Digital Modulation Techniques

- Digital band-pass modulation techniques
  - Amplitude-shift keying
  - Phase-shift keying
  - Frequency-shift keying
- Receivers
  - Coherent detection
    - *The receiver is synchronized to the transmitter with respect to carrier phases*
  - Noncoherent detection
    - *The practical advantage of reduced complexity but at the cost of degraded performance*
- Lesson 1: Each digital band-pass modulation scheme is defined by a transmitted signal with a unique phasor representation.
- Lesson 2 : At the receiving end, digital demodulation techniques encompass different forms, depending on whether the receiver is coherent or noncoherent
- Lesson 3 : Two ways of classifying digital modulation schemes are (a) by the type of modulation used, and (b) whether the transmitted data stream is in binary or M-ary form.

7.1 Some Preliminaries

- Given a binary source
  - The modulation process involves switching ore keying the amplitude, phase, or frequency of a sinusoidal carrier wave between a pair of possible values in accordance with symbols 0 and 1.

$$c(t) = A_c \cos(2\pi f_c t + \phi_c) \quad (7.1)$$

  - All three of them are examples of a band-pass process

1. Binary amplitude shift-keying (BASK)
   - *The carrier amplitude is keyed between the two possible values used to represent symbols 0 and 1*
2. Binary phase-shift keying (BPSK)
   - *The carrier phase is keyed between the two possible values used to represent symbols 0 and 1.*
3. Binary frequency-shift keying (BFSK)
   - *The carrier frequency is keyed between the two possible values used to represent symbols 0 and 1.*

- Decreasing the bit duration T_b has the effect of increasing the transmission bandwidth requirement of a binary modulated wave.

$$A_c = \sqrt{\frac{2}{T_b}} \quad (7.2)$$

$$c(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t + \phi_c) \quad (7.3)$$

- Differences that distinguish digital modulation from analog modulation.
  - *The transmission bandwidth requirement of BFSK is greater than that of BASK for a given binary source.*
  - *However, the same does not hold for BPSK.*

- Band-Pass Assumption
  - The spectrum of a digital modulated wave is centered on the carrier frequency f_c
  - Under the assumption $f_c >> W$,
    - *There will be no spectral overlap in the generation of s(t)*
    - *The spectral content of the modulated wave for positive frequency is essentially separated from its spectral content for negative frequencies.*

$$s(t) = b(t)c(t) \quad (7.4)$$

$$s(t) = \sqrt{\frac{2}{T_b}}\, b(t)\cos(2\pi f_c t) \quad (7.5)$$

  - The transmitted signal energy per bit

$$E_b = \int_0^{T_b} |s(t)|^2 dt$$

$$= \frac{2}{T_b}\int_0^{T_b} |b(t)|^2 \cos^2(2\pi f_c t)dt \quad (7.6)$$

$$\cos^2(2\pi f_c t) = \frac{1}{2}[1 + \cos(4\pi f_c t)]$$

$$E_b = \frac{1}{T_b}\int_0^{T_b} |b(t)|^2 dt + \frac{1}{T_b}\int_0^{T_b} |b(t)|^2 \cos^2(4\pi f_c t)dt \quad (7.7)$$

- The band-pass assumption implies that $|b(t)|^2$ is essentially constant over one complete cycle of the sinusoidal wave $\cos(4\pi f_c t)$

$$\int_0^{T_b} |b(t)|^2 \cos^2(4\pi f_c t)dt \approx 0$$

$$E_b \approx \frac{1}{T_b}\int_0^{T_b} |b(t)|^2 dt \quad (7.8)$$

- For linear digital modulation schemes governed by Eq.(7.5), the transmitted signal energy is a scaled version of the energy in the incoming binary wave responsible for modulating the sinusoidal carrier.

7.2 Binary Amplitude-Shift Keying

- The ON-OFF signaling variety

$$b(t) = \begin{cases} \sqrt{E_b}, & \text{for binary symbol } 1 \\ 0, & \text{for binary symbol } 0 \end{cases} \quad (7.9)$$

$$s(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_c t), & \text{for symbol } 1 \\ 0, & \text{for symbol } 0 \end{cases} \quad (7.10)$$

- The average transmitted signal energy is (the two binary symbols must by equiprobable)

$$E_{\text{av}} = \frac{E_b}{2} \quad (7.11)$$

- Generation and Detection of ASK Signals
  - Generation of ASK signal : by using a produce modulator with two inputs
    - *The ON-OFF signal of Eq. (7.9)*

$$b(t) = \begin{cases} \sqrt{E_b}, & \text{for binary symbol } 1 \\ 0, & \text{for binary symbol } 0 \end{cases} \qquad (7.9)$$

    - *The sinusoidal carrier wave*

$$c(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

  - Detection of ASK signal
    - *The simplest way is to use an envelope detector, exploiting the nonconstant-envelope property of the BASK signal*

Fig. 7.1

Fig.7.1

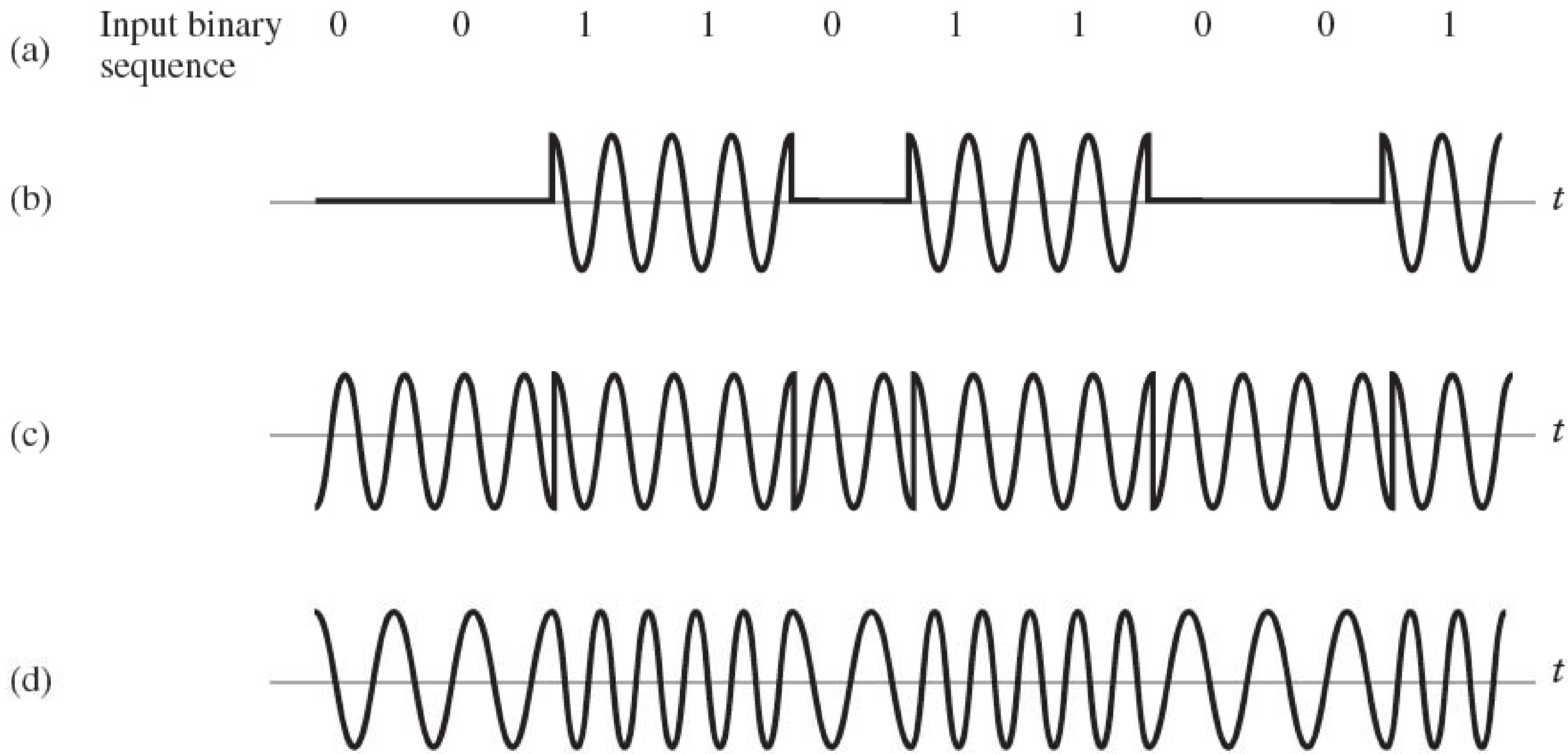

FIGURE 7.1 The three basic forms of signaling binary information. (*a*) Binary data stream. (*b*) Amplitude-shift keying. (*c*) Phase-shift keying. (*d*) Frequency-shift keying with continuous phase.

- Computation Experiment I: Spectral Analysis of BASK
  - The objective
    1) *To investigate the effect of varying the carrier frequency fc on the power spectrum of the BASK signal s(t), assuming that the wave is fixed.*
    2) *Recall that the power spectrum of a signal is defined as 10 times the logarithm of the squared magnitude spectrum of the signal*
    3) *To investigate the effect of varying the frequency of the square wave on the spectrum of the BASK signal, assuming that the sinusoidal carrier wave is fixed.*

Fig. 7.2(a)

Fig. 7.2(b)

  - The two parts of Fig. 7.2 correspond to objective 1)

Fig. 7.3(a)

Fig. 7.3(b)

  - The two parts of Fig. 7.3 correspond to objective 2)

Fig.7.2(a)

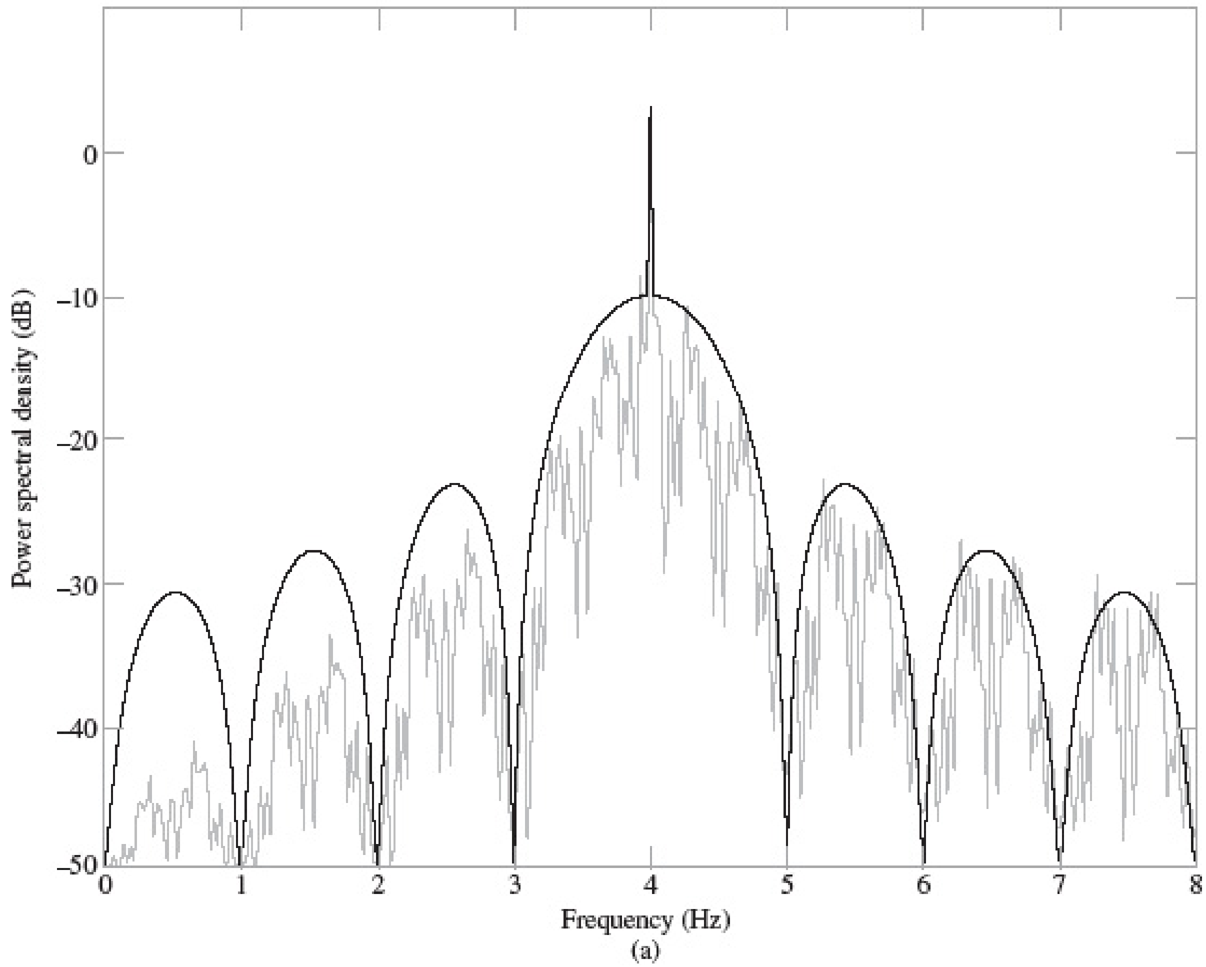

FIGURE 7.2 Power spectra of BASK signal produced by square wave as the modulating signal for varying modulation frequency: (*a*) $f_c = 4$ Hz and $T_b = 1$ s; (*b*) $f_c = 8$ Hz and $T_b = 1$ s.

Fig.7.2(b)

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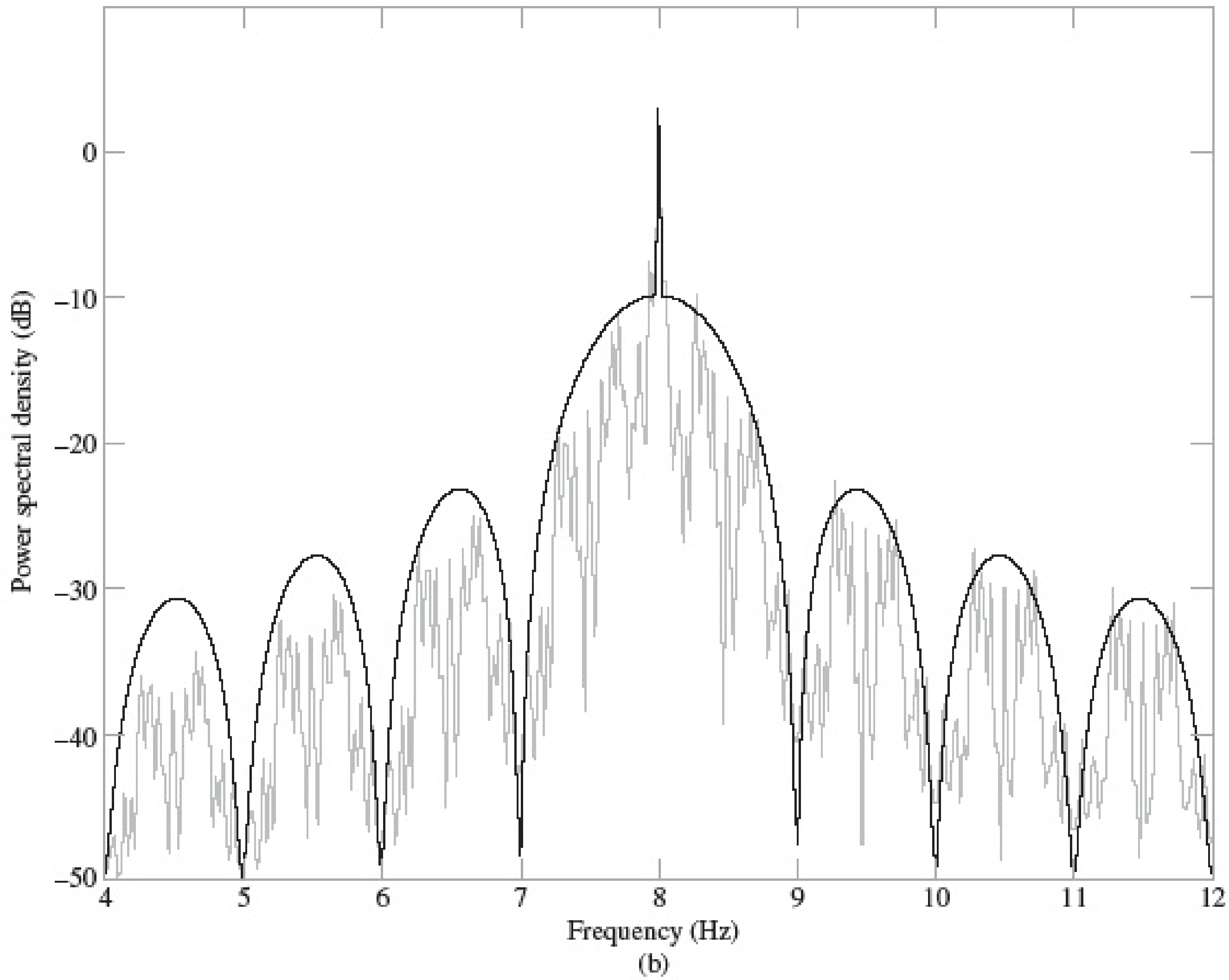

FIGURE 7.2 Power spectra of BASK signal produced by square wave as the modulating signal for varying modulation frequency: (*a*) $f_c = 4$ Hz and $T_b = 1$ s; (*b*) $f_c = 8$ Hz and $T_b = 1$ s.

Fig.7.3(a)

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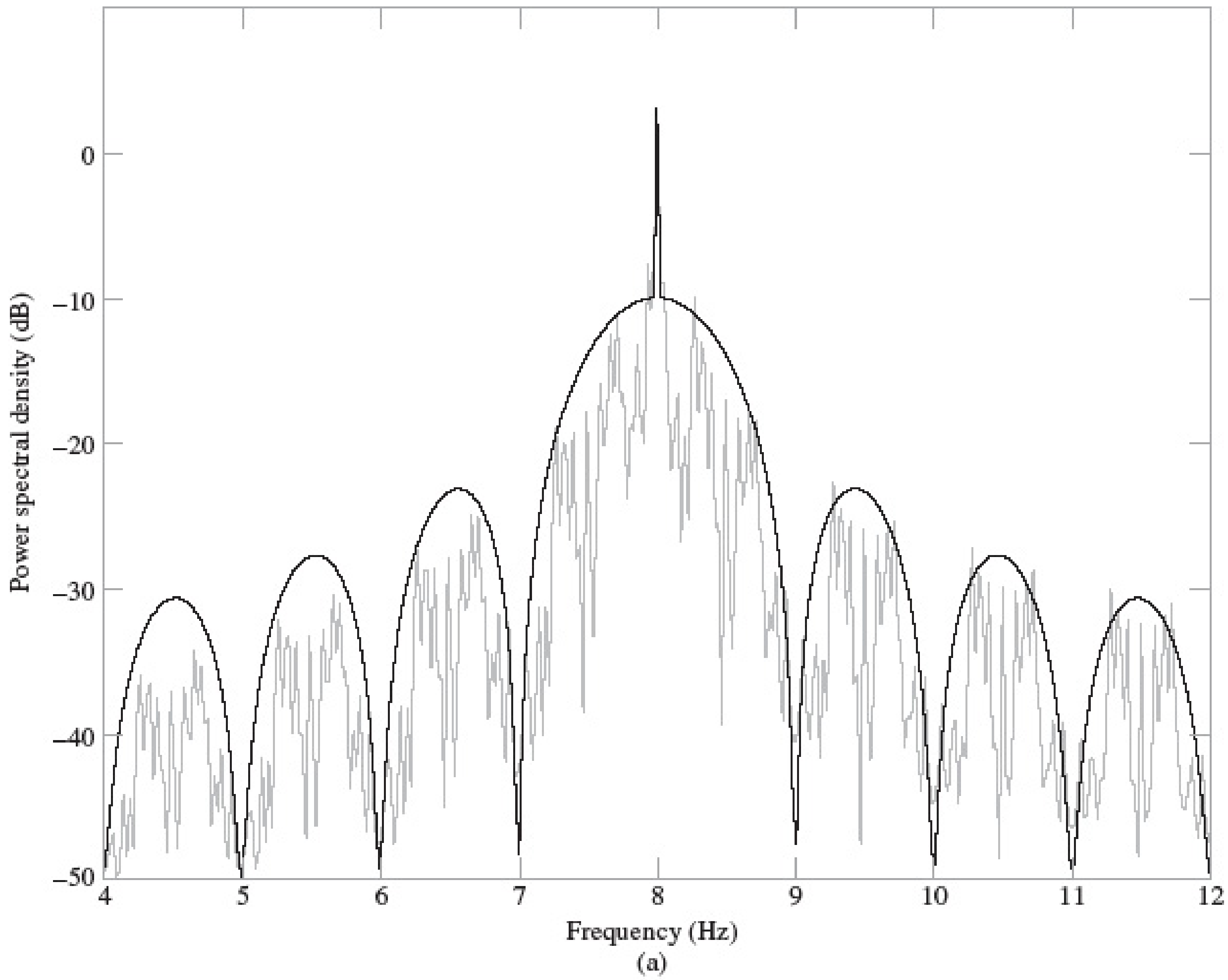

FIGURE 7.3 Power spectra of BASK signal produced by square wave as the modulating signal for varying bit duration: (*a*) f_c = 8 Hz and T_b = 1 s; (*b*) f_c = 8 Hz and T_b = 1/2 s

Fig.7.3(b)

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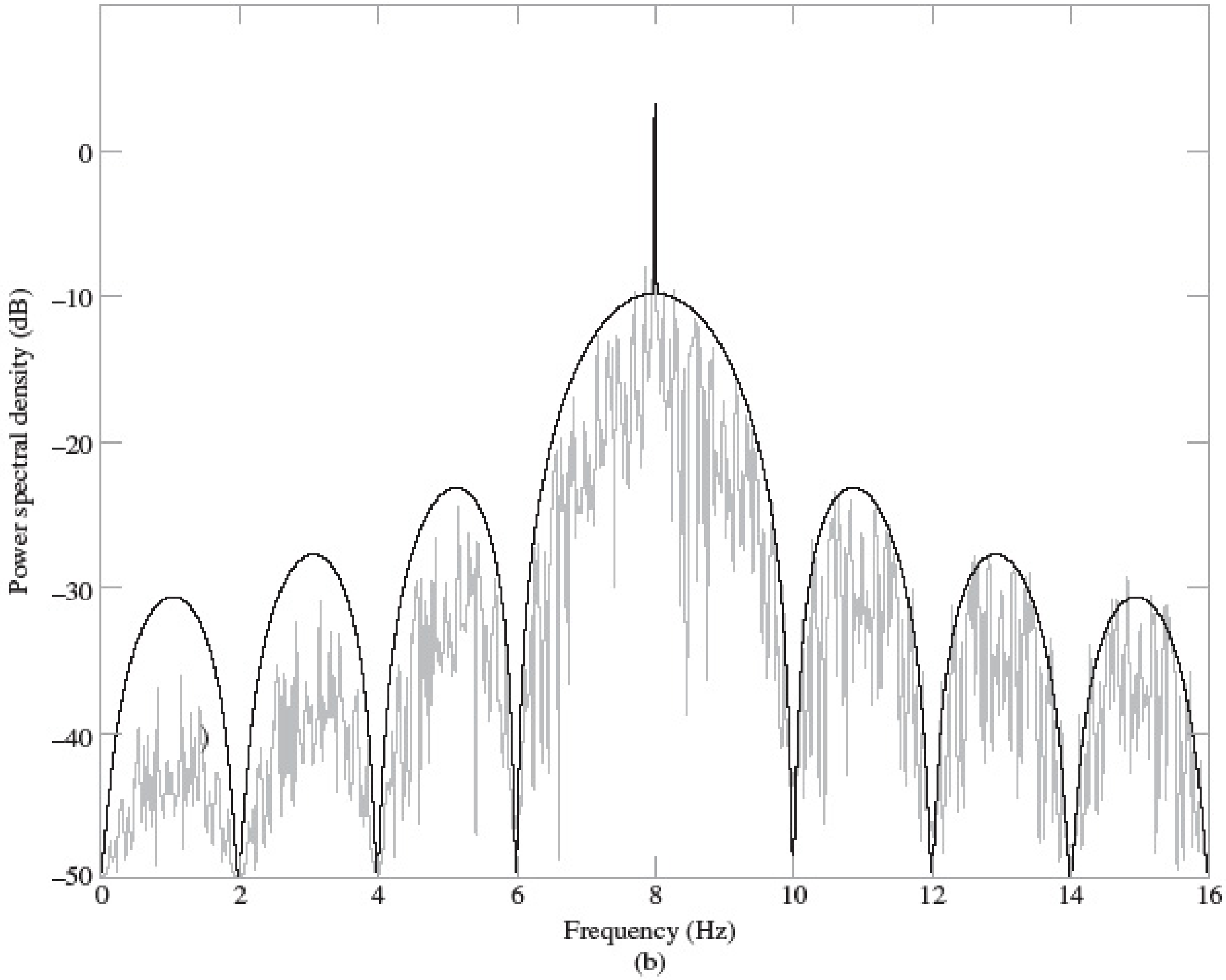

Figure 7.3 Power spectra of BASK signal produced by square wave as the modulating signal for varying bit duration: (*a*) $f_c = 8$ Hz and $T_b = 1$ s; (*b*) $f_c = 8$ Hz and $T_b = 1/2$ s

1. The spectrum of the BASK signal contains a line component at $f=f_c$
2. When the square wave is fixed and the carrier frequency is doubled, the mid-band frequency of the BASK signal is likewise doubled.
3. When the carrier is fixed and the bit duration is halved, the width of the main lobe of the sinc function defining the envelope of the BASK spectrum is doubled, which, in turn, means that the transmission bandwidth of the BASK signal is doubled.
4. The transmission bandwidth of BASK, measured in terms of the width of the main lobe of its spectrum, is equal to $2/T_b$, where T_b is the bit duration.

7.3 Phase-Shift Keying

- Binary Phase-Shift Keying (BPSK)
  - The special case of double-sideband suppressed-carried (DSB-SC) modulation
  - The pair of signals used to represent symbols 1 and 0,

$$s_i(t)=\begin{cases}\sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t), & \text{for symbol 1 corresponding to } i=1\\ \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t+\pi)=-\sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t), & \text{for symbol 0 corresponding to } i=2\end{cases} \quad (7.12)$$

  - An antipodal signals
    - *A pair of sinusoidal wave, which differ only in a relative phase-shift of π radians.*

1. The transmitted energy per bit, E_b is constant, equivalently, the average transmitted power is constant.
2. Demodulation of BPSK cannot be performed using envelope detection, rather, we have to look to coherent detection as described next.

- Generation and Coherent Detection of BPSK Signals

1. Generation

- A product modulator consisting of two component

   1) *Non-return-to-zero level encoder*
      - *The input binary data sequence is encoded in polar form with symbols 1 and 0 represented by the constant-amplitude levels ; $\sqrt{E_b}$ and - $\sqrt{E_b}$,*

   2) *Product modulator*
      - *Multiplies the level-encoded binary wave by the sinusoidal carrier c(t) of amplitude $\sqrt{2/T_b}$ to produce the BPSK signal*

Fig. 7.4

2. Detection

- A receiver that consists of four sections
  1) *Product modulator; supplied with a locally generated reference signal that is a replica of the carrier wave c(t)*
  2) *Low-pass filter; designed to remove the double-frequency components of the product modulator output*
  3) *Sampler ; uniformly samples the output of the low-pass filter, the local clock governing the operation of the sampler is synchronized with the clock responsible for bit-timing in the transmitter.*
  4) *Decision-making device ; compares the sampled value of the low-pass filter's output to an externally supplied threshold. If the threshold is exceed, the device decides in favor of symbol 1, otherwise, it decides in favor of symbol 0.*
- What should the bandwidth of the filter be ?
  - *The bandwidth of the low-pass filter in the coherent BPSK receiver has to be equal to or greater than the reciprocal of the bit duration T_b for satisfactory operation of the receiver.*

Fig. 7.4

Fig.7.4

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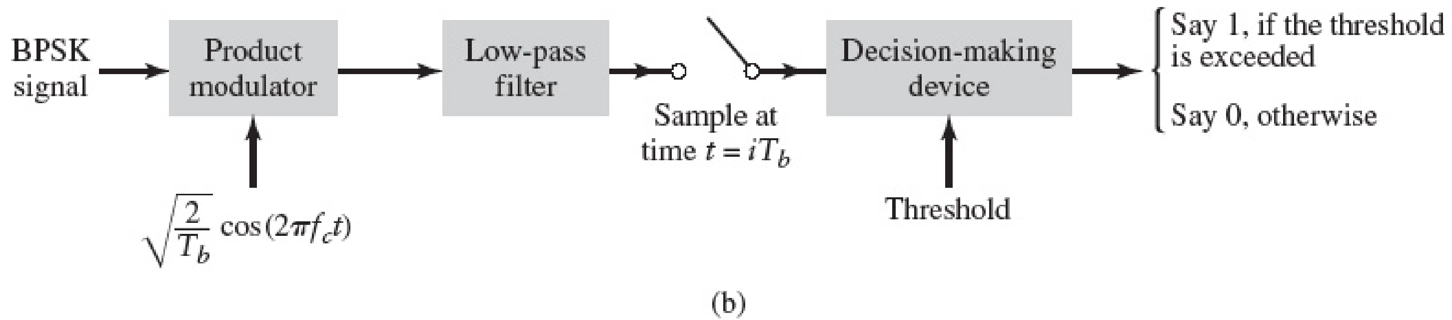

FIGURE 7.4 (*a*) BPSK modulator. (*b*) Coherent detector for BPSK; for the sampler, integer $i = 0, \pm 1, \pm 2, \ldots$.

Computer Experiment II: Spectral Analysis of BPSK

- The objectives
  1. *To evaluate the effect of varying the carrier frequency fc on the power spectrum of the BPSK signal, for a fixed square modulating wave.*
  2. *To evaluate the effect of varying modulation frequency on the power spectrum of the BPSK signal, for a fixed carrier frequency.*

Fig. 7.5(a)

Fig. 7.5(b)

Fig. 7.6(a)

Fig. 7.6(b)

Fig.7.5(a)

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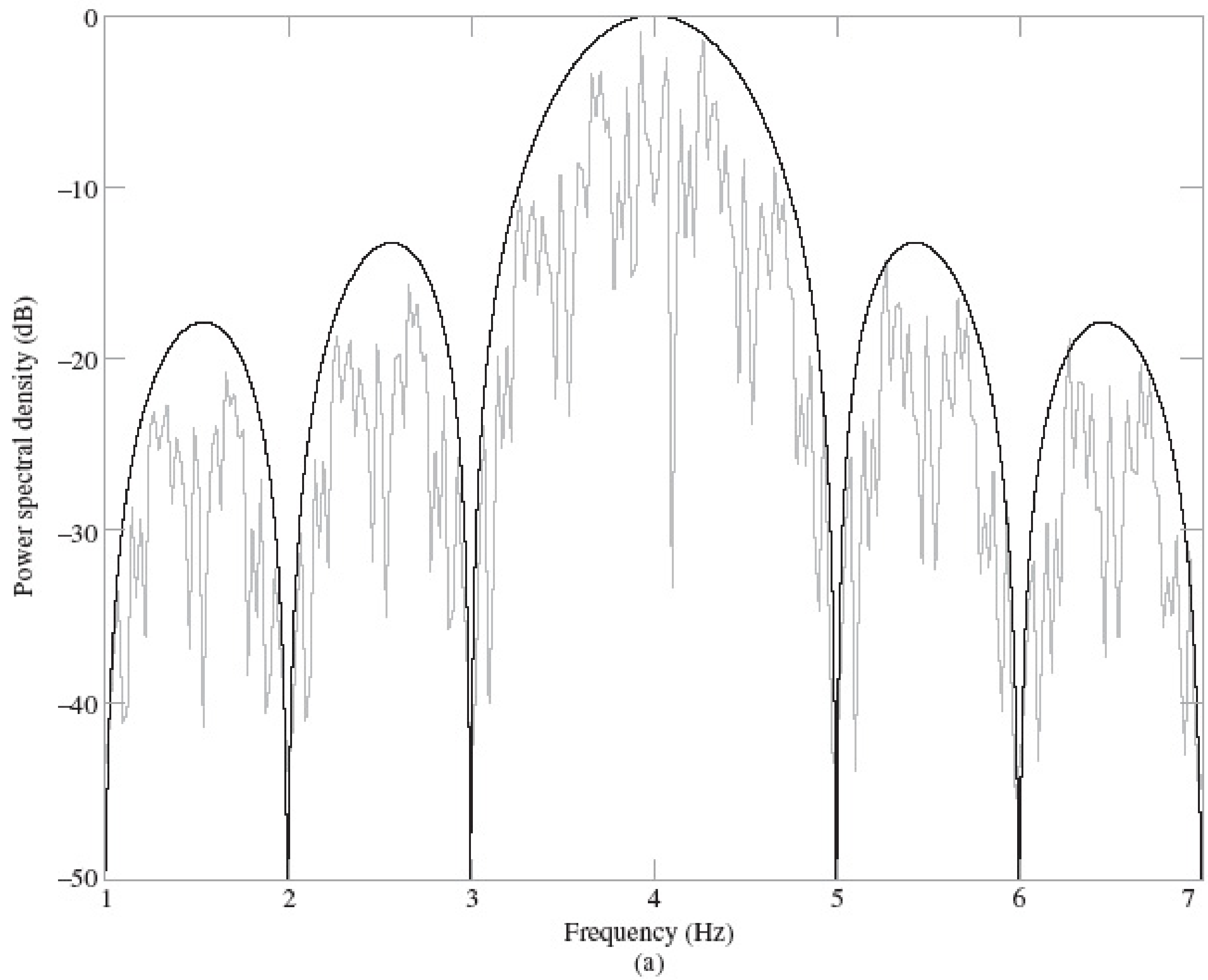

FIGURE 7.5 Power spectra of BPSK signal produced by square wave as the modulating signal for varying modulation frequency: (*a*) $f_c = 4$ Hz and $T_b = 1$ s; (*b*) $f_c = 8$ Hz and $T_b = 1$ s.

Fig.7.5(b)

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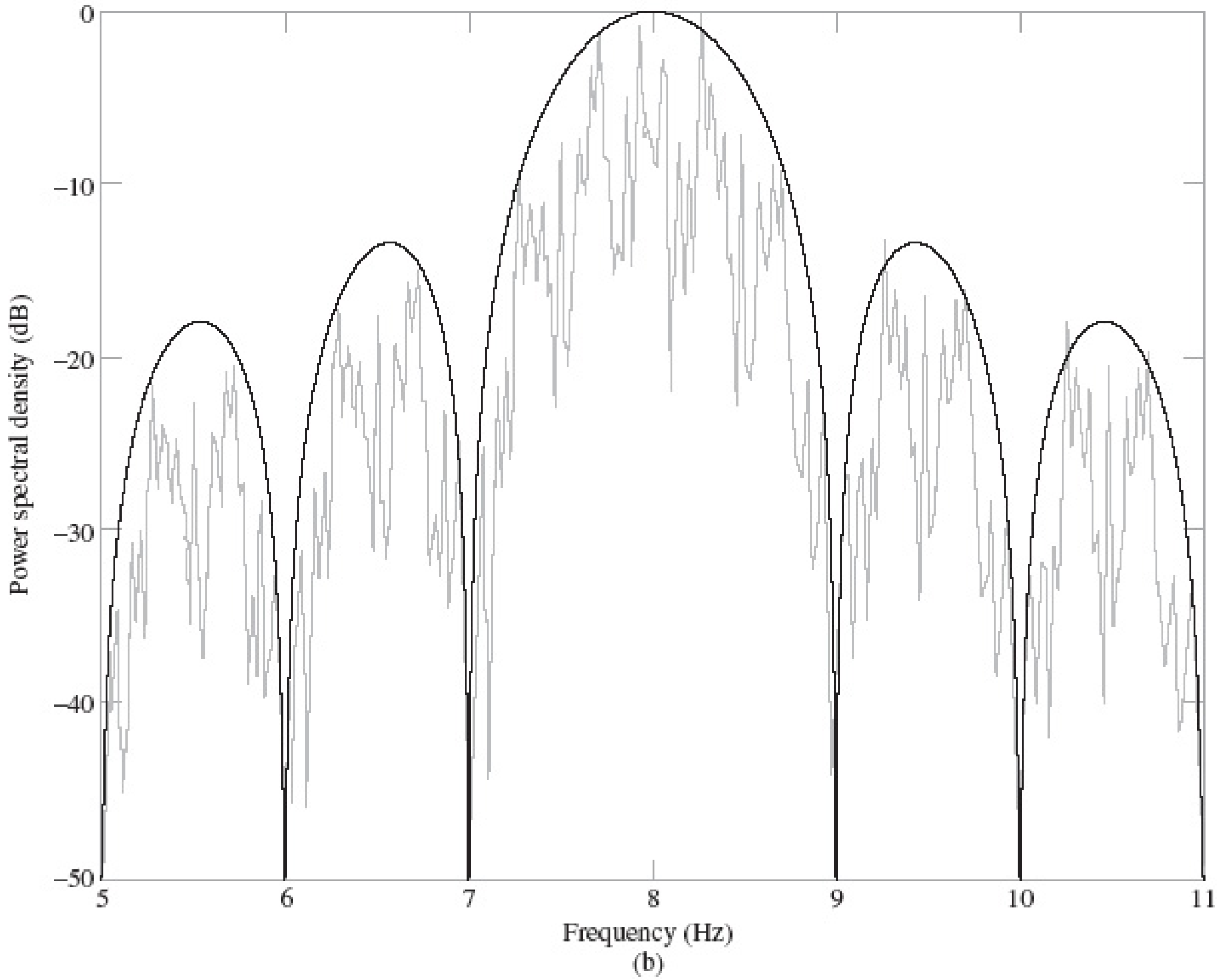

FIGURE 7.5 Power spectra of BPSK signal produced by square wave as the modulating signal for varying modulation frequency: (*a*) f_c = 4 Hz and T_b = 1 s; (*b*) f_c = 8 Hz and T_b = 1 s.

Fig.7.6(a)

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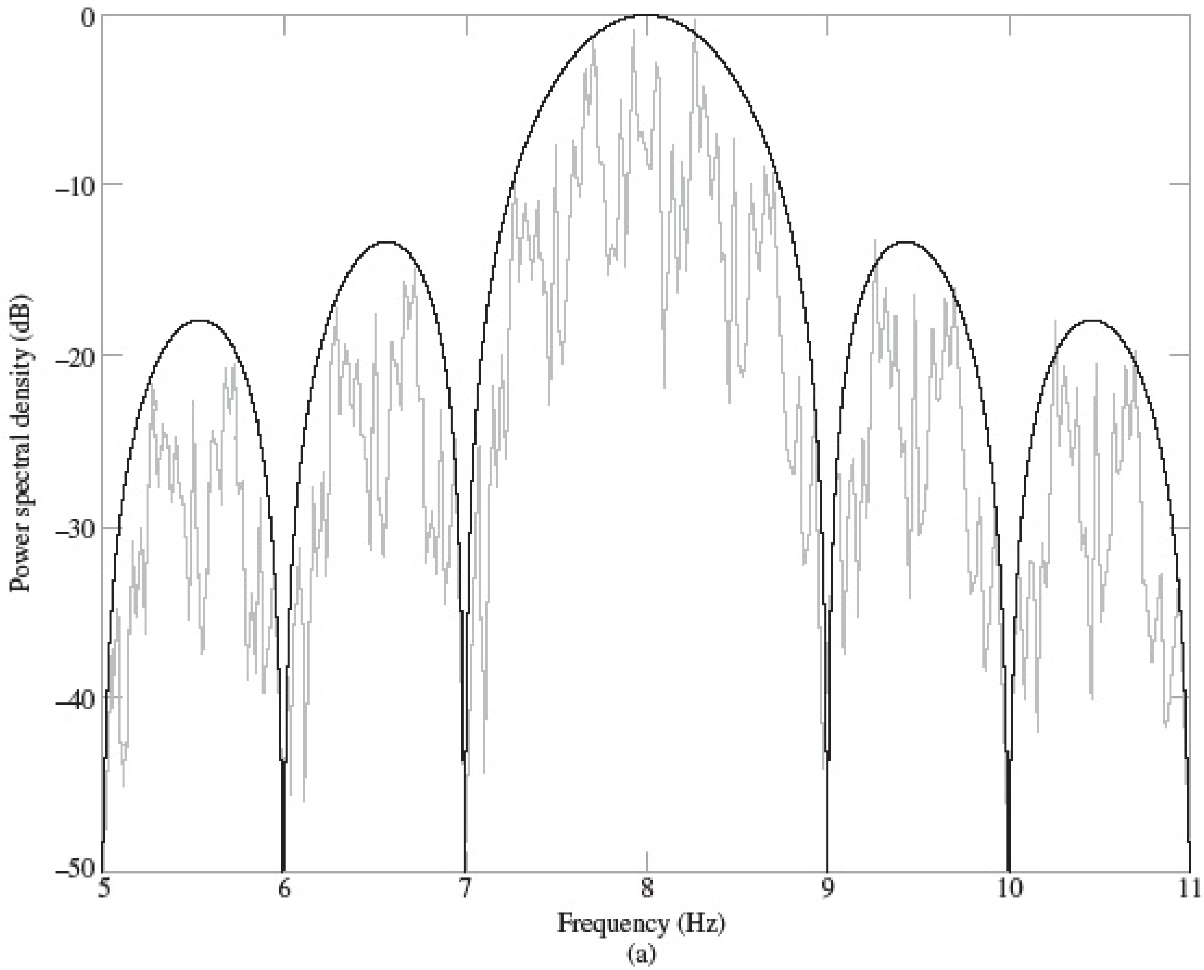

FIGURE 7.6 Power spectra of BPSK signal produced by square wave as the modulating signal for varying bit duration: (*a*) f_c = 8 Hz and T_b = 1 s; (*b*) f_c = 8 Hz and T_b = 1/2 s.

Fig.7.6(b)

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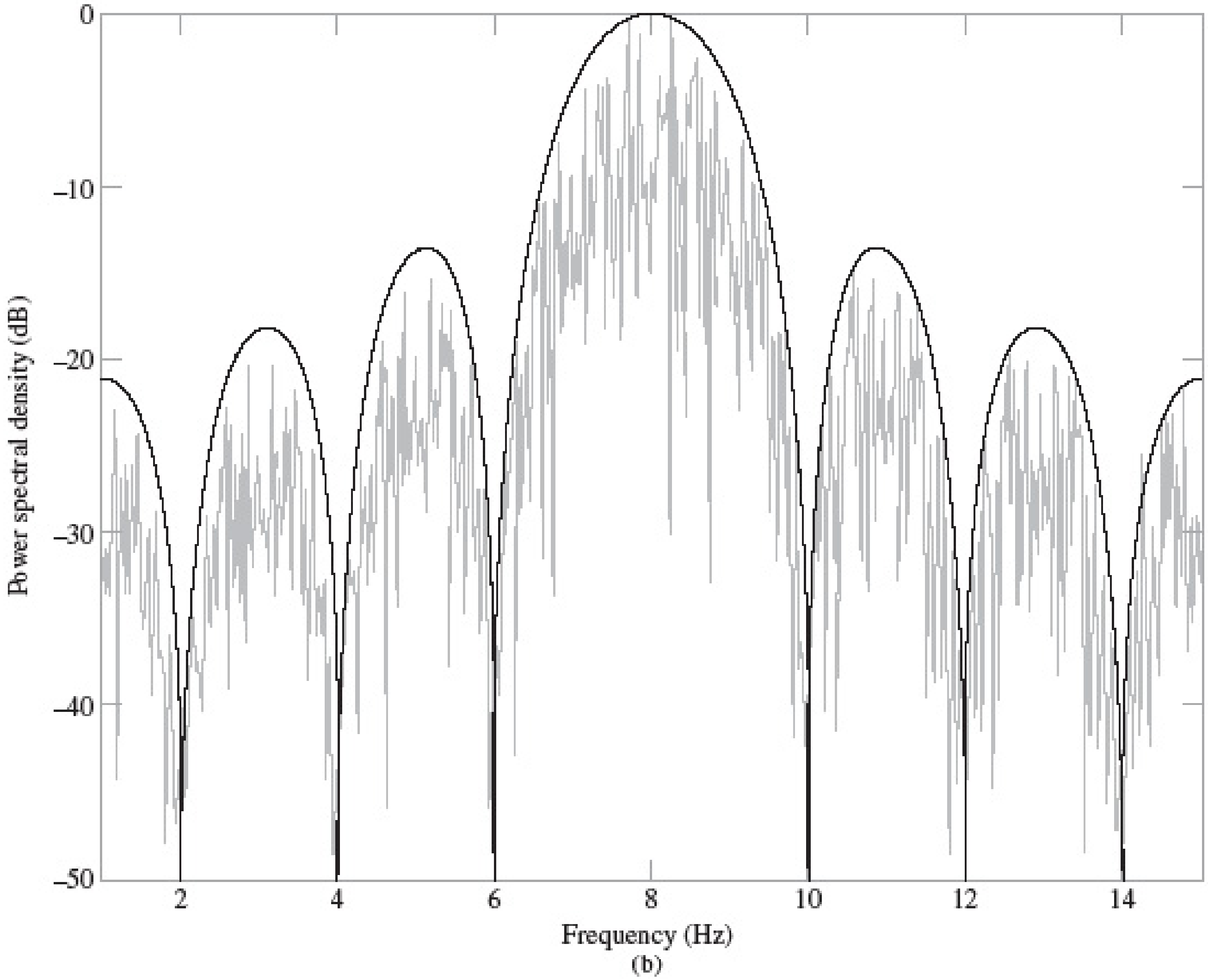

FIGURE 7.6 Power spectra of BPSK signal produced by square wave as the modulating signal for varying bit duration: (*a*) $f_c = 8$ Hz and $T_b = 1$ s; (*b*) $f_c = 8$ Hz and $T_b = 1/2$ s.

- Comparing these two figures, we can make two important observations

1. BASK and BPSK signals occupy the same transmission bandwidth, which defines the width of the main lobe of the sinc-shaped power spectra.
2. The BASK spectrum includes a carrier component, whereas this component is absent from the BPSK spectrum. With this observation we are merely restating the fact that BASK is an example of amplitude modulation, whereas BPSK is an example of double sideband-suppressed carrier modulation
   - *The present of carrier in the BASK spectrum means that the binary data stream can be recovered by envelope detection of the BASK signal.*
   - *On the other hand, suppression of the carrier in the BPSK spectrum mandates the use of coherent detection for recovery of the binary data stream form the BASK signal*

- Quadriphase-Shift Keying
  - An important goal of digital communication is the efficient utilization of channel bandwidth
  - In QPSK (Quadriphase-shift keying)
    - *The phase of the sinusoidal carrier takes on one of the four equally spaced values, such as π/4, 3π/4, 5π/4, and 7π/4*

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right], & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases} \qquad (7.13)$$

    - *Each one of the four equally spaced phase values corresponds to a unique pair of bits called* ***dibit***

$$T = 2T_b \qquad (7.14)$$

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t) \qquad (7.15)$$

1. In reality, the QPSK signal consists of the sum of two BPSK signals
2. One BPSK signal, represented by the first term defined the product of modulating a binary wave by the sinusoidal carrier

   this binary wave has an amplitude equal to ±√E/2

$$\sqrt{2E/T}\cos\left[(2i-1)\frac{\pi}{4}\right]\cos(2\pi f_c t),$$

$$\sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]=\begin{cases}\sqrt{E/2} & \text{for } i=1,4\\ -\sqrt{E/2} & \text{for } i=2,3\end{cases} \quad (7.16)$$

3. The second binary wave also has an amplitude equal to ±√E/2

$$-\sqrt{2E/T}\sin\left[(2i-1)\frac{\pi}{4}\right]\sin(2\pi f_c t),$$

$$-\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]=\begin{cases}-\sqrt{E/2} & \text{for } i=1,2\\ \sqrt{E/2} & \text{for } i=3,4\end{cases} \quad (7.17)$$

4. The two binary waves defined in Eqs (7.16) and (7.17) share a common value for the symbol duration
5. The two sinusoidal carrier waves identified under points 2 and 3 are in phase quadrature with respect to each other. They both have unit energy per symbol duration. These two carrier waves constitute an ortho-normal pair of basis functions
6. Eqs. (7.16) and (7.17) identity the corresponding dibit, as outlined in Table 7.1

Table.7.1

Table 7.1

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TABLE 7.1 *Relationship Between Index i And Identity of Corresponding Dibit, and Other Related Matters*

Index i	Phase of QPSK signal (radians)	Amplitudes of constituent binary waves: Binary wave 1 $a_1(t)$	Amplitudes of constituent binary waves: Binary wave 2 $a_2(t)$	Input dibit $0 \leq t \leq T$
1	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$	10
2	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$	00
3	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$	01
4	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$	11

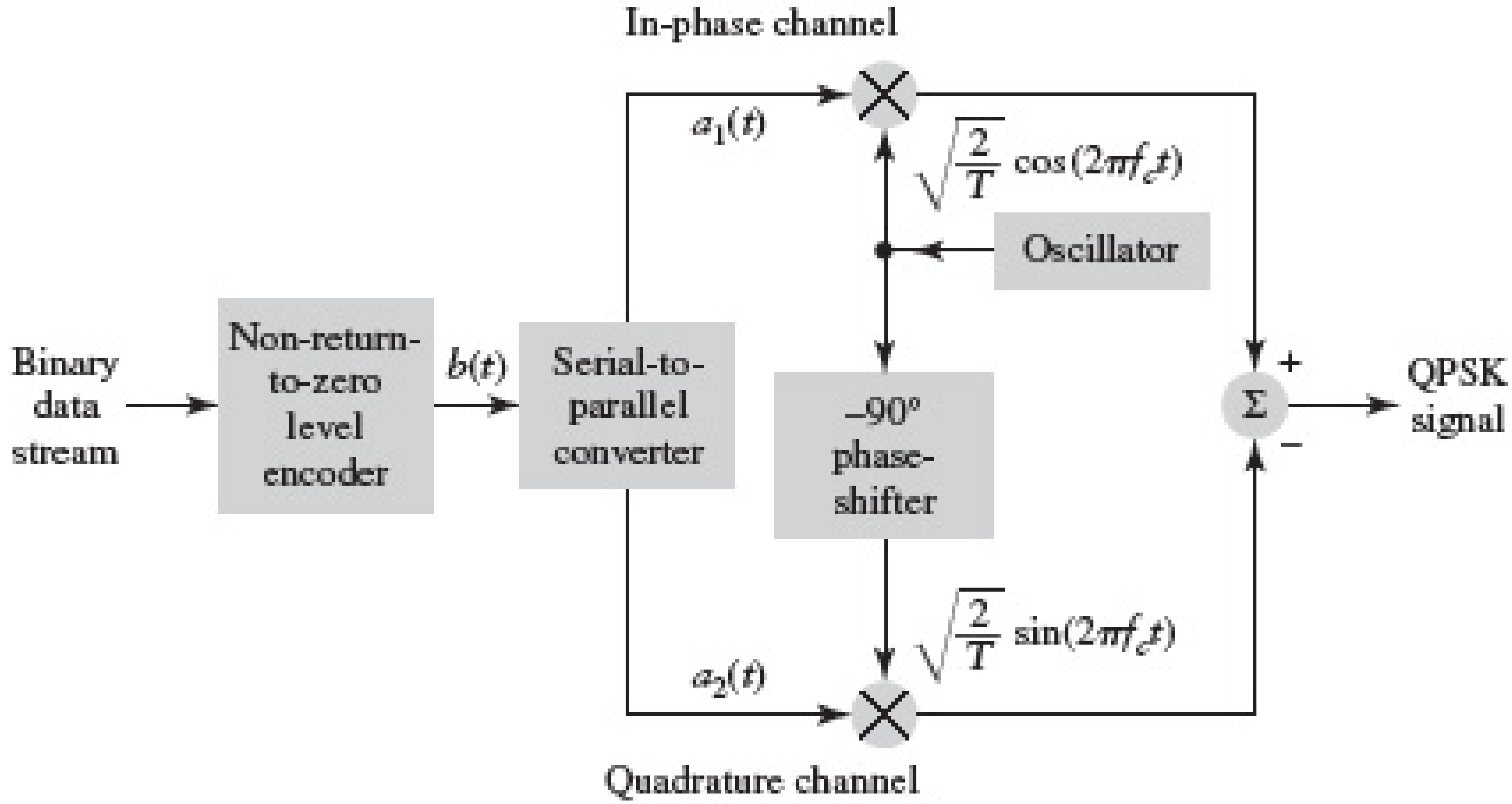
Generation and Coherent Detection of QPSK Signals

1. Generation

- The incoming binary data stream is first converted into polar form by a non-return-to-zero level encoder
- The resulting binary wave is next divided by means of a demultiplexer into two separate binary waves consisting of the odd- and even-mumbered input bits of *b(t)* – these are referred to as the demultiplexed components of the input binary wave.
- The two BPSK signals are subtracted to produce the desired QPSK signals

Fig. 7.7(a)

Fig.7.7(a)


In-phase channel
$a_1(t)$
$\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$
Oscillator
Binary data stream
Non-return-to-zero level encoder
$b(t)$
Serial-to-parallel converter
–90° phase-shifter
Σ
+
–
QPSK signal
$\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$
$a_2(t)$
Quadrature channel
(a)

2. Detection

- The QPSK receiver consists of an In-phase and quadrature with a common input.
- Each channel is made up of a product modulator, low-pass filter, sampler, and decision-making device.
- The I- and Q-channles of the receiver, recover the demultiplexed components $a_1(t)$ and $a_2(t)$
- By applying the outputs of these two channels to a multiplexer, the receiver recovers the original binary sequence

- Each of the two low-pass filters must be assigned a bandwidth equal to or greater than the reciprocal of the symbol duration T

Fig. 7.7(b)

Fig.7.7(b)

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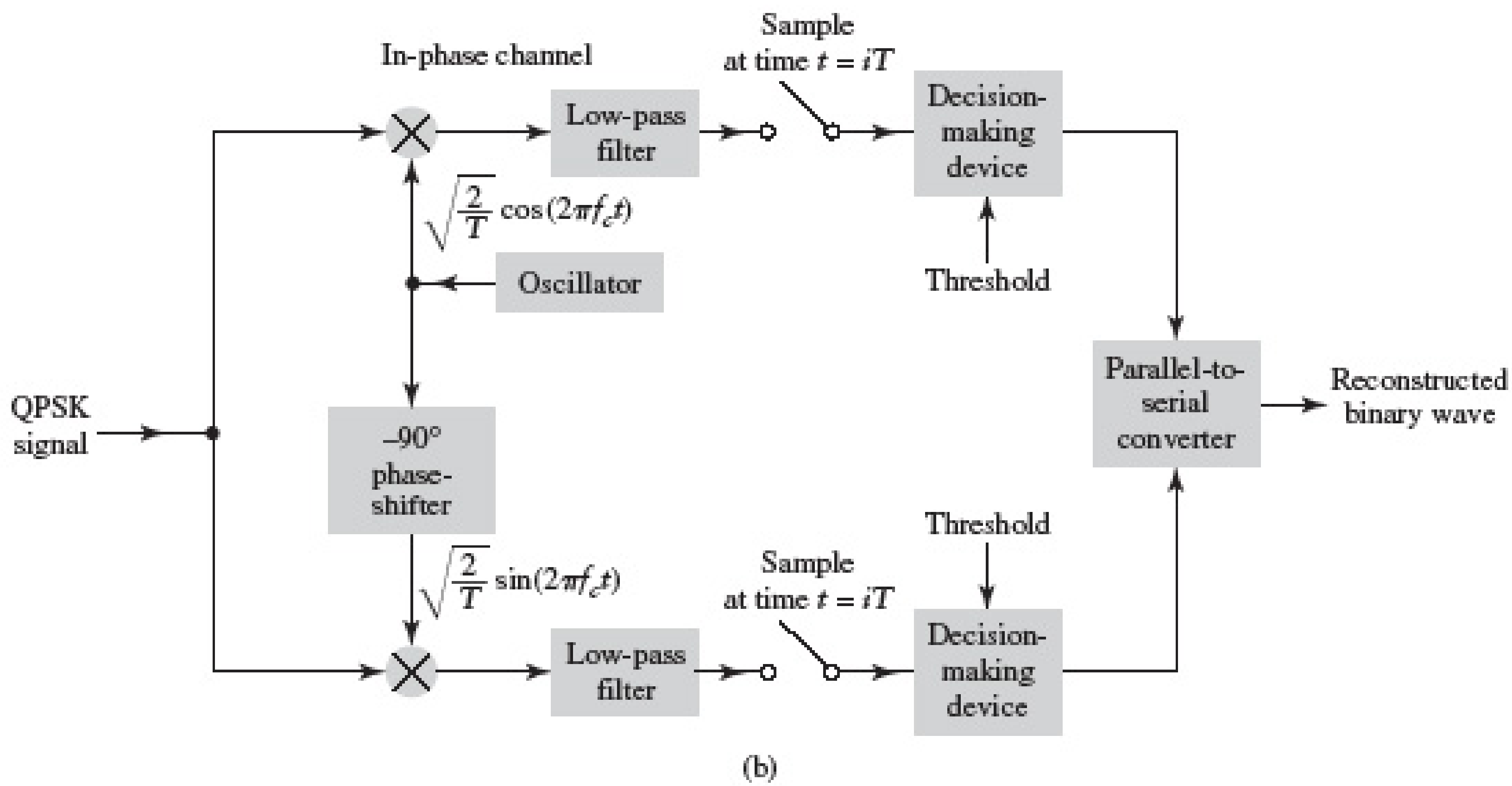

(b)

FIGURE 7.7 Block diagrams of (*a*) QPSK transmitter and (*b*) coherent QPSK receiver; for the two synchronous samplers, integer $i = 0, \pm 1, \pm 2, \ldots$.

- Offset Quadriphase-Shift Keying (OQPSK)
  - The extent of amplitude fluctuations exhibited by QPSK signals may be reduced by using a variant of quadriphase-shift keying
  - The demultiplexed binary wave labeled $a_2(t)$ is delayed by one bit duration with respect to the other demultiplexed binary wave labled $a_1(t)$
  - $\pm 90^\circ$ phase transitions occur twice as frequency but with a reduced range of amplitude fluctuations.
  - Amplitude fluctuations in OQPSK due to filtering have a smaller amplitude than in QPSK.

EXAMPLE 7.1 Phase transitions

Parts (*a*) and (*b*) of Fig. 7.8 depict the waveforms of QPSK and OQPSK, both of which are produced by the binary data stream 0011011001 with the following composition over the interval $0 \le t \le 10T_b$:

- The input dibit (i.e., the pair of adjacent bits in the binary data stream) changes in going from the interval $0 \le t \le 2T_b$ to the next interval $2T_b \le t \le 4T_b$.
- The dibit changes again in going from the interval $2T_b \le t \le 4T_b$ to the next interval $4T_b \le t \le 6T_b$.
- The dibit changes yet again in going from the interval $4T_b \le t \le 6T_b$ to the next interval $6T_b \le t \le 8T_b$.
- Finally, the dibit is changed one last time in going from the interval $6T_b \le t \le 8T_b$ to the interval $8T_b \le t \le 10T_b$.

Examining the two waveforms of Fig. 7.8, we find the following:

(i) In QPSK, the carrier phase undergoes jumps of 0°, ±90°, or ±180° every $2T_b$ seconds.

(ii) In OQPSK, on the other hand, the carrier phase experiences only jumps of 0° or ±90° every T_b seconds.

Fig. 7.8

Fig.7.8

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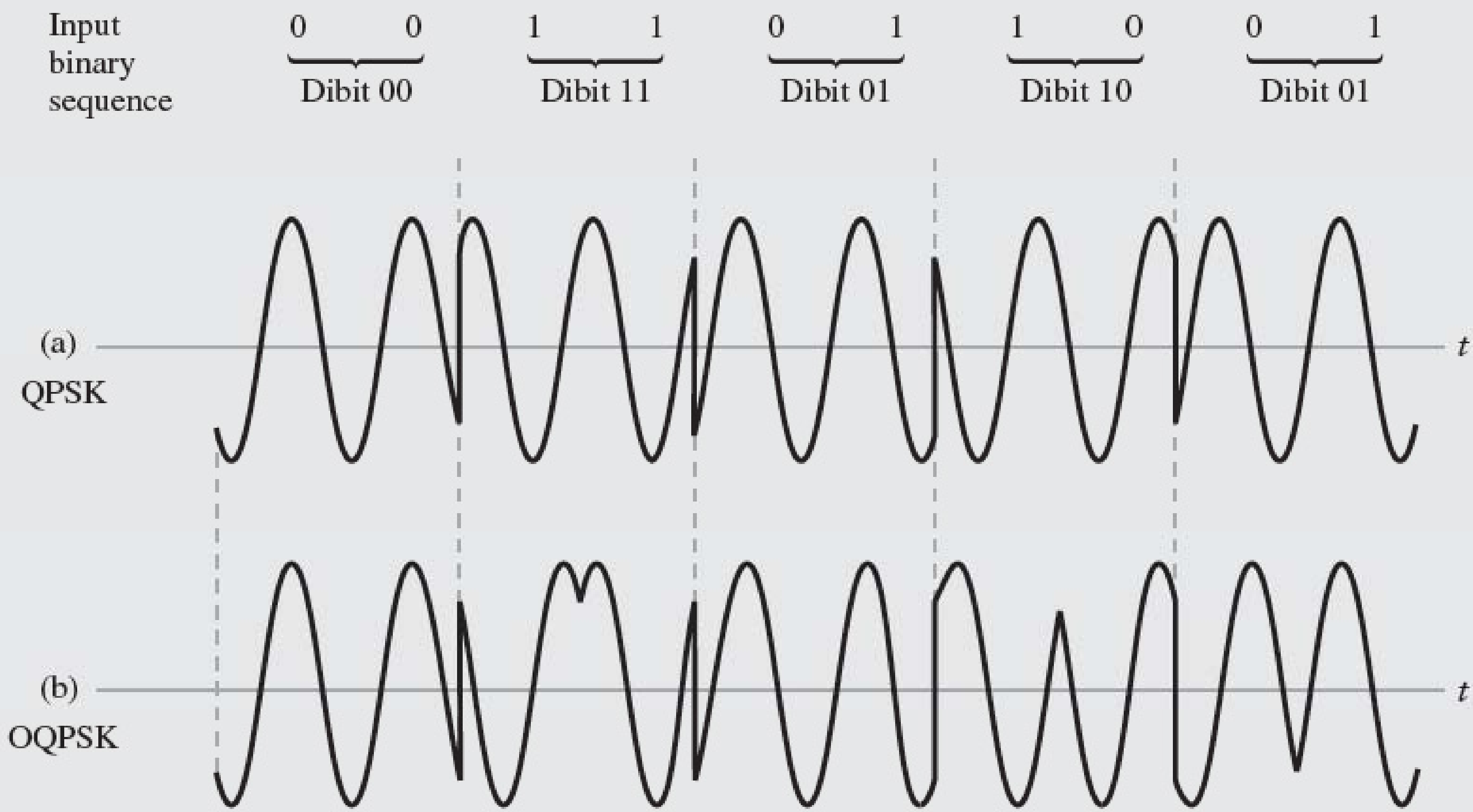

FIGURE 7.8 Graphical comparison of phase transitions in QPSK and OQPSK.

- Computer Experiment III : QPSK and OPQSK Spectra
  - QPSK Spectra

$$\text{Carrier Frequency, } f_c = 8Hz$$

$$\text{Bit duration, } T_b = \begin{cases} 1s \text{ for part (a) of the figure} \\ 0s \text{ for part (b) of the figure} \end{cases}$$

  - OQPSK Spectra
    - *For the same parameters used for QPSK*
  - QPSK occupies a bandwidth equal to one half that of BPSK

Fig. 7.9(a)

Fig. 7.9(b)

Fig. 7.10(a)

Fig. 7.10(b)

Fig.7.9(a)

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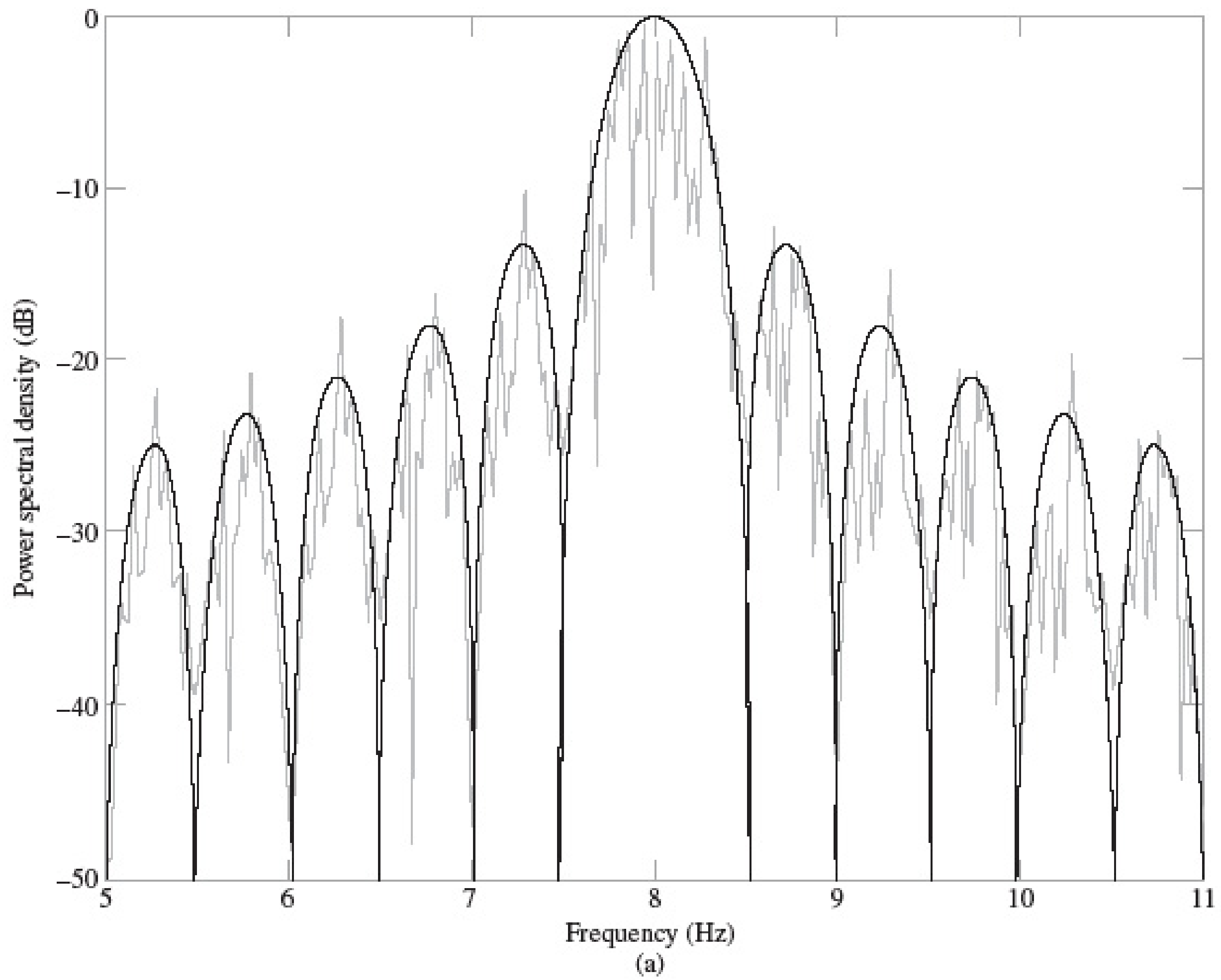

FIGURE 7.9 Power spectra of QPSK produced by square wave as the modulating signal for fixed carrier frequency and varying bit duration: (*a*) $f_c = 8$ Hz and $T_b = 1$ s; (*b*) $f_c = 8$ Hz and $T_b = 1/2$ s.

Fig.7.9(b)

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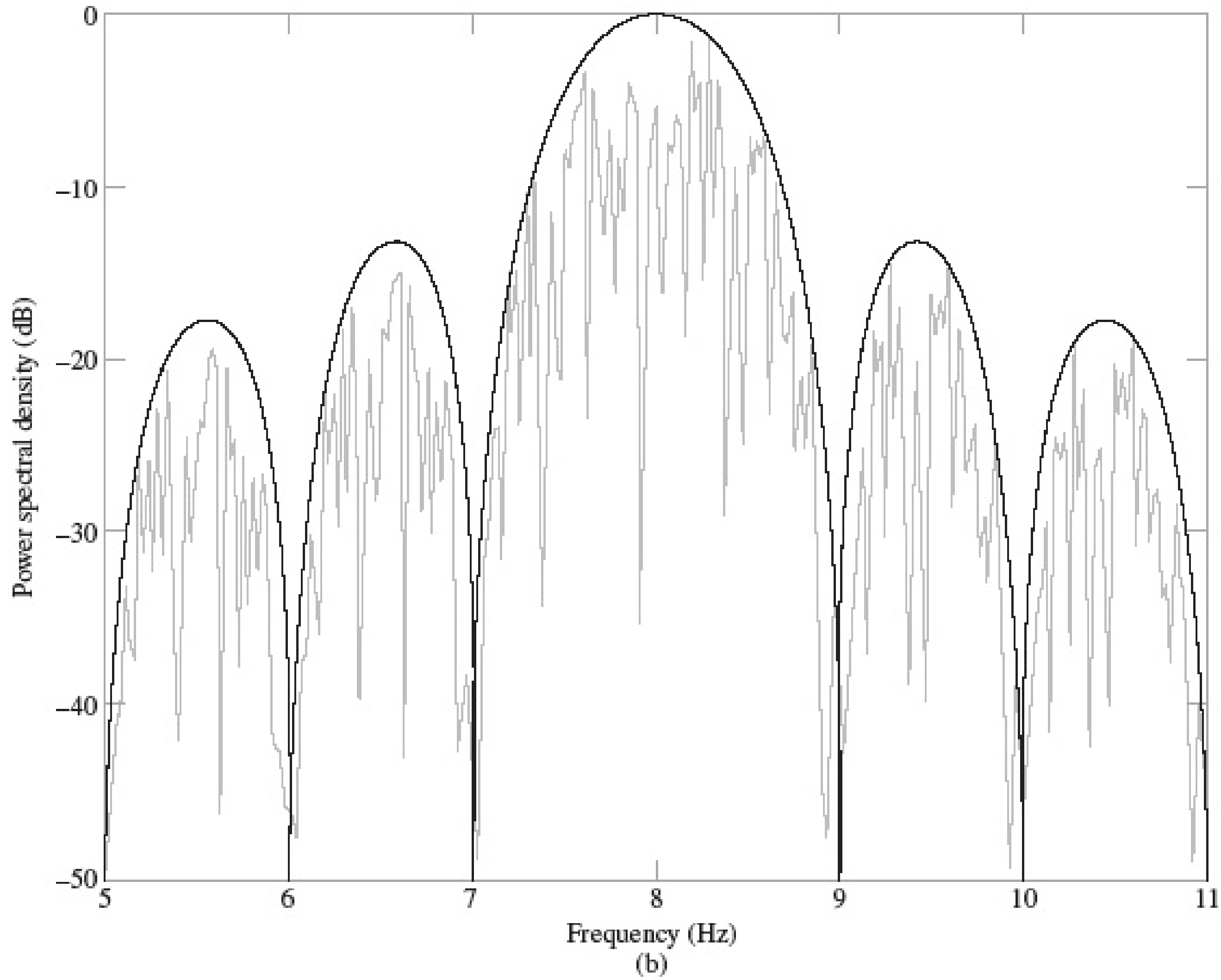

FIGURE 7.9 Power spectra of QPSK produced by square wave as the modulating signal for fixed carrier frequency and varying bit duration: (*a*) $f_c = 8$ Hz and $T_b = 1$ s; (*b*) $f_c = 8$ Hz and $T_b = 1/2$ s.

Fig.7.10(a)

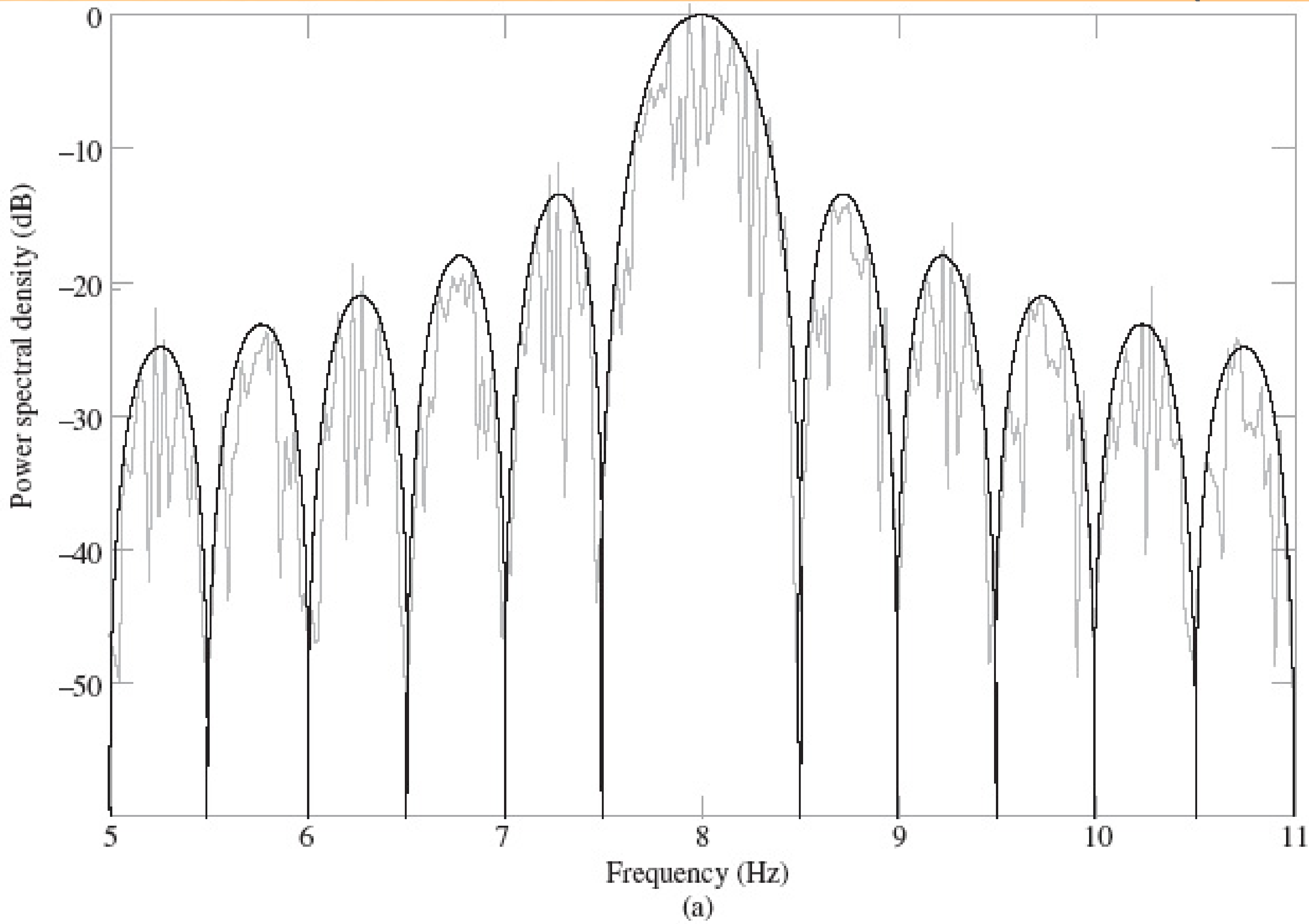

FIGURE 7.10 Power spectra of OQPSK produced by square wave as the modulating signal for fixed carrier frequency and varying bit duration: (*a*) $f_c = 8$ Hz and $T_b = 1$ s; (*b*) $f_c = 8$ Hz and $T_b = 1/2$ s.

Fig.7.10(b)

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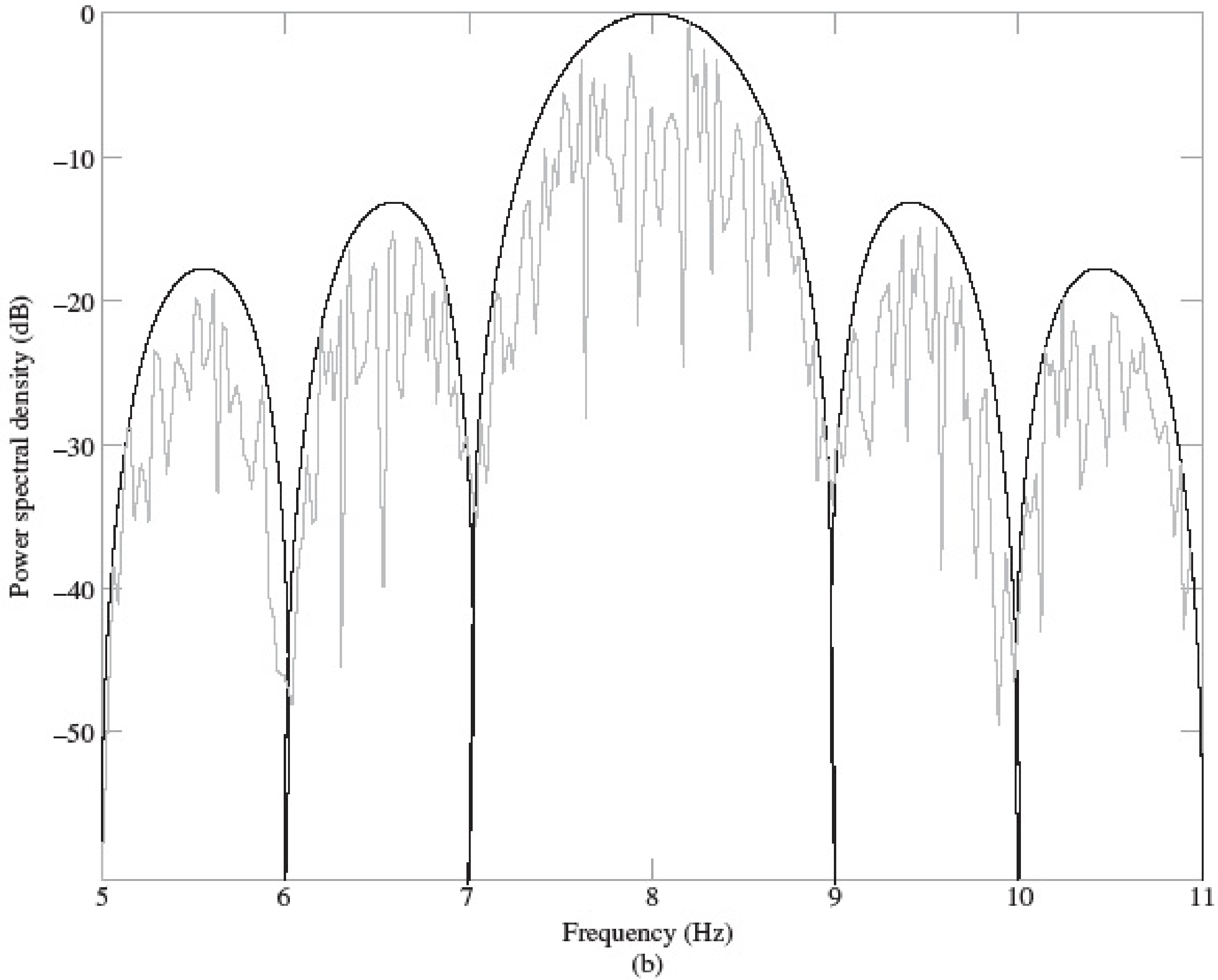

FIGURE 7.10 Power spectra of OQPSK produced by square wave as the modulating signal for fixed carrier frequency and varying bit duration: (*a*) $f_c = 8$ Hz and $T_b = 1$ s; (*b*) $f_c = 8$ Hz and $T_b = 1/2$ s.

7.4 Frequency-Shift Keying

- Binary Frequency-Shift Keying (BFSK)
  - Each symbols are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_1 t), & \text{for symbol 1 corresponding to } i = 1 \\ \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_2 t), & \text{for symbol 0 corresponding to } i = 2 \end{cases} \quad (7.18)$$

  - Sunde's BFSK
    - *When the frequencies f_1 and f_2 are chosen in such a way that they differ from each other by an amount equal to the reciprocal of the bit duration T_b*

- Computer Experiment IV : Sunde's BFSK

1. Waveform

- Input binary sequence 0011011001 for a bit duration $T_b=1s$
- The latter part of the figure clearly displays the phase-continuous property of Sunde's BFSK

Fig. 7.11

2. Spectrum

$$\text{Bit duration, } \quad T_b = 1s$$

$$\text{Carrier frequency, } f_c = 8Hz$$

1. *The spectrum contains two line components at the frequency $f=f_c \pm 1(2T_b)$; which equal 7.5Hz and 8.5Hz for $f_c=8$ Hz and $T_b=1s$*
2. *The main lobe occupies a band of width equal to $(3/T_b)=3Hz$, centered on the carrier frequency $f_c=8$ Hz*
3. *The largest sidelobe is about 21 dB below the main lobe.*

Fig. 7.12

Fig.7.11

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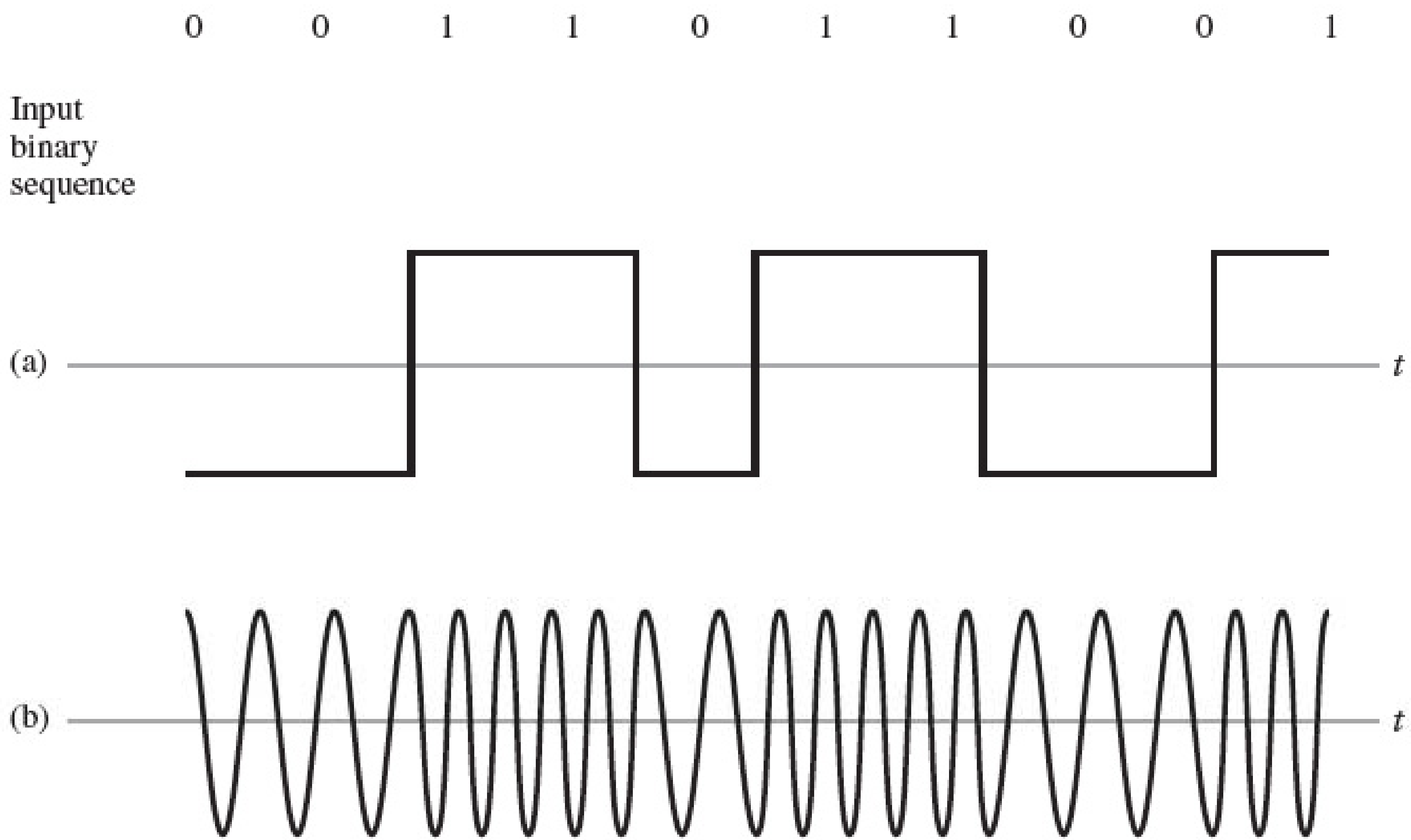

FIGURE 7.11 (*a*) Binary sequence and its non-return-to-zero level-encoded waveform. (*b*) Sunde's BFSK signal.

Fig.7.12

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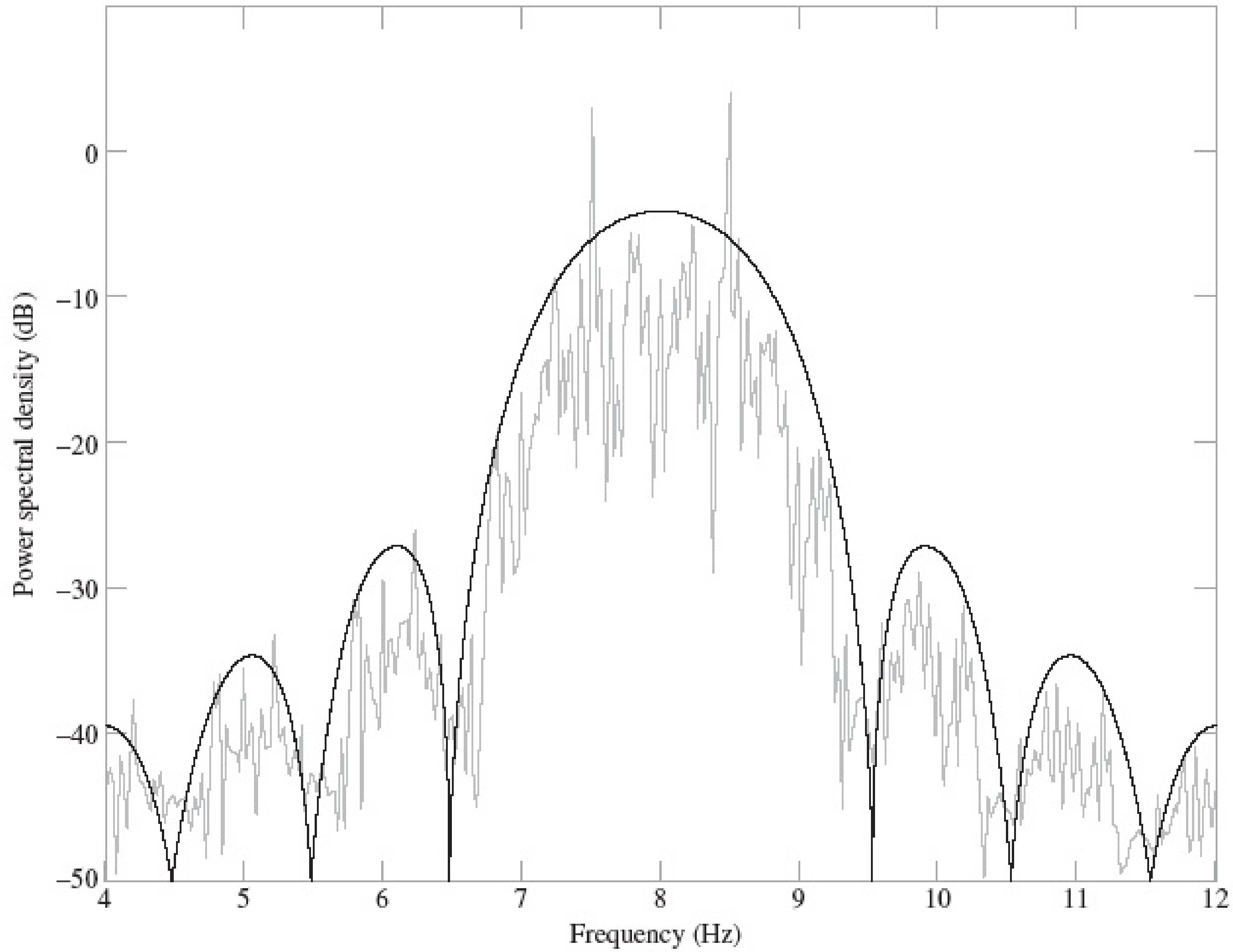

FIGURE 7.12 Power spectrum of Sunde's BFSK produced by square wave as the modulating signal for the following parameters: $f_c = 8$ Hz and $T_b = 1$ s.

- Continuous-phase Frequency-Shift Keying
  - The modulated wave maintains phase continuity at all transition points, even though at those points in time the incoming binary data stream switches back and forth
  - Sunde's BFSK, the overall excursion δf in the transmitted frequency from symbol 0 to symbol 1, is equal to the bit rate of the incoming data stream.
  - MSK (Minimum Shift Keying)
    - The special form of CPFSK
    - Uses a different value for the frequency excursion δf, with the result that this new modulated wave offers superior spectral properties to Sunde's BFSK.

- Minimum-Shift Keying
  - Overall frequency excursion δf from binary symbol 1 to symbol 0, is one half the bit rate

$$\delta f = f_1 - f_2$$

$$= \frac{1}{2T_b} \qquad (7.19)$$

$$f_c = \frac{1}{2}(f_1 + f_2) \qquad (7.20)$$

$$f_1 = f_c + \frac{\delta f}{2}, \quad \text{for symbol} \quad 1 \quad (7.21)$$

$$f_2 = f_c - \frac{\delta f}{2}, \quad \text{for symbol} \quad 0 \quad (7.22)$$

  - Define the MSK signal as the angle-modulated wave

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos[2\pi f_c t + \theta(t)] \quad (7.23)$$

- Sunde's BFSK has no memory; in other words, knowing which particular change occurred in the previous bit interval provides no help in the current bit interval.

$$\theta(t) = 2\pi\left(\frac{\delta f}{2}\right)t$$

$$= \frac{\pi t}{2T_b}, \qquad \text{for symbol 1} \quad (7.24)$$

$$\theta(t) = 2\pi\left(-\frac{\delta f}{2}\right)t$$

$$= -\frac{\pi t}{2T_b}, \qquad \text{for symbol 0} \quad (7.25)$$

EXAMPLE 7.2: Relationship Between OQPSK and MSK Waveforms

The purpose of this example is to illustrate the relationship that exists between OQPSK and MSK waveforms. Figures 7.13 and 7.14 bear out this fundamental relationship:

Fig. 7.13

- The five waveforms of Fig. 7.13 plot the components of the OQPSK signal for the input binary data stream 0011011001.
- The corresponding five waveforms of Fig. 7.14 plot the components of the MSK signal for the same input binary data stream 0011011001.

Comparing the results plotted in Figs. 7.13 and 7.14, we may make the following observation. Although the OQPSK and MSK are derived from different modulation principles, the MSK from frequency-shift keying and the OQPSK from phase-shift keying, these two digitally modulated waves are indeed closely related. The basic difference between them lies merely in the way in which the binary symbols in their in-phase and quadrature components are level-encoded. In OQPSK, the level-encoding is based on rectangular pulses, with one binary wave shifted from the other binary wave by one bit duration. On the other hand, in MSK, the level-encoding is based on the half cycle of a cosinusoid.

Fig. 7.14

Fig.7.13

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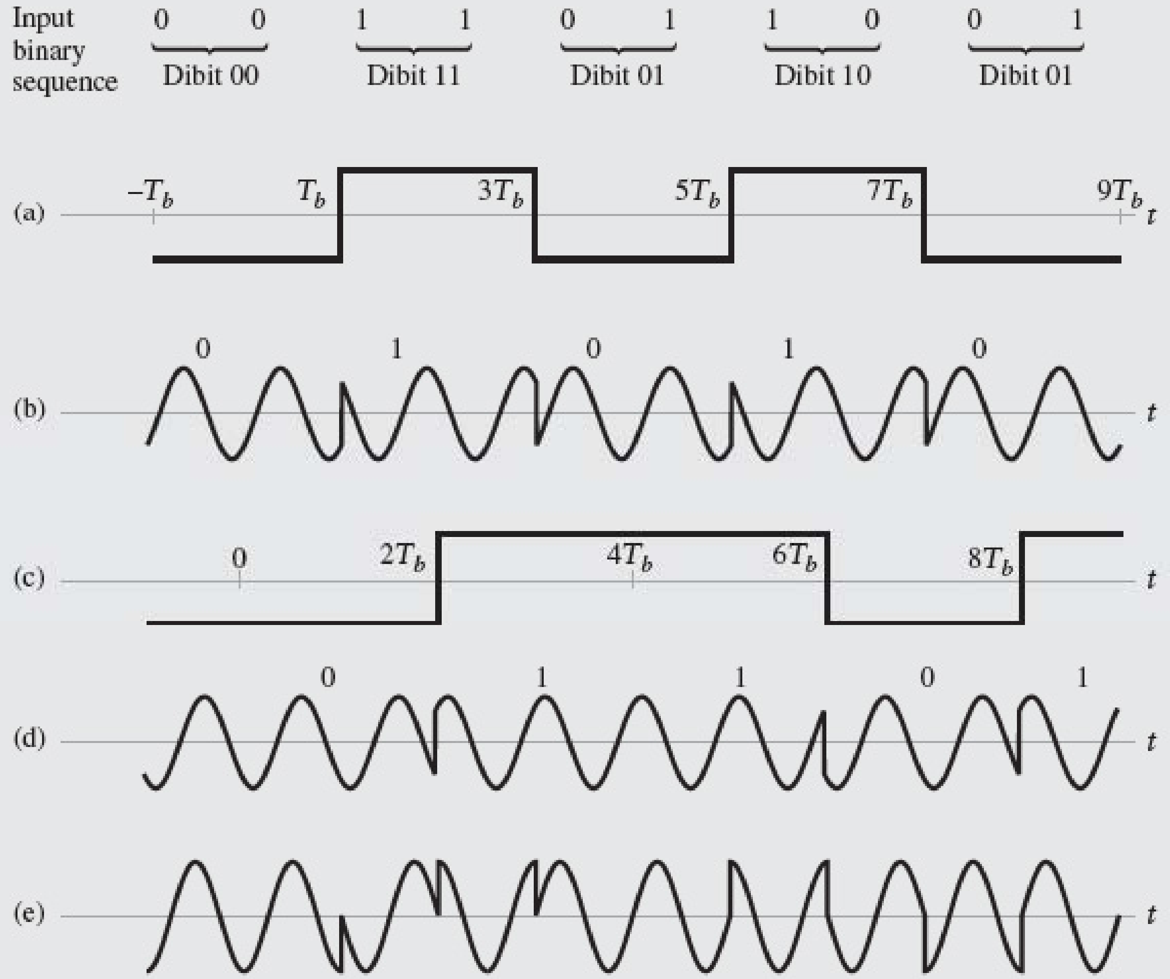

FIGURE 7.13 OQPSK signal components: (*a*) Modulating signal for in-phase component. (*b*) Modulated waveform of in-phase component. (*c*) Modulating signal for quadrature component. (*d*) Modulated waveform of quadrature component. (*e*) Waveform of OQPSK signal obtained by subtracting (*d*) from (*b*).

Fig.7.14

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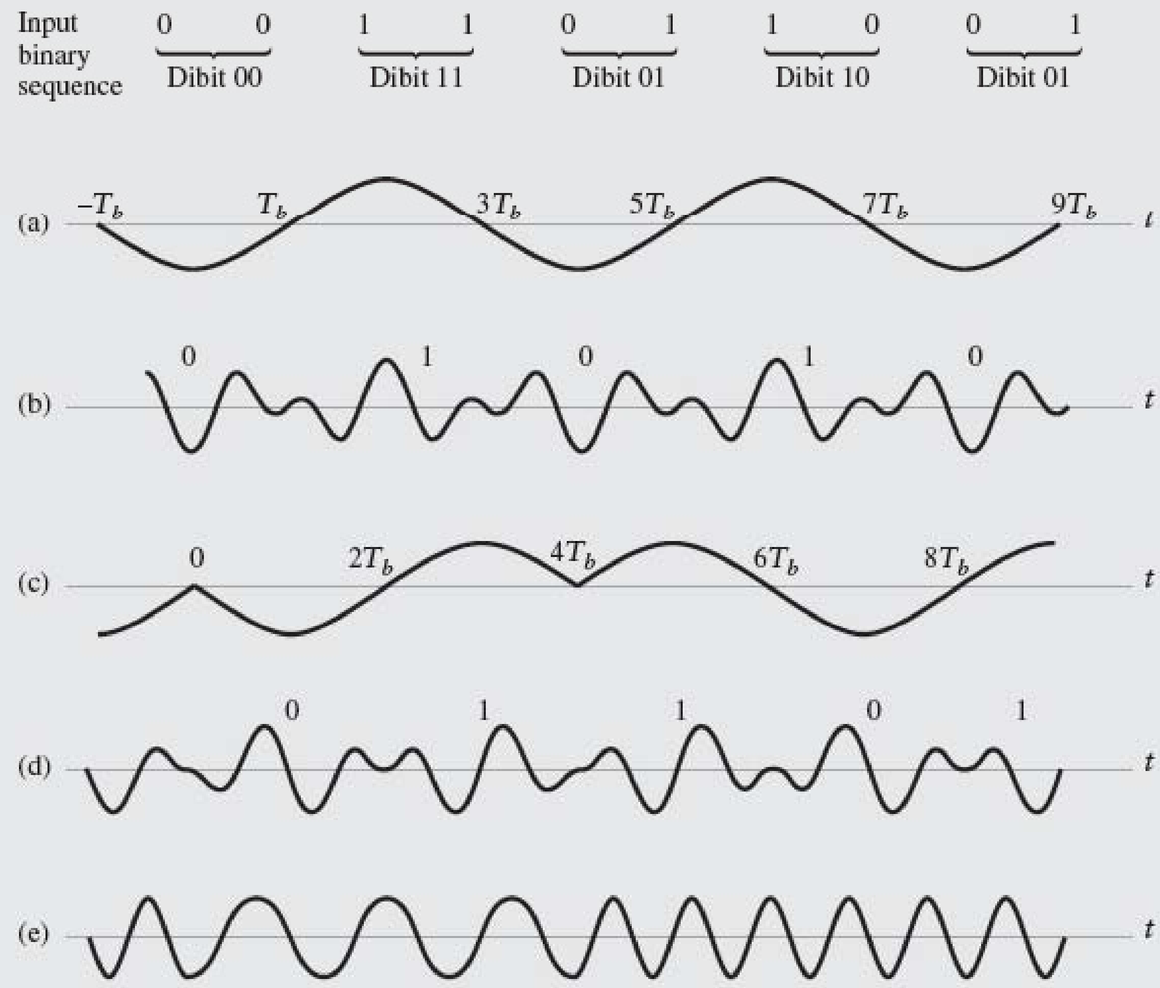

FIGURE 7.14 MSK signal components: (*a*) Modulating signal for in-phase component. (*b*) Modulated waveform of in-phase (*c*) Modulating signal for quadrature component (*d*) Modulated quadrature component. (*e*) Waveform of MSK signal obtained by subtracting (*d*) from (*b*).

- Formulation of Minimum-Shift Keying

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(\theta(t)) \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}} \sin(\theta(t)) \sin(2\pi f_c t) \quad (7.26)$$

(i) $s_I(t) = \sqrt{E_b} \cos(\theta(t))$ is the in-phase (I) component associated with the carrier $\sqrt{2/T_b}\cos(2\pi f_c t)$. (7.27)

(ii) $s_Q(t) = \sqrt{E_b} \sin(\theta(t))$ is the quadrature (Q) component associated with the 90°-phase-shifted carrier. (7.28)

$$s_I(t) = a_1(t) \cos(2\pi f_0 t) \quad (7.29)$$

$$s_Q(t) = a_2(t) \sin(2\pi f_0 t) \quad (7.30)$$

$$\theta(t) = -\tan^{-1}\left[\frac{s_Q(t)}{s_I(t)}\right] = -\tan^{-1}\left[\frac{a_2(t)}{a_1(t)} \tan(2\pi f_0 t)\right] \quad (7.31)$$

1. $a_2(t)=a_1(t)$

   This scenario arises when two successive binary symbols in the incoming data stream are the same

$$\begin{aligned}\theta(t) &= -\tan^{-1}[\tan(2\pi f_0 t)] \\ &= -2\pi f_0 t \qquad (7.32)\end{aligned}$$

2. $a_2(t)=-a_1(t)$

   This second scenario arises when two successive binary symbols in the incoming data stream are different

$$\begin{aligned}\theta(t) &= -\tan^{-1}[-\tan(2\pi f_0 t)] \\ &= 2\pi f_0 t \qquad (7.33)\end{aligned}$$

$$f_0 = \frac{1}{4T_b} \qquad (7.34)$$

- Given a non-return-to-zero level encoded binary wave *b(t)* of prescribed bit duration T_b and a sinusoidal carrier wave of frequency f_c, we may formulate the MSK signal by proceeding as follows
  1. *Use the given binary wave b(t) to construct the binary demultiplexed-offset waves* $a_1(t)$ *and* $a_2(t)$
  2. *Use Eq. (7.34) to determine the frequency* f_0
  3. *Use Eq. (7.29) and (7.30) to determine the in-phase component* $s_I(t)$ *and quadrature component* $s_Q(t)$*, respectively from which the MSK signal s(t) follows*

- Computer Experinment V : MSK Spectrum
  - The parameters

$$\text{Bit duration,} \quad T_b = 1s$$
$$\text{Carrier frequency,} \; f_c = 8Hz$$

Fig. 7.15

1. MSK versus QPSK
   - *The main lobe of MSK occupies a frequency band whose width is* $1.5/T_b$=1.5Hz
   - *The transmission bandwidth of MSK is 50 percent larger than that of QPSK*
   - *The sidelobes of MSK are considerably smaller than those of QPSK*
2. MSK versus Sunde's BFSK
   - *The transmission bandwidth of MSK is one half that of Sunde's BFSK*
   - *Sunde's BFSK exhibits two line components at* $f=f_c \pm 1/(2T_b)$
   - *The spectrum of MSK is continuous across the whole frequency band*

Fig.7.15

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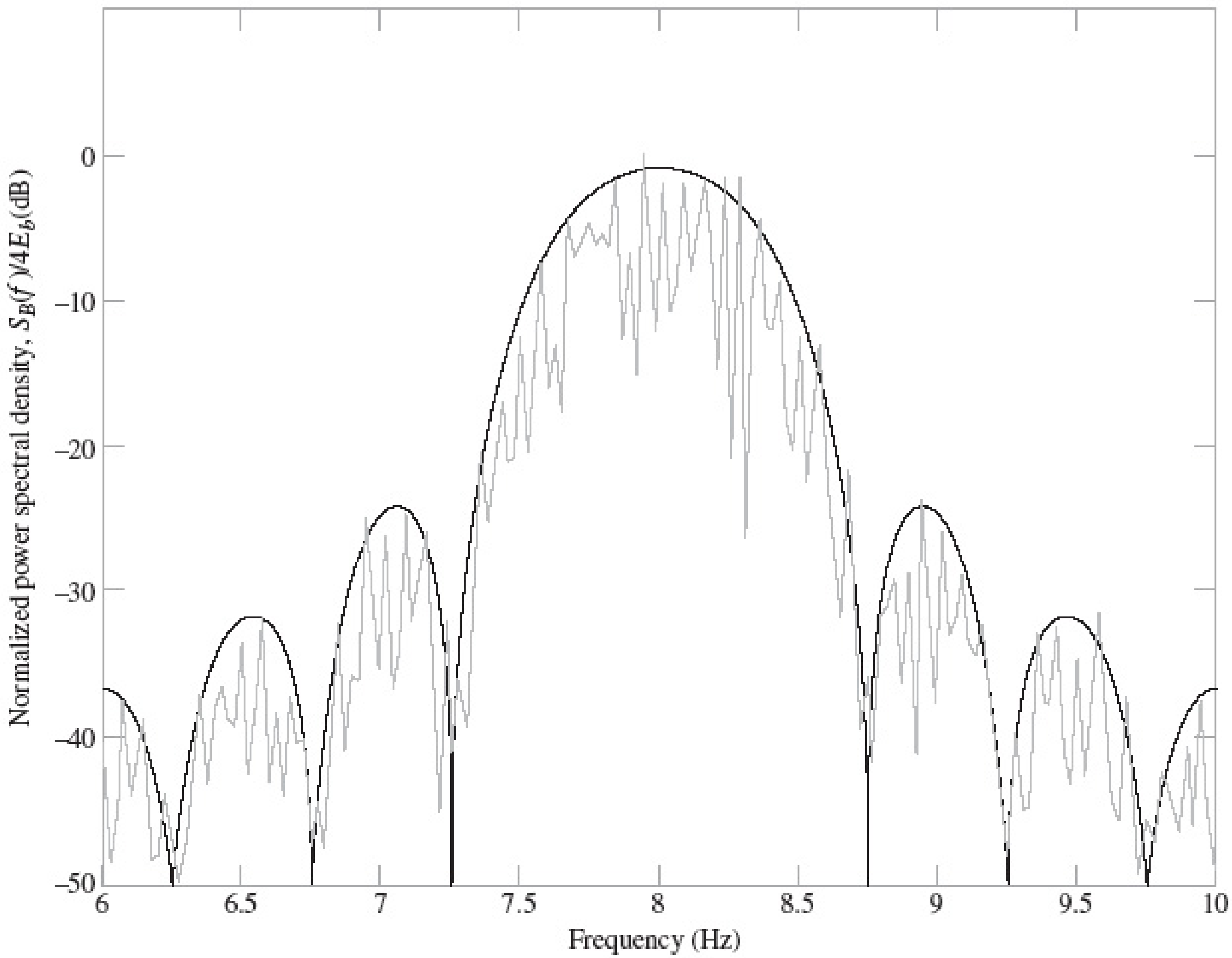

FIGURE 7.15 Power spectrum of MSK produced by square wave as the modulating signal for the following parameters: $f_c = 8$ Hz and $T_b = 1$ s.

- Although the carrier frequency is not high enough to completely eliminate spectral overlap, the overlap is relatively small as evidenced by
  - *The small value of the spectrum at zero frequency*
  - *The small degree of asymmetry about the carrier frequency* f_c*=8Hz*

Fig. 7.16

Fig.7.16

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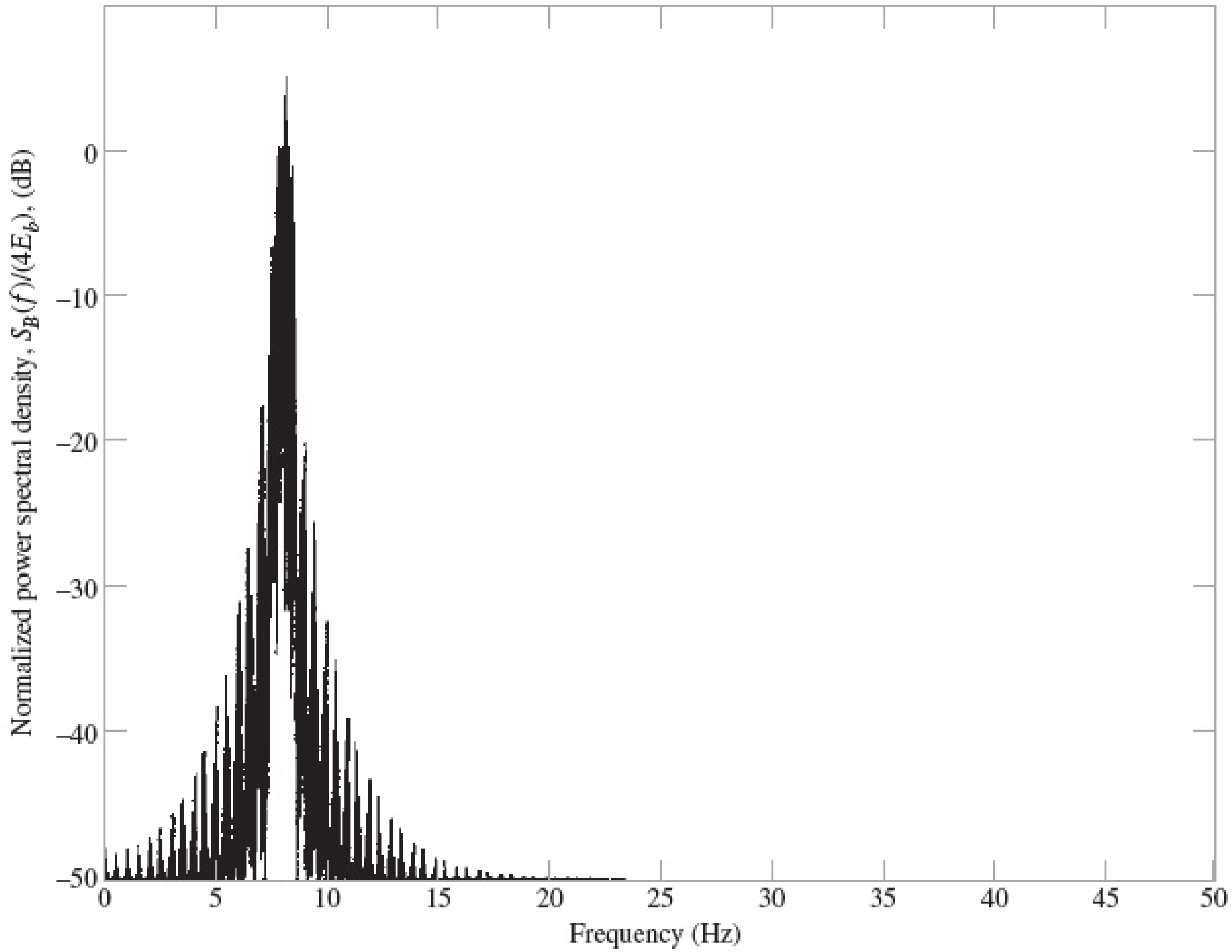


FIGURE 7.16 Illustrative confirmation that for MSK produced by a square modulating wave, the carrier frequency $f_c = 8$ Hz is high enough to produce insignificant spectral overlap for bit duration $T_b = 1$ s.

7.5 Summary of Three Binary Signaling Schemes

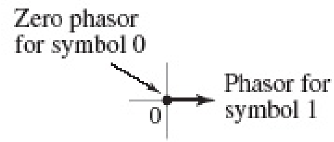
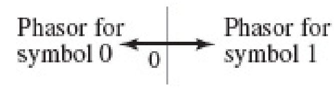
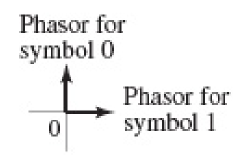
1. BASK, BPSK, and BFSK are the digital counterparts of amplitude modulation, phase modulation, and frequency modulation
2. Both BASK and BPSK exhibit discontinuity. It is possible to configure BFSK in such a way that phase continuity is maintained across the entire input binary data stream. The BFSK waveform plotted in part (d) of the figure is an example of minimum-shift keying

- Table 7.2 presents a summary of the three binary modulation schemes

Table 7.2

Table 7.2

Table 7.2 *Summary of Three Basic Binary Modulation Schemes*

Type of modulation scheme	*Variable parameter*	*Definition of modulated wave* $s_1(t)$ *or* $s_2(t)$, *for* $0 \le t \le T_b$	*Phasor representation of modulated wave*
1. Binary amplitude-shift keying (BASK)	$\begin{pmatrix}\text{Carrier amplitude}\\ A_c\end{pmatrix} = \begin{cases}\sqrt{\dfrac{2}{T_b}} & \text{for symbol 1}\\ 0 & \text{for symbol 0}\end{cases}$	$s_1(t) = \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t)$ for symbol 1 $s_2(t) = 0$ for symbol 0	
2. Binary phase-shift keying (BPSK)	$\begin{pmatrix}\text{Carrier phase}\\ \phi_c\end{pmatrix} = \begin{cases}0 & \text{for symbol 1}\\ \pi & \text{for symbol 0}\end{cases}$	$s_1(t) = \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t)$ for symbol 1 $s_2(t) = \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t + \pi)$ for symbol 0	
3. Binary frequency-shift keying (BFSK)	$\begin{pmatrix}\text{Carrier frequency}\\ f_c\end{pmatrix} = \begin{cases}f_1 & \text{for symbol 1}\\ f_2 & \text{for symbol 0}\end{cases}$	$s_1(t) = \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_1 t)$ for symbol 1 $s_2(t) = \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_2 t)$ for symbol 0	

Notations

T_b = bit duration

E_b = transmitted signal energy per bit

Carrier: $c(t) = A_c \cos(2\pi f_c t + \phi_c)$

The carrier phase ϕ_c is set equal to zero for both BASK and BFSK.

7.6 Noncoherent Digital Modulations Schemes

- Both BASK and BPSK are examples of linear modulation, with increasing complexity in going from BASK and BPSK.
- BFSK is in general an example of nonlinear modulation

Fig. 7.17

❖ Noncoherent Detection of BASK Signal

- The system designer would have knowledge of two system parameters
  - *The carrier frequency* f_c
  - *The transmission bandwidth, which is determined by the bit duration* T_b.
- The band-pass filter is designed to have a mid-band frequency equal to the carrier frequency f_c and a bandwidth equal to the transmission bandwidth of the BASK signal.
- The rise time and decay time of the response of the filter to a rectangular pulse be short compared to the bit duration T_b

1. Band-pass filter produces a pulsed sinusoid for symbol 1, no output for symbol 0.
2. Envelope detector traces the envelope of the filtered version of the BASK signal.
3. Decision-making device working in conjunction with the sampler, regenerates the original binary data stream

Fig.7.17

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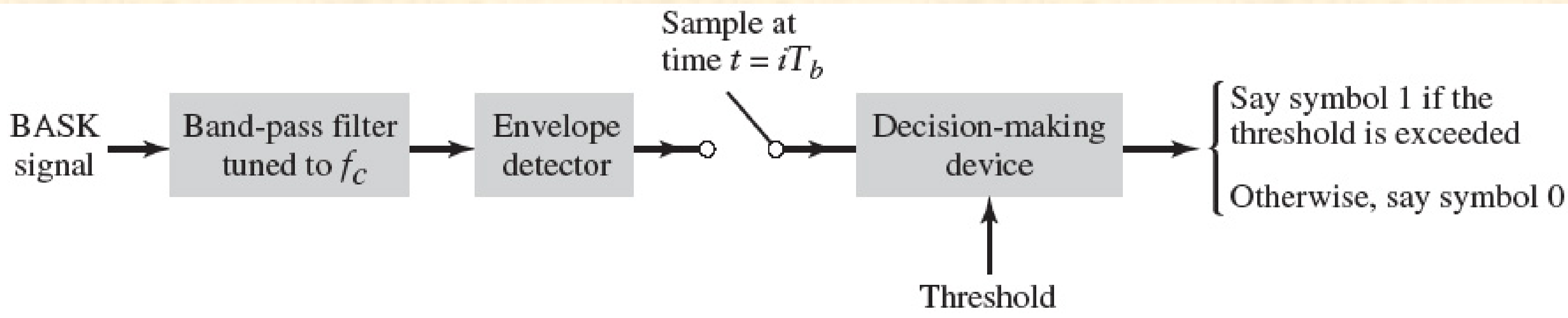

Figure 7.17 Noncoherent BASK receiver; the integer i for the sampler equals $0, \pm 1, \pm 2, \ldots$.

Noncoherent Detection of BFSK Signals

- The receiver consists of two paths
  - *Path 1 : uses a band-pass filter of mid-band frequency f_1. produce the output v_1*
  - *Path 2 : uses a band-pass filter of mid-band frequency f_2. produce the output v_2*
- The output of the two paths are applied to a comparator

Fig. 7.18

Fig.7.18

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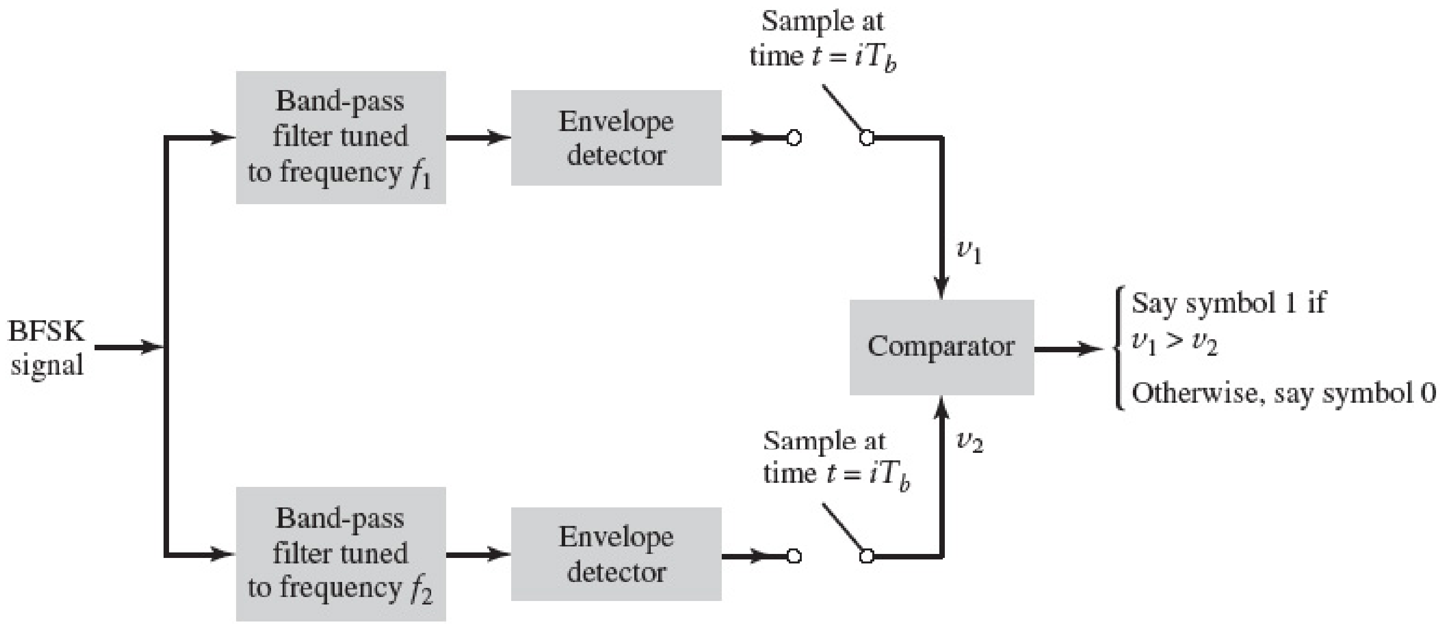

FIGURE 7.18 Noncoherent BFSK receiver; the two samplers operate synchronously, with $i = 0, \pm 1, \pm 2, \ldots$.

- Differential Phase-Shift Keying
  - In the case of phase-shift keying, we cannot have noncoherent detection in the traditional sense because the term "noncoherent" means having t to without carrier-phase information
  - We employ a "pseudo PSK" technique (differential phase-shift keying)
  - DPSK eliminates the need for a coherent reference signal at the receiver by combination two basic operations at the transmitter
    - *Differential encoding of the input binary wave*
    - *Phase-shift keying*
  - The receiver is equipped with a storage capability designed to measure the relative phase difference between the waveforms received during two successive bit intervals.
  - The phase difference between waveforms received in two succssive bit intervals will be essentially independent of θ.

1. Generation

- The differential encoding process at the transmitter input starts with an arbitrary first bit, serving merely as reference
  - *If the incoming binary symbol b_k is 1, then the symbol d_k is unchanged with respect to the previous symbol d_{k-1}*
  - *If the incoming binary symbol b_k is 0, then the symbol d_k is changed with respect to the previous symbol d_{k-1}*

2. Detection

- The phase-modulated pulses pertaining to two successive bits are identical except for a possible sign reversal
- The incoming pulse is multiplied by the preceding pulse
- The preceding pulse serves the purpose of a locally generated reference signal
- Applying the sampled output of the low-pass filter to a decision-making device supplied with a prescribed threshold, detection of the DPSK signal is accomplished.

Fig. 7.19 Table 7.3

Fig.7.19

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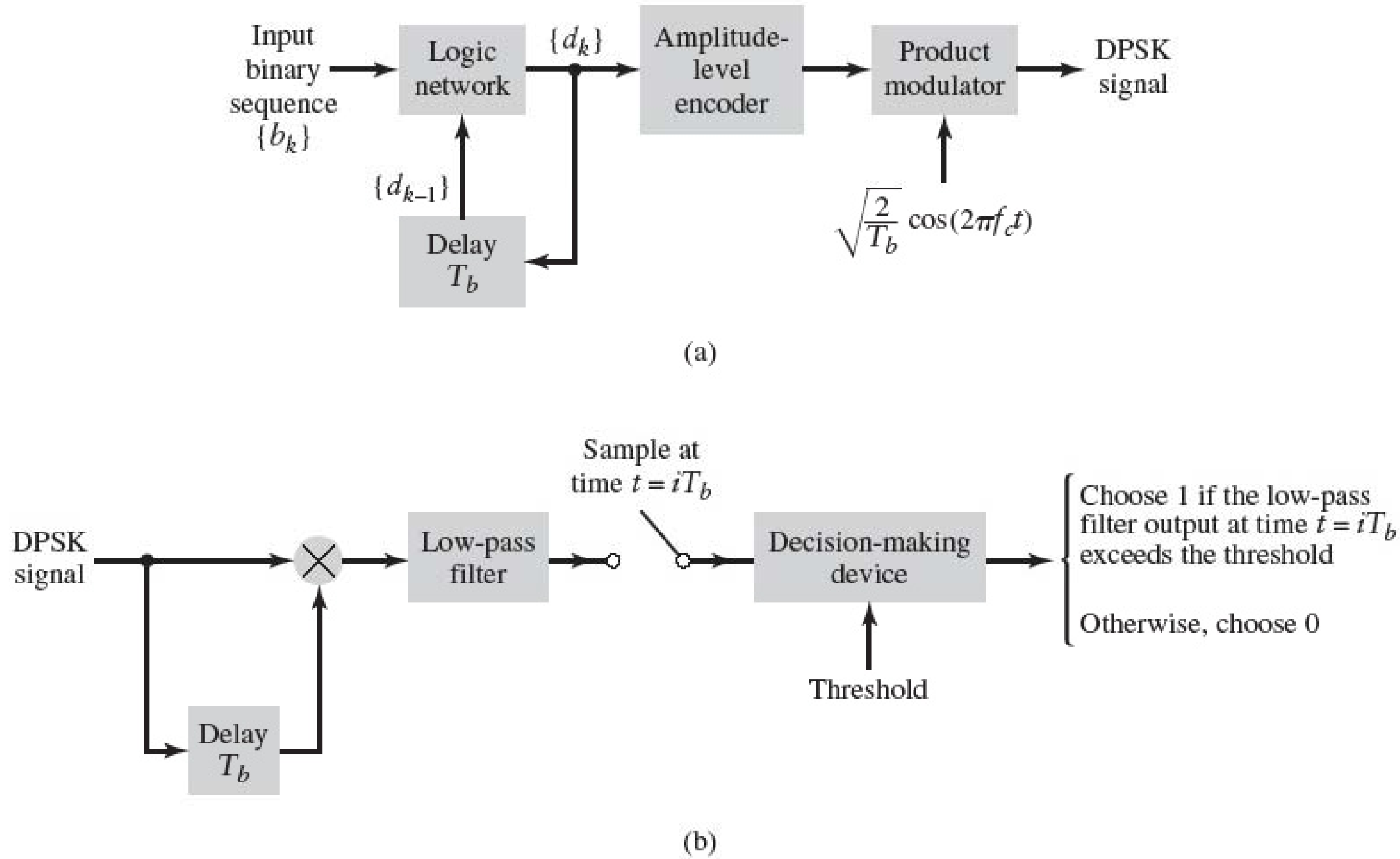

FIGURE 7.19 Block diagrams for (*a*) DPSK transmitter and (*b*) DPSK receiver; for the sampler, integer $i = 0, \pm 1, \pm 2, \ldots$.

table.7.3

TABLE 7.3 *Illustration of the Generation and Detection of DPSK Signal*

$\{b_k\}$		1	0	0	1	0	0	1	1
$\{d_{k-1}\}$		1	1	0	1	1	0	1	1
Differentially encoded sequence $\{d_k\}$	1	1	0	1	1	0	1	1	1
Transmitted phase (radians)	0	0	π	0	0	π	0	0	0
Sampler's output (polarity)		+	−	−	+	−	−	+	+
Binary symbol at decision-maker's output		1	0	0	1	0	0	1	1

Note: The symbol 1 inserted at the beginning of the differentially encoded sequence d_k is the reference bit.

7.7 M-ary Digital Modulation Schemes

- we send any one of *M* possible signals during each signaling interval of duration *T*
- The requirement is to conserve bandwidth at the expense of both increased power and increased system complexity
- When the bandwidth of the channel is less than the required value, we resort to an M-ary modulation scheme for maximum bandwidth conservation

❖ M-ary Phase-Shift Keying

- If we take blocks of m bits to produce a symbol and use an M-ary PSK scheme with $M=2^m$ and symbol duration $T=mT_b$
- The bandwidth required is proportional to $1/(mT_b)$
- The use of M-ary PSK provides a reduction in transmission bandwidth by a factor by a factor $m=\log_2 M$

- The discrete coefficients are respectively referred to as the in-phase and quadrature components of the M-ary PSK singal

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M} i\right), \quad \begin{matrix} i = 0,1,\ldots,M-1 \\ 0 \le t \le T \end{matrix} \qquad (7.35)$$

$$s_i(t) = \left[\sqrt{E}\cos\left(\frac{2\pi}{M} i\right)\right]\left[\sqrt{\frac{2}{T}}\cos(2\pi f_c t)\right] - \left[\sqrt{E}\sin\left(\frac{2\pi}{M} i\right)\right]\left[\sqrt{\frac{2}{T}}\sin(2\pi f_c t)\right], \quad \begin{matrix} i = 0,1,\ldots,M-1 \\ 0 \le t \le T \end{matrix} \qquad (7.36)$$

$$\left\{\left[\sqrt{E}\cos\left(\frac{2\pi}{M} i\right)\right]^2 + \left[\sqrt{E}\sin\left(\frac{2\pi}{M} i\right)\right]^2\right\}^{1/2} = \sqrt{E}, \quad \text{for all } i \qquad (7.37)$$

- Signal-Space Diagram
  - Pair of orthogonal functions

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t), \quad 0 \le t \le T \quad (7.38)$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t), \quad 0 \le t \le T \quad (7.39)$$

  - Figure 7.20

1. *M-ary PSK is described in geometric terms by a constellation of M signal points distributed uniformly on a circle of radius √E*
2. *Each signal point in the figure corresponds to the signal $s_i(t)$ of Eq. (7.35) for a particular value of the index i.*
3. *The squared length from the origin to each signal point is equal to the signal energy E.*

Fig. 7.20

Fig.7.20

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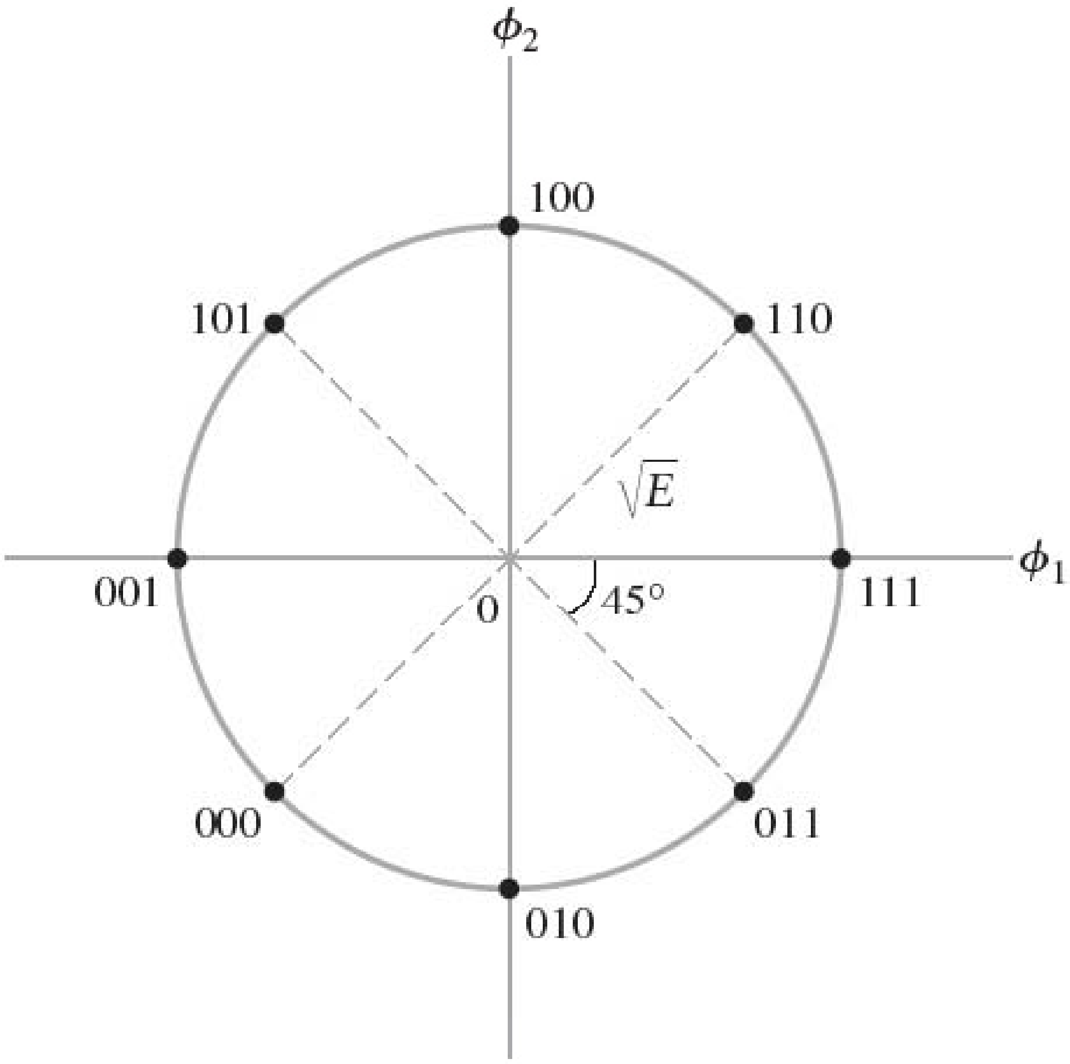

FIGURE 7.20 Signal-space diagram of 8-PSK.

- M-ary Quadrature Amplitude Modulation (QAM)
  - The mathematical description of the new modulated signal

$$s_i(t) = \sqrt{\frac{2E_0}{T}} a_i \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}} b_i \sin(2\pi f_c t), \quad \begin{matrix} i = 0,1,\ldots,M-1 \\ 0 \le t \le T \end{matrix} \qquad (7.40)$$

  - The level parameter for in-phase component and quadrature component are independent of each other for all I
  - M-ary QAM is a hybrid form of M-ary modulation
  - M-ary amplitude-shift keying (M-ary ASK)
    - *If $b_i=0$ for all i, the modulated signal $s_i(t)$ of Eq. (7.40) reduces to*

$$s_i(t) = \sqrt{\frac{2E_0}{T}} a_i \cos(2\pi f_c t) \quad i = 0,1,\ldots,M-1$$

  - M-ary PSK
    - *If E0=E and the constraint is satisfied*

$$(Ea_i^2 + Eb_i^2)^{1/2} = \sqrt{E}, \quad \text{for all } i$$

- Signal-Space Diagram
  - Figure 7.21 is the signal-space representation of M-ary QAM for M=16
  - Unlike M-ary PSK, the different signal points of M-ary QAM are characterized by different energy levels
  - Each signal point in the constellation corresponds to a specific quadbit

Fig. 7.21

Fig.7.21

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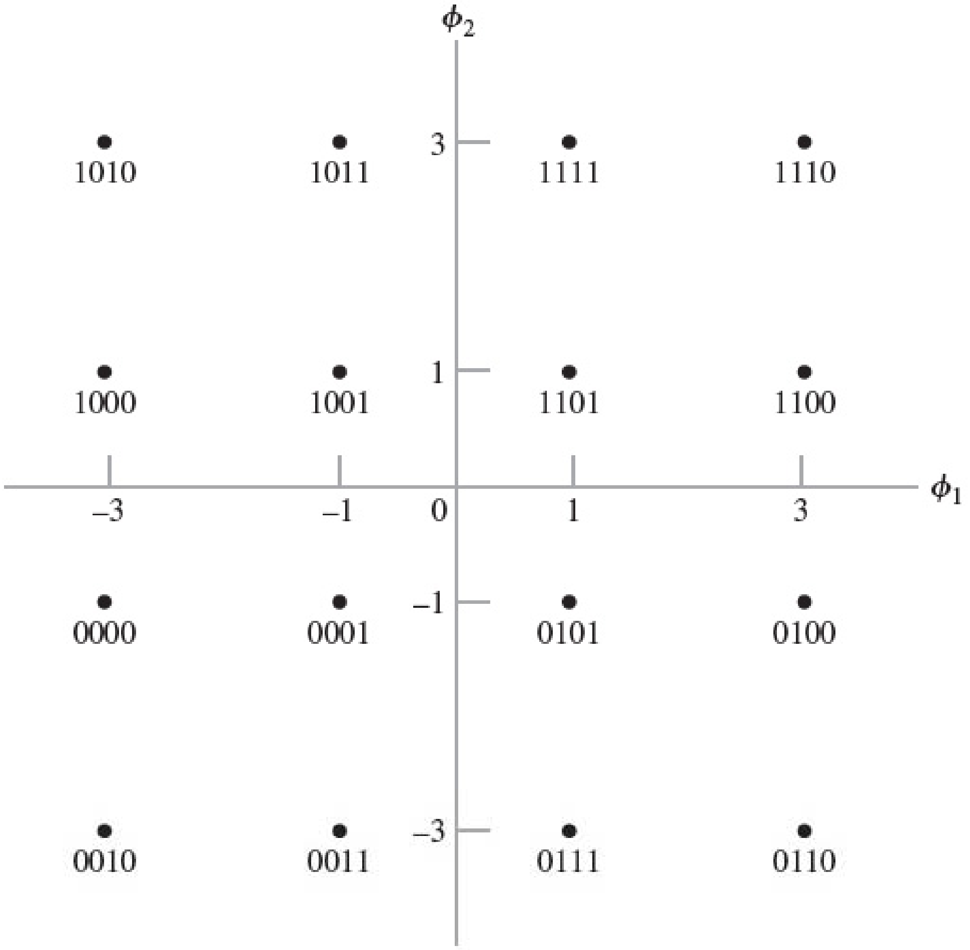

FIGURE 7.21 Signal-space diagram of Gray-encoded M-ary QAM for $M = 16$.

- M-ary Frequency-Shift Keying
  - In one form of M-ary FSK, the transmitted signals are defined for some fixed integer *n* as

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[\frac{\pi}{T}(n+i)t\right], \quad \begin{matrix} i = 0,1,\ldots,M-1 \\ 0 \le t \le T \end{matrix} \qquad (7.41)$$

  - Like M-ary PSK, the envelope of M-ary FSK is constant for all M

$$\int_0^T s_i(t)s_j(t)dt = \begin{cases} E \text{ for } i = j \\ 0 \text{ for } i \ne j \end{cases} \qquad (7.42)$$

  - Signal-Space Diagram
    - *Unlike M-ary PSK and M-ary QAM, M-ary FSK is described by an M-dimensional signal-space diagram*

$$\phi_i(t) = \frac{1}{\sqrt{E}} s_i(t) \quad \begin{matrix} i = 0,1,\ldots,M-1 \\ 0 \le t \le T \end{matrix} \qquad (7.43)$$

Fig. 7.22

Fig.7.22

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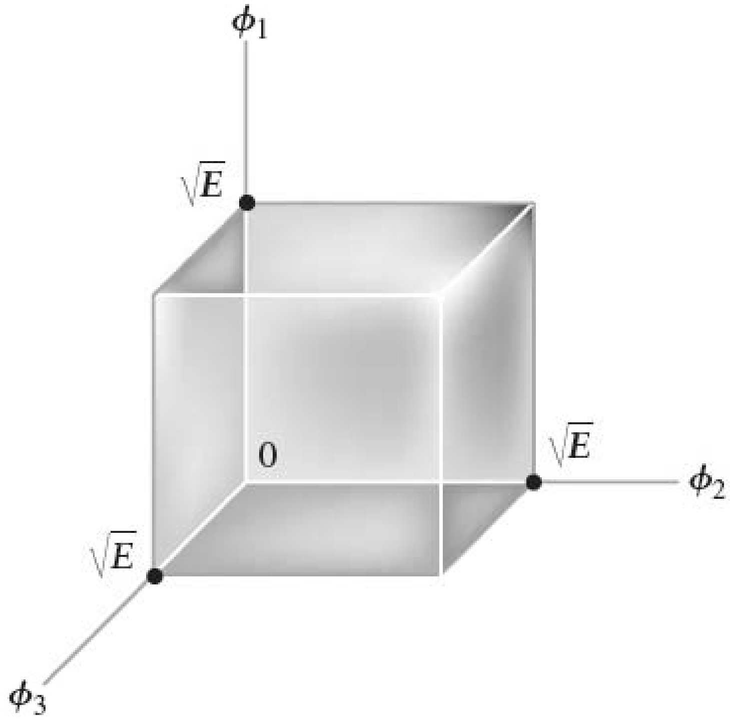

FIGURE 7.22 Signal constellation for M-ary FSK for $M = 3$.

7.8 Mapping of Digitally Modulation Waveforms Onto Constellations of Signal Points

- for a specific method of digital modulation, the geometric representation is pictured in the form of a constellation of points in the signal-space diagram
- The purpose of this section is
  - *Consolidate the idea of a signal-space diagram pictorially*
  - *Discuss what this idea teaches us in the analysis of noise in digital communication systems, which we treat later in the book*
- The signal-space representation of BPSK is simple, involving a single basis function

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \quad (7.44)$$

- The use of correlation provides another way of designing a receiver for the coherent detection of BPSK

1. Correlating the signal

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad \text{for symbol 1} \quad (7.45)$$

$$s_1 = \int_0^{T_b} \phi_1(t) s_1(t)\, dt$$

$$= \int_0^{T_b} \frac{2}{T_b} \sqrt{E_b} \cos^2(2\pi f_c t)\, dt \quad (7.46)$$

under the band-pass assumption,

$$s_1 = \sqrt{E_b} \quad (7.47)$$

2. We may show that the signal

$$s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad \text{for symbol 0} \quad (7.48)$$

$$s_2 = -\sqrt{E_b} \quad (7.49)$$

Fig. 7.23

Fig.7.23

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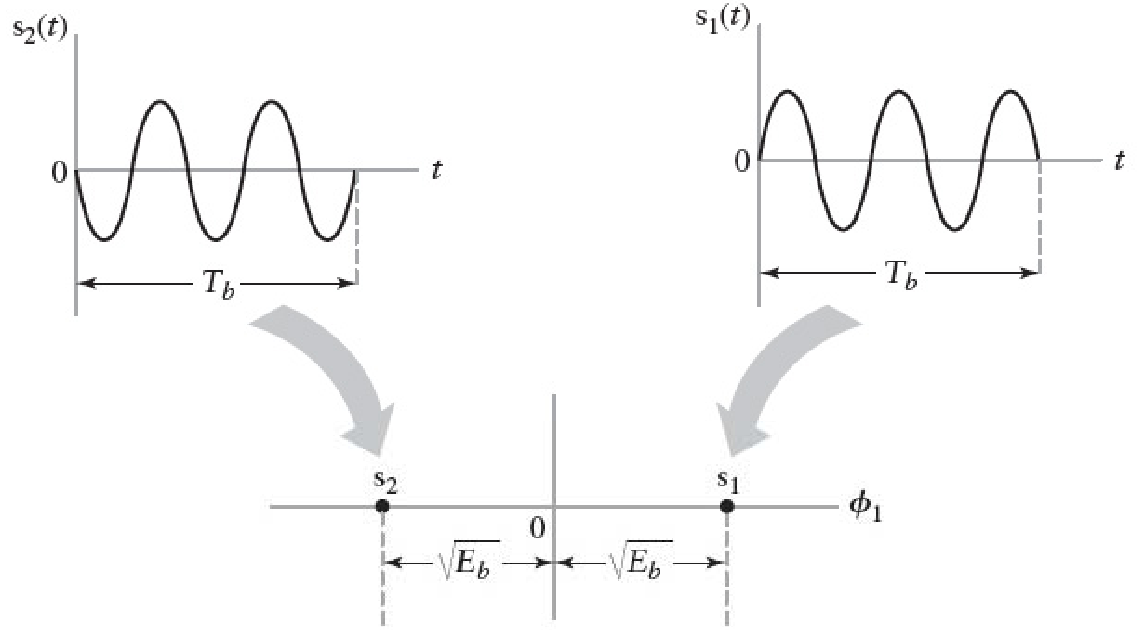

FIGURE 7.23 Mapping of BPSK signal onto a one-dimensional signal-space diagram.

- As with BPSK, the signal-space diagram consists of two transmitted signal points

$$s_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \quad (7.50) \qquad s_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix} \quad (7.51)$$

- Fig. 7.23 and 7.24 differ in one important respect : dimensionality

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t) \quad (7.52) \qquad \phi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t) \quad (7.53)$$

1. The separation between the transmitted signal points for BPSK is √2 times that for BFSK
2. The received signal point lies inside a "cloud" centered on the transmitted signal point

Fig. 7.24

Fig.7.24

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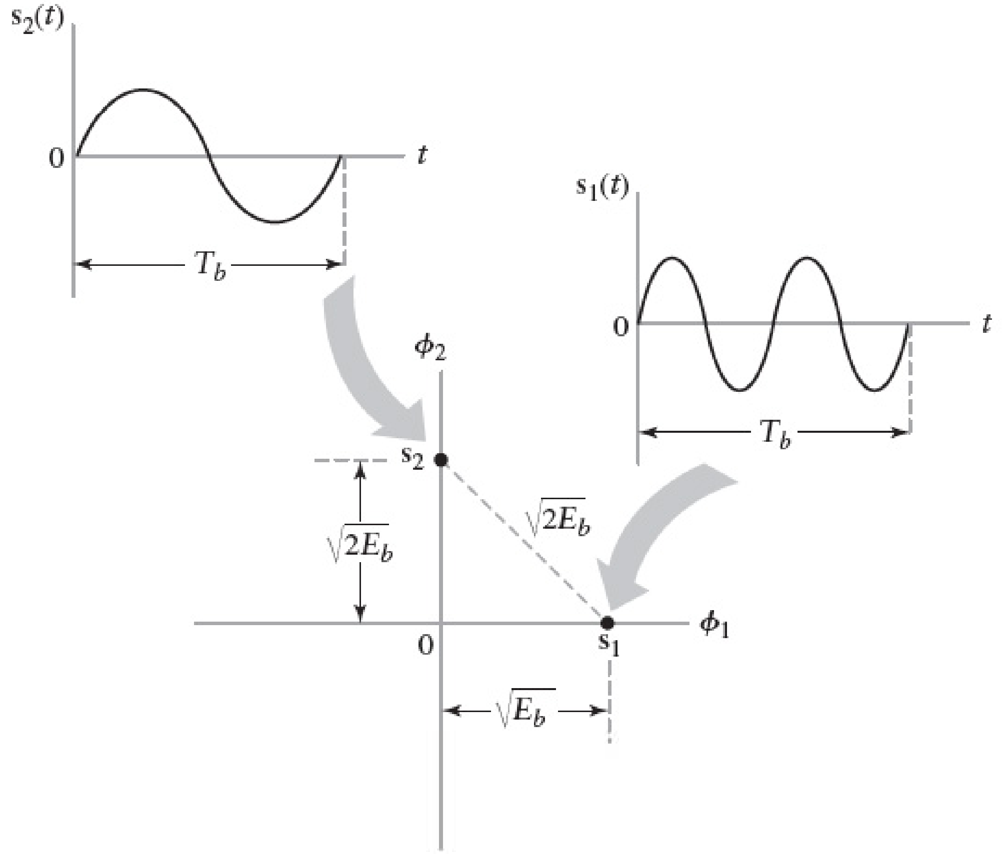

FIGURE 7.24 Mapping of BFSK onto two-dimensional signal-space diagram.