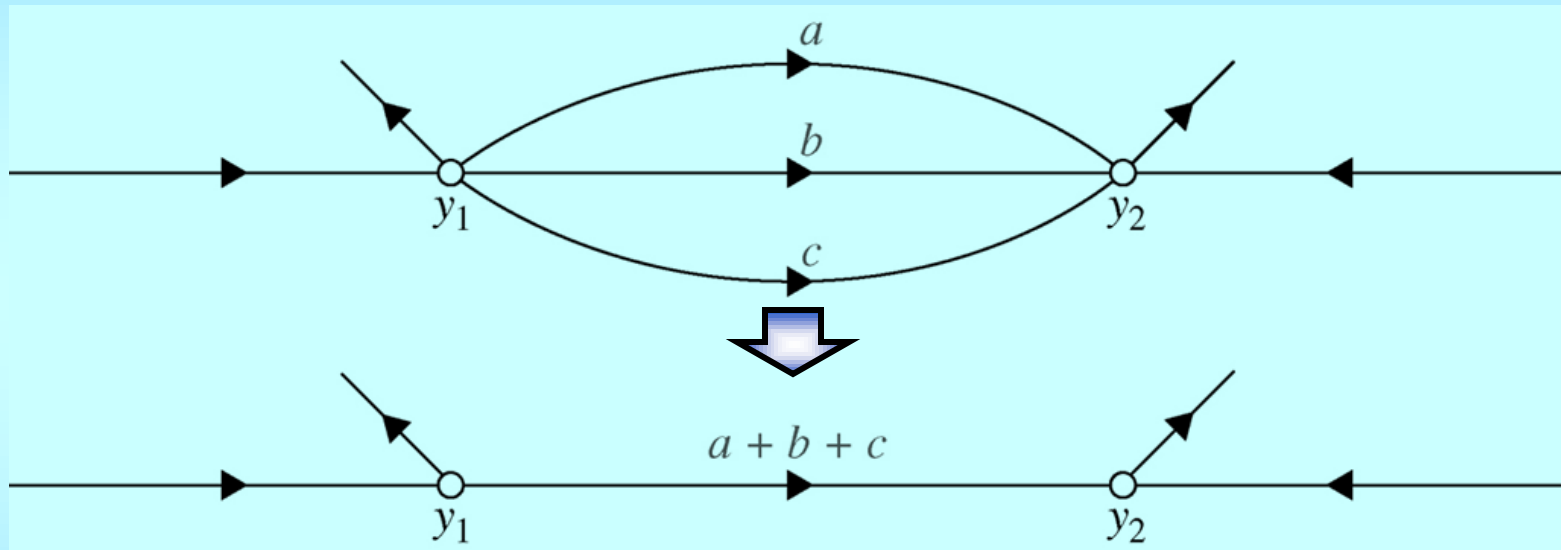


An input node is a node with only out going arrows.

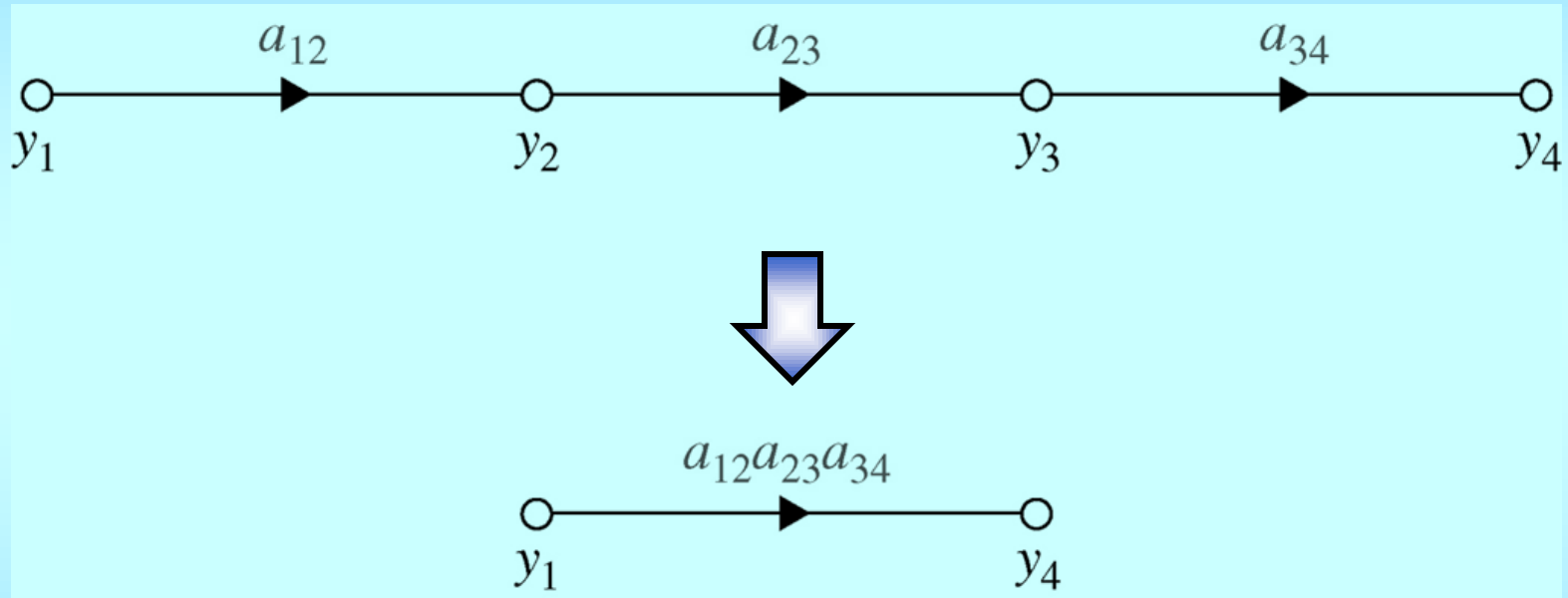
R , N and r are input nodes.



Parallel branches can be summed to form a single branch

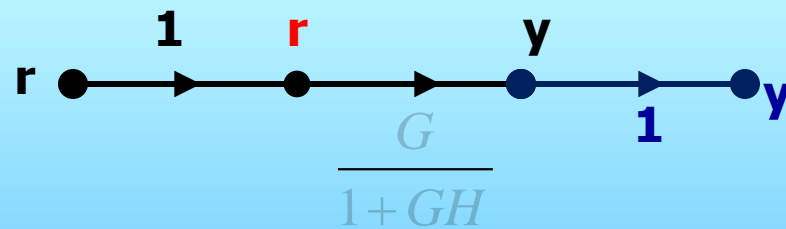
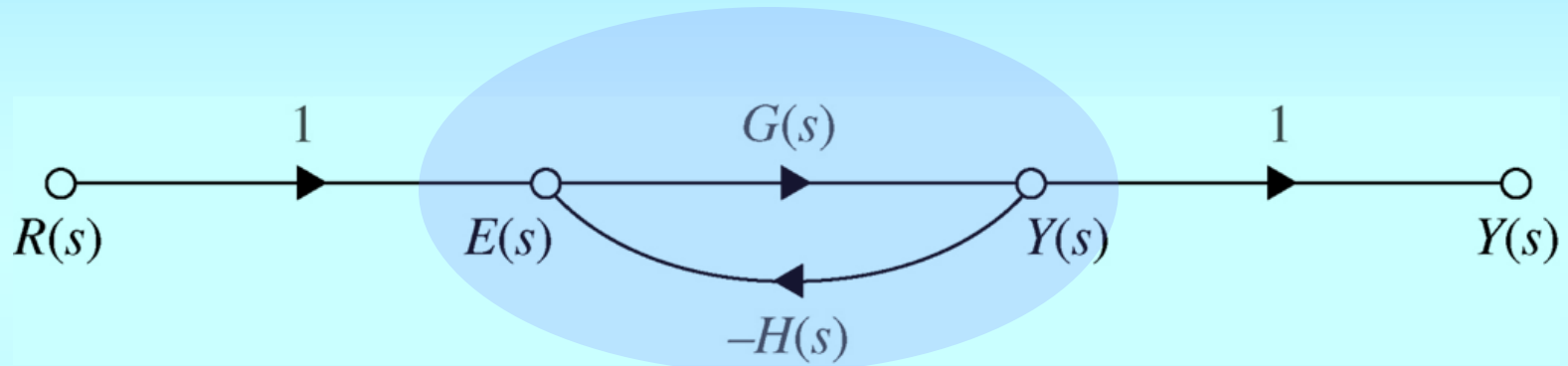


Series branches can be multiplied to form a single branch

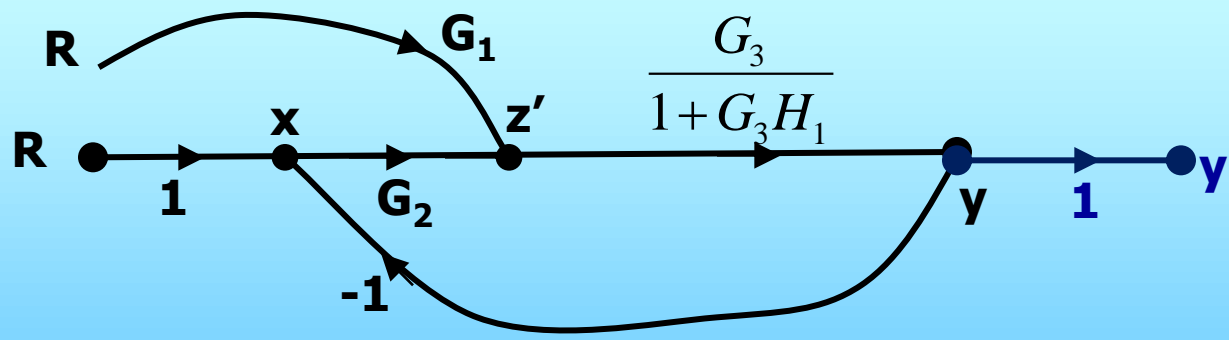
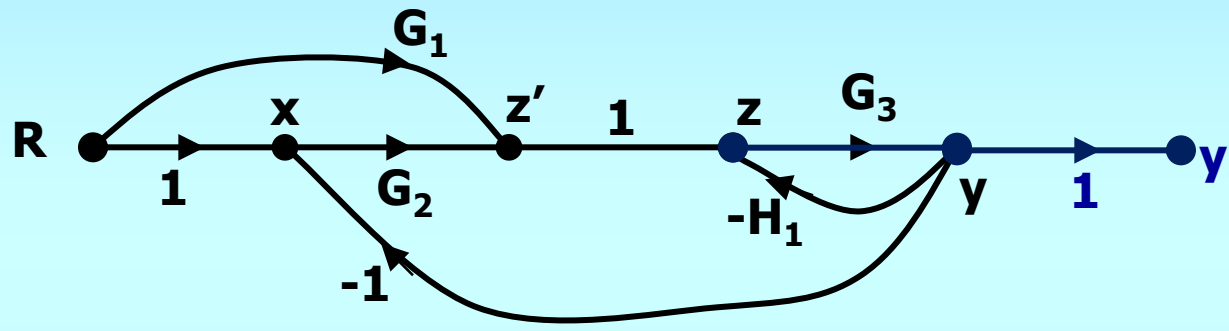
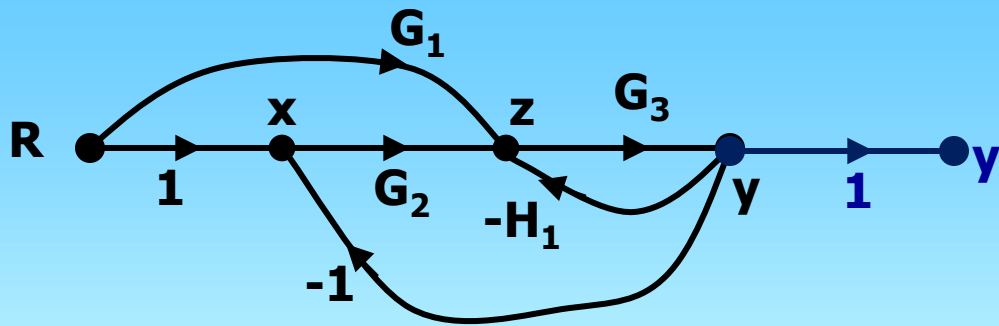


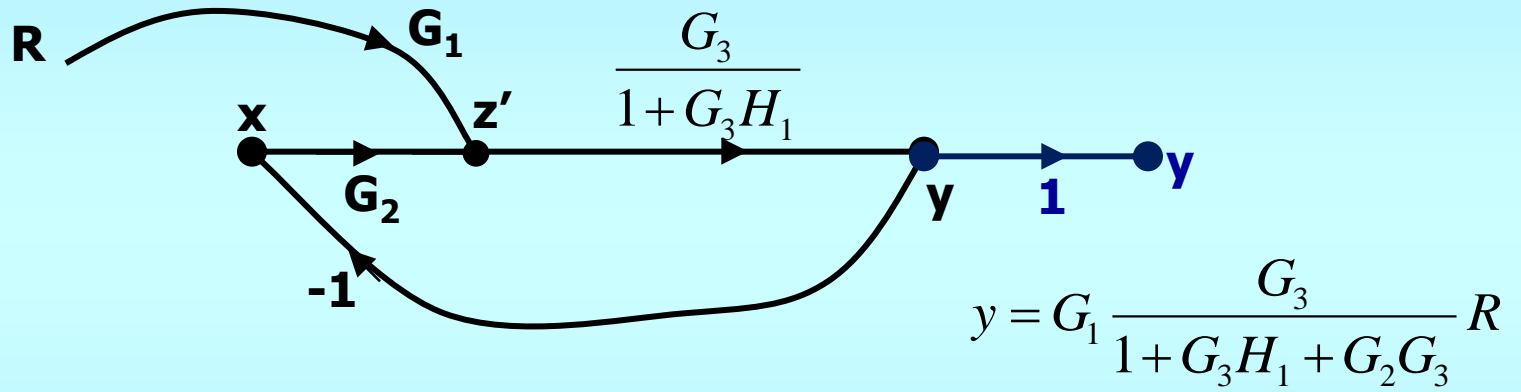
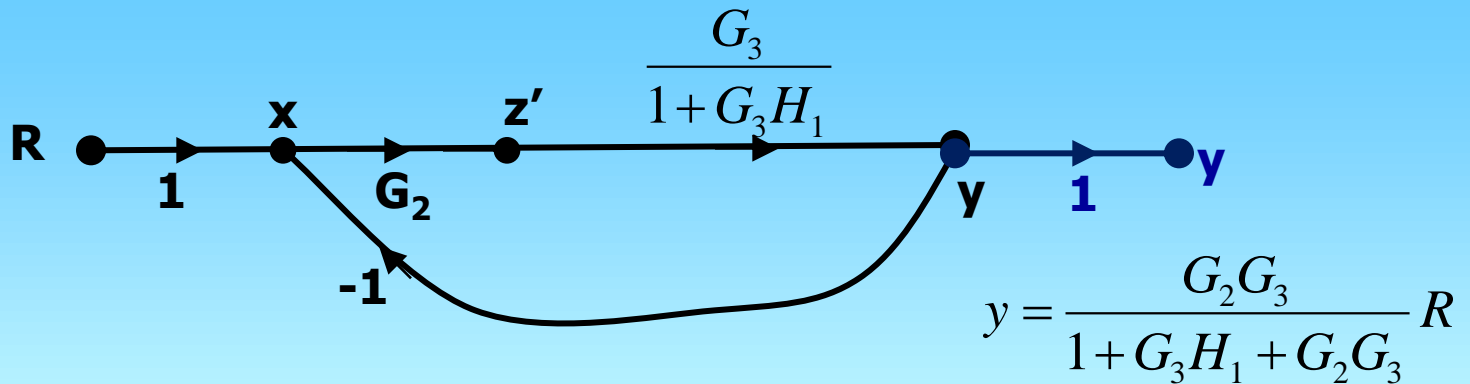
fig_03_31

Feedback connections can be simplified into a single branch



Note: the internal node E is lost!





Overall:

$$y = \left(\frac{G_2 G_3}{1 + G_3 H_1 + G_2 G_3} + \frac{G_1 G_3}{1 + G_3 H_1 + G_2 G_3} \right) R$$

$$= \frac{G_2 G_3 + G_1 G_3}{1 + G_3 H_1 + G_2 G_3} R$$

Mason's Gain Formula

- A forward path: a path from input to output
- Forward path gain M_k : total product of gains along the path
- A loop is a closed path in which you can start at any point, follow the arrows, and come back to the same point
- A loop gain L_i : total product of gains along a loop
- Loop i and loop j are non-touching if they do not share any nodes or branches

- The determinant Δ :

$$\Delta = 1 - \sum_{\text{all } i} L_i + \sum_{\text{all non-touching pairs of loops}} L_i \cdot L_j - \sum_{\text{all n.t. 3 loops}} L_i \cdot L_j \cdot L_k + \sum_{\text{all n.t. 4 loops}} L_i \cdot L_j \cdot L_k \cdot L_m - \dots$$

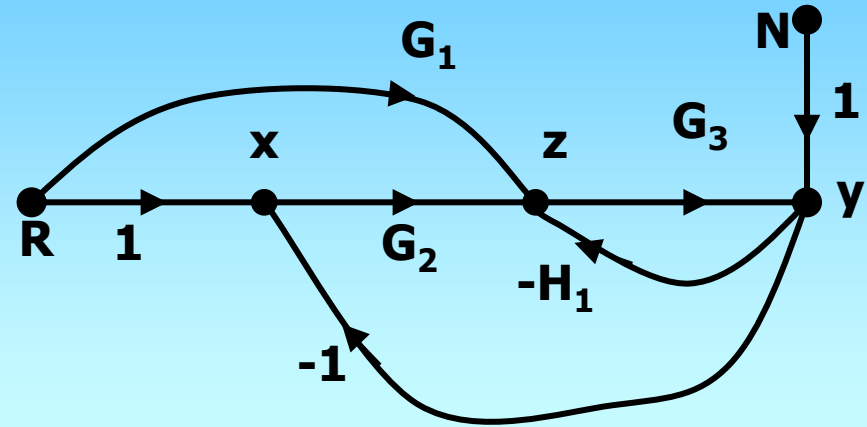
- Δ_k : The determinant of the S.F.G. after removing the k-th forward path
- Mason's Gain formula:

$$I/O \text{ T.F.} = \frac{y_o}{y_i} = \sum_{\text{all forward path } k} \frac{M_k \cdot \Delta_k}{\Delta}$$

Get T.F. from N to y

1 forward path: $N \rightarrow y$
 $M = 1$

2 loops: $L_1 = -H_1 G_3$
 $L_2 = -G_2 G_3$



$$\Delta = 1 - \sum_{all} L_i + \sum_{N.T.} L_i \cdot L_j = 1 + G_3 H_1 + G_2 G_3$$

Δ_1 : remove nodes N, y, and branch $N \rightarrow y$

All loops broken: $\rightarrow \Delta_1 = 1$

$$\frac{y}{N} = \sum \frac{M_k \Delta_k}{\Delta} = \frac{M_1 \Delta_1}{\Delta} = \frac{1}{\Delta} = \frac{1}{1 + G_3 H_1 + G_2 G_3}$$

Get T.F. from R to y

$$2 \text{ f.p.}: \quad R \longrightarrow x \longrightarrow z \longrightarrow y : \quad M_1 = G_2 G_3$$

$$R \longrightarrow z \longrightarrow y : \quad M_2 = G_1 G_3$$

$$2 \text{ loops}: \quad L_1 = -G_3 H_1$$

$$L_2 = -G_2 G_3$$

$$\Delta = 1 - \sum_{\text{all}} L_i + \sum_{\text{N.T.}} L_i \cdot L_j = 1 + G_3 H_1 + G_2 G_3$$

Δ_1 : remove M_1 and compute Δ

$$\Delta_1 = 1$$

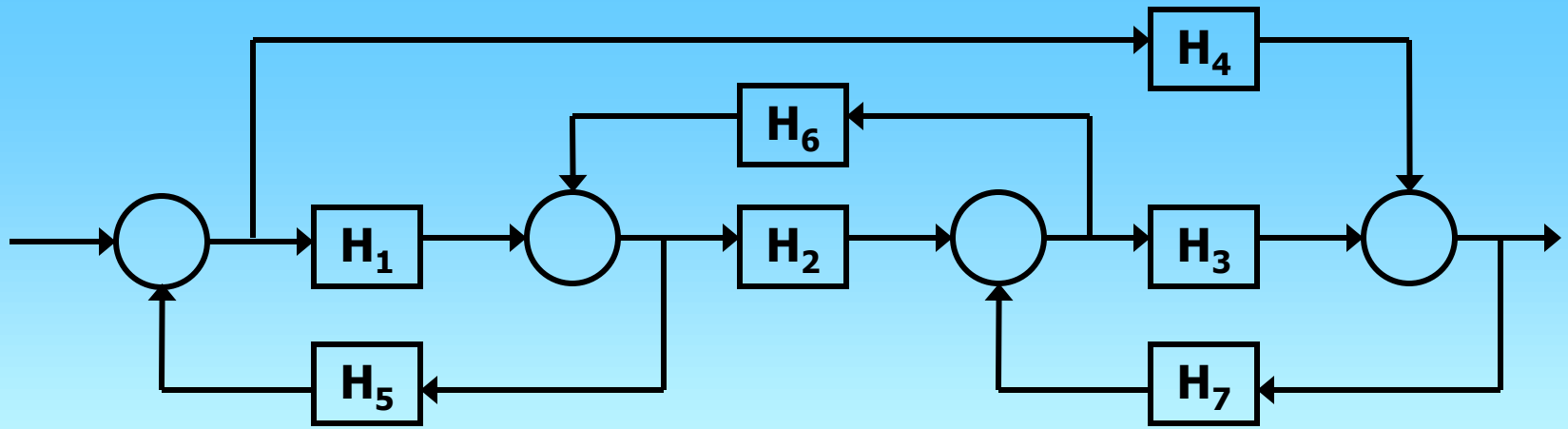
Δ_2 : remove M_2 and compute Δ

$$\Delta_2 = 1$$

$$H = \frac{y}{R} = \sum \frac{M_k \Delta_k}{\Delta} = \sum \frac{M_k}{\Delta} = \frac{G_2 G_3 + G_1 G_3}{1 + G_3 H_1 + G_2 G_3}$$

Overall:

$$y = \frac{G_2 G_3 + G_1 G_3}{1 + G_3 H_1 + G_2 G_3} R + \frac{1}{1 + G_3 H_1 + G_2 G_3} N$$



Forward path:

$$M_1 = H_1 H_2 H_3$$

$$M_2 = H_4$$

Loops:

$$L_1 = H_1 H_5$$

$$L_2 = H_2 H_6$$

$$L_3 = H_3 H_7$$

$$L_4 = H_4 H_7 H_6 H_5$$

L_1 and L_3 are non-touching

$$\begin{aligned}\Delta &= 1 - \sum L_i + L_1 L_3 \\ &= 1 - H_1 H_5 - H_2 H_6 - H_3 H_7 - H_4 H_7 H_6 H_5 \\ &\quad + H_1 H_5 H_3 H_7\end{aligned}$$

Δ_1 : If M_1 is taken out, all loops are broken.

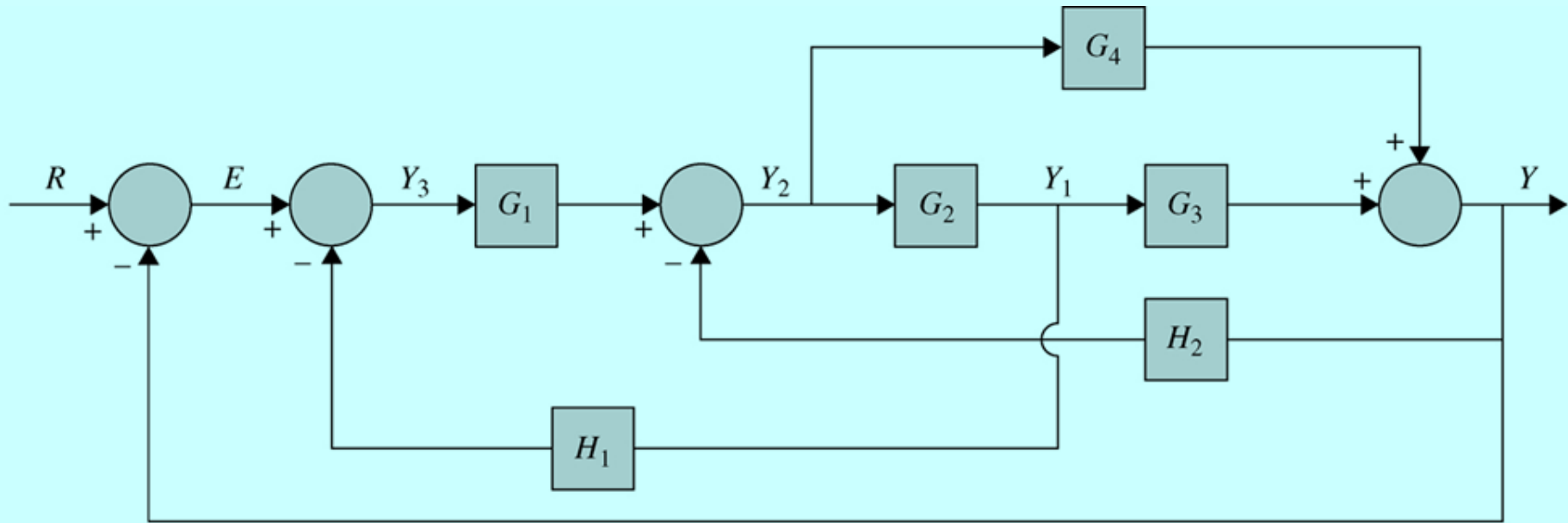
therefore $\Delta_1 = 1$

Δ_2 : If M_2 is taken out, the loop in the middle (L_2) is still there.

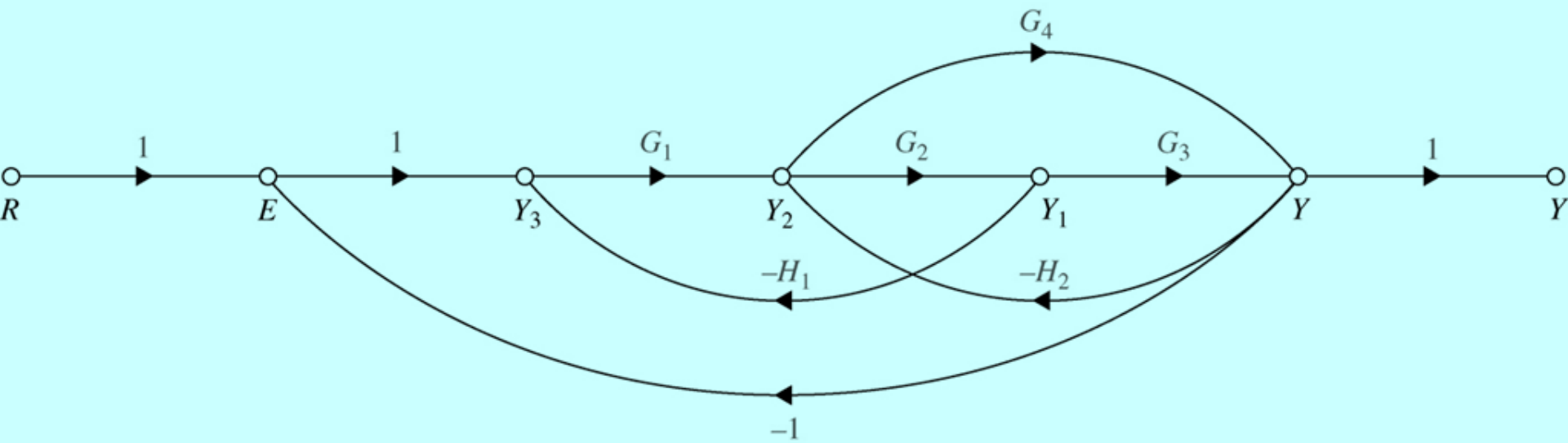
therefore $\Delta_2 = 1 - L_2 = 1 - H_2 H_6$

Total T.F.:

$$\begin{aligned}H &= \sum \frac{M_k \Delta_k}{\Delta} = \frac{M_1 + M_2 (1 - H_2 H_6)}{\Delta} \\ &= \frac{H_1 H_2 H_3 + H_4 - H_4 H_2 H_6}{1 - H_1 H_5 - H_2 H_6 - H_3 H_7 - H_4 H_7 H_6 H_5 + H_1 H_5 H_3 H_7}\end{aligned}$$



(a)



(b)

Two forward paths:

$$M_1 = G_1 G_2 G_3$$

$$M_2 = G_1 G_4$$

Five loops:

$$G_1 G_2 H_1 \quad G_2 G_3 H_2 \quad G_4 H_2$$

$$G_1 G_2 G_3 \quad G_1 G_4$$

Determinant :

$$\Delta = 1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2 + G_1G_2G_3 + G_1G_4$$

After removing forward path 1, no loops.

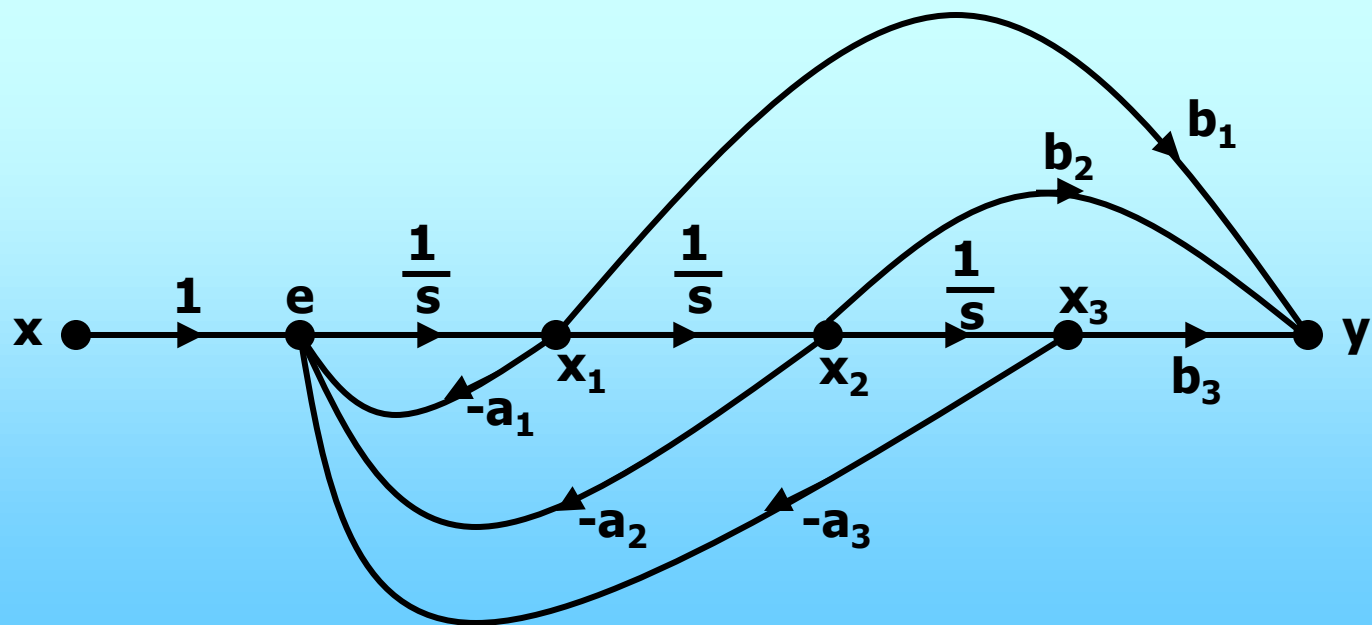
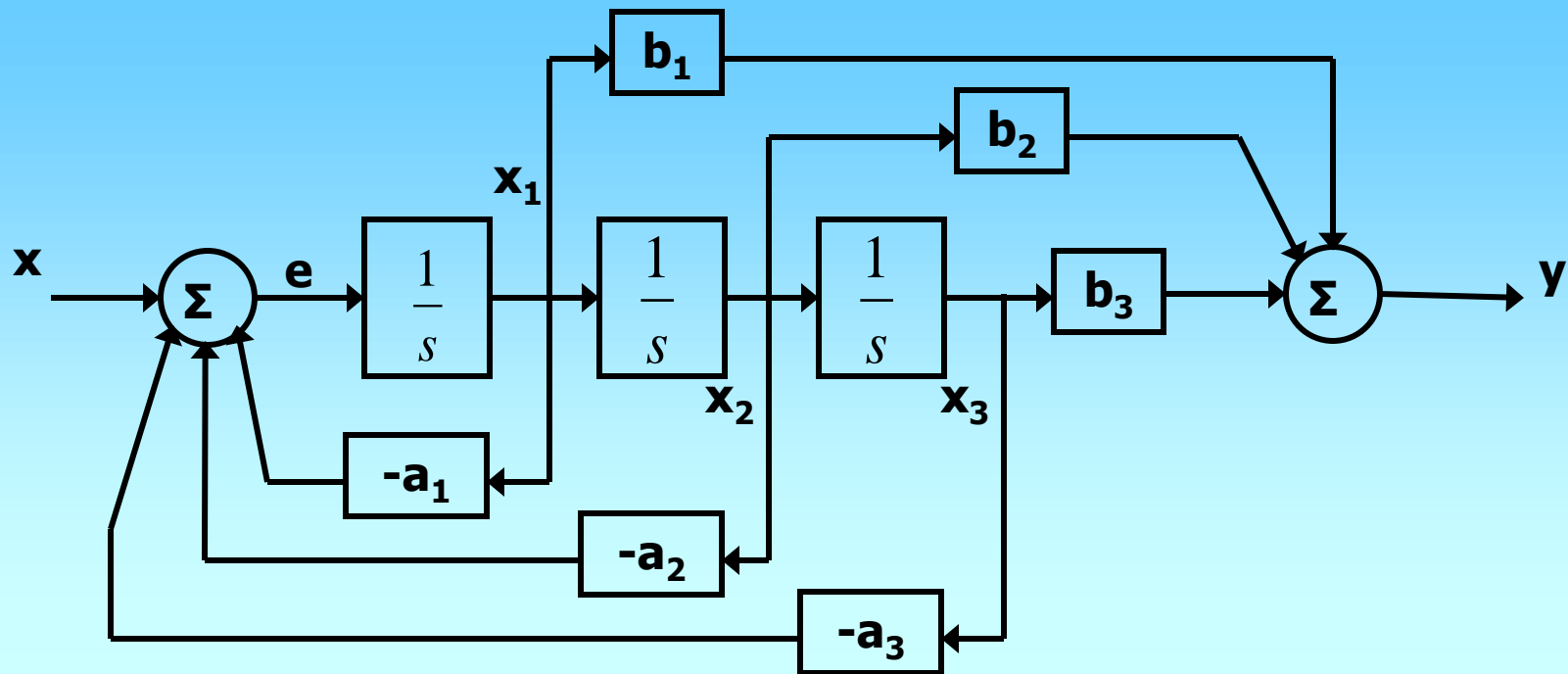
$$\therefore \Delta_1 = 1$$

After removing forward path 2, no loops.

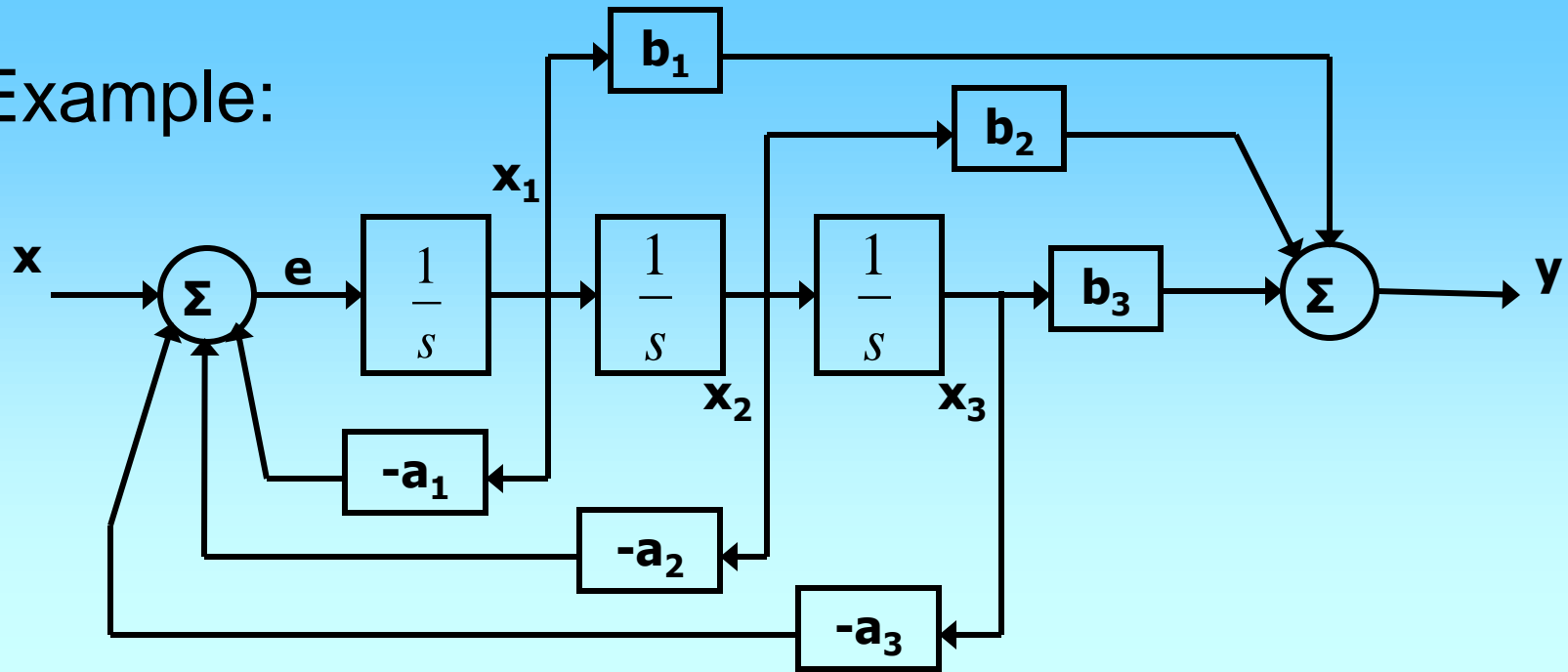
$$\therefore \Delta_2 = 1$$

Total gain :

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2 + G_1G_2G_3 + G_1G_4}$$



Example:



- Forward paths:

$$M_1 = \frac{b_3}{s^3}$$

$$M_2 = \frac{b_2}{s^2}$$

$$M_3 = \frac{b_1}{s}$$

- Loops:

$$L_1 = -\frac{a_1}{s}$$

$$L_2 = -\frac{a_2}{s^2}$$

$$L_3 = -\frac{a_3}{s^3}$$

Determinant:

$$\Delta = 1 - \sum_{\text{all } i} L_i = 1 + \frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{s^3}$$

Δ_1 : If M_1 is taken out, all loops are broken.

therefore $\Delta_1 = 1$

Δ_2 : If M_2 is taken out, all loops are broken.

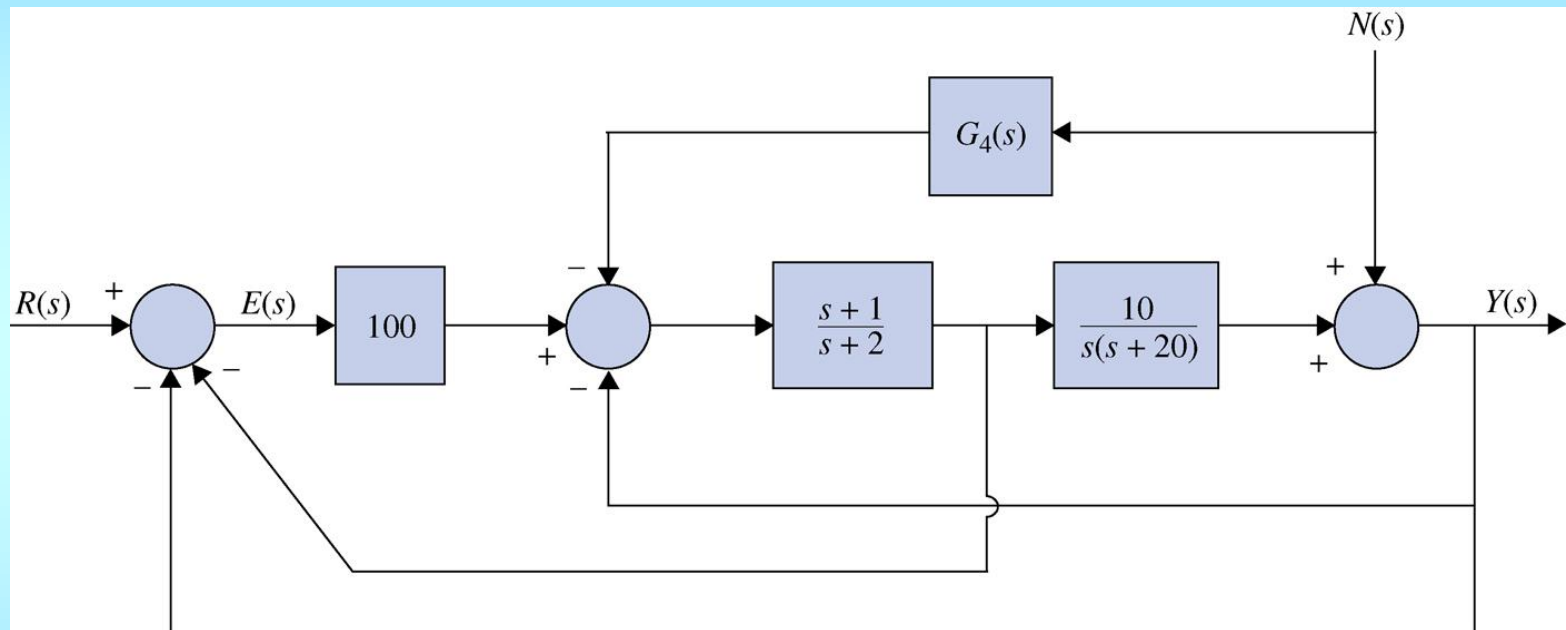
therefore $\Delta_2 = 1$

Δ_3 : Similarly, $\Delta_3 = 1$

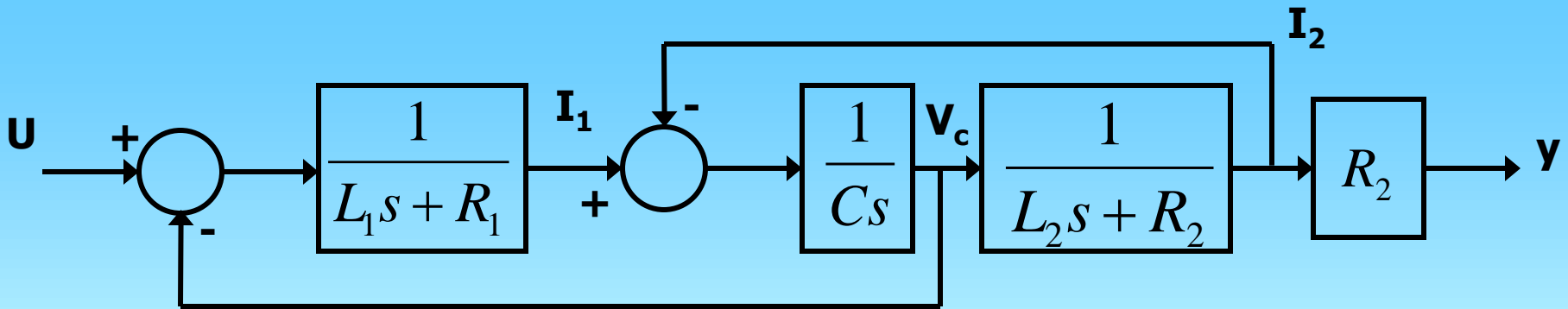
$$\therefore T.F. = \sum \frac{M_i \Delta_i}{\Delta} = \frac{M_1 M_2 M_3}{\Delta} = \frac{\frac{b_1}{s} + \frac{b_2}{s^2} + \frac{b_3}{s^3}}{1 + \frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{s^3}}$$

$$\Delta = 1 + 100 \frac{s+1}{s+2} \frac{10}{s(s+20)} + 100 \frac{s+1}{s+2} + \frac{s+1}{s+2} \frac{10}{s(s+20)}$$

$$= 1 + \frac{s+1}{s+2} \left(\frac{1000}{s(s+20)} + 1 + \frac{10}{s(s+20)} \right)$$



$$y = \frac{\frac{s+1}{s+2} \frac{1000}{s(s+20)}}{\Delta} R + \frac{\frac{s+1}{s+2} \frac{10}{s(s+20)} G_4 + 1 * \left(1 + 100 \frac{s+1}{s+2} \right)}{\Delta} N$$



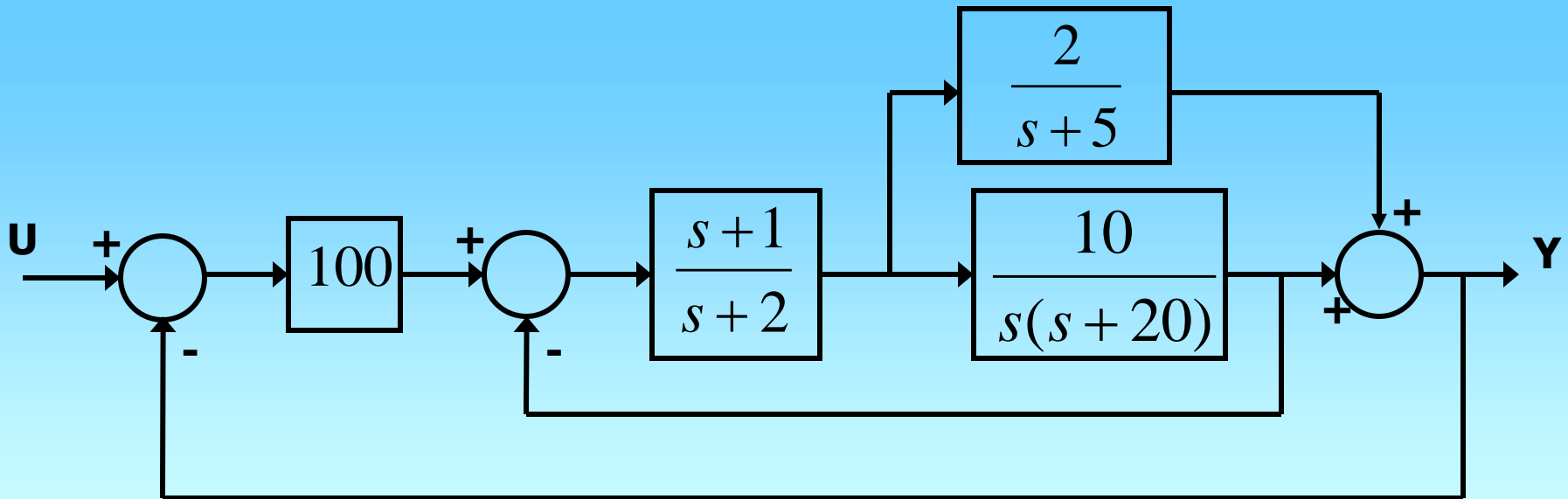
One forward path, two loops, no non-touching loops.

$$L1 = -\frac{1}{L_1s + R_1} \frac{1}{Cs}; \quad L2 = -\frac{1}{Cs} \frac{1}{L_2s + R_2}$$

$$M1 = \frac{1}{L_1s + R_1} \frac{1}{Cs} \frac{1}{L_2s + R_2} R_2; \quad \Delta1 = 1$$

$$T.F. = \frac{M1 \Delta1}{1 - L1 - L2}$$

$$T.F. = \frac{R_2}{Cs(L_1s + R_1)(L_2s + R_2) + L_2s + R_2 + L_1s + R_1}$$



Two forward paths, three loops, no non-touching loops.

$$M1 = 100 \frac{s+1}{s+2} \frac{10}{s(s+20)}; \quad \Delta1 = 1$$

$$M2 = 100 \frac{s+1}{s+2} \frac{2}{s+5}; \quad \Delta2 = 1$$

$$L1 = -100 \frac{s+1}{s+2} \frac{10}{s(s+20)};$$

$$L2 = -\frac{s+1}{s+2} \frac{10}{s(s+20)}; \quad L3 = -100 \frac{s+1}{s+2} \frac{2}{s+5}$$