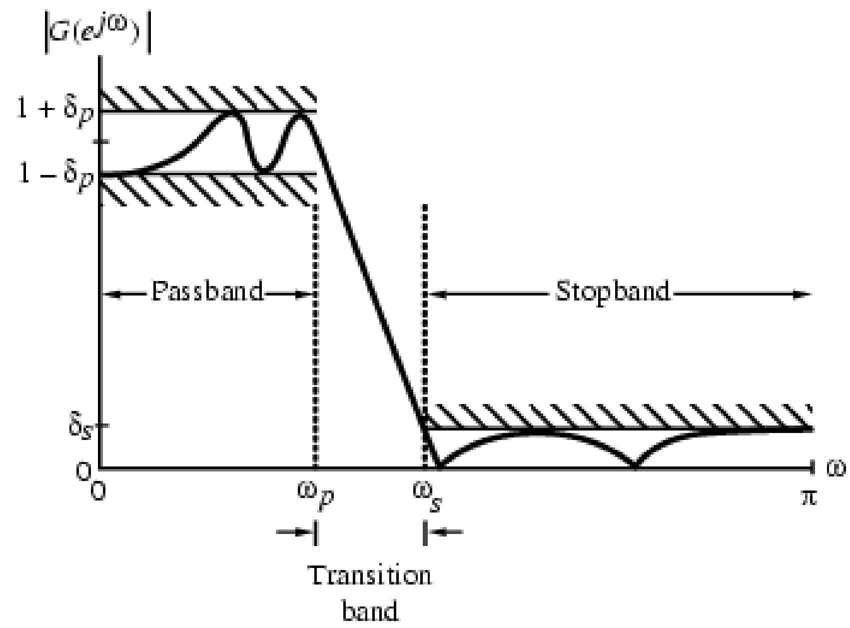
Digital Filter Specifications

The magnitude response of a digital lowpass filter may be given as indicated below



Digital Filter Specifications

Filter specification parameters

- ω_p - **passband edge frequency**
- ω_s - **stopband edge frequency**
- δ_p - **peak ripple value** in the passband
- δ_s - **peak ripple value** in the stopband

Digital Filter Specifications

- Practical specifications are often given in terms of **loss function (in dB)**
- $$\mathrm{G}(\omega) = -20\log_{10}\left|G(e^{j\omega})\right|$$
- **Peak passband ripple**

$$\alpha_p = -20\log_{10}(1-\delta_p) \quad \text{dB}$$

- **Minimum stopband attenuation**

$$\alpha_s = -20\log_{10}(\delta_s) \text{ dB}$$

Digital Filter Specifications

- In practice, passband edge frequency F_p and stopband edge frequency F_s are specified in Hz
- For digital filter design, normalized bandedge frequencies need to be computed from specifications in Hz using

$$\omega_p = \frac{\Omega_p}{F_T} = \frac{2\pi F_p}{F_T} = 2\pi F_p T$$

$$\omega_s = \frac{\Omega_s}{F_T} = \frac{2\pi F_s}{F_T} = 2\pi F_s T$$

Digital Filter Specifications

- Example - Let $F_p = 7$ kHz, $F_s = 3$ kHz, and $F_T = 25$ kHz
- Then

$$\omega_p = \frac{2\pi(7\times 10^3)}{25\times 10^3} = 0.56\pi$$

$$\omega_s = \frac{2\pi(3\times 10^3)}{25\times 10^3} = 0.24\pi$$

Selection of Filter Type

- The transfer function $H(z)$ meeting the specifications must be a causal transfer function
- For IIR real digital filter the transfer function is a real rational function of

$$H(z) = \frac{p_0 + p_1 z^{-1} + p_2 z^{-2} + \cdots + p_M z^{-M}}{d_0 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_N z^{-N}}$$

- $H(z)$ must be stable and of lowest order N or M for reduced computational complexity

Selection of Filter Type

- FIR real digital filter transfer function is a polynomial in z^{-1} (order *N*) with real coefficients

$$H(z) = \sum_{n=0}^{N} h[n] z^{-n}$$

- For reduced computational complexity, degree *N* of *H*(*z*) must be as small as possible
- If a linear phase is desired then we must have:

$$h[n] = \pm h[N-n]$$

- (More on this later)

Constraints and Performance Measures

- BIBO stability:
  - If $|x(n)| < \infty$, it is required that $|y(n)| < \infty$.
  - Poles should be inside unit circle: $|p_j| < 1$ (for causal systems where $h(n)=0$ for $n < 0$.)
- Dynamic range overflow:
  - Intermediate or final result should not cause overflow
- Quantization error:
  - Should be bounded.
  - Should not cause instability.
- Speed:
  - Throughput rate and number of operations per data sample
- Hardware:
  - Memory I/O, address calculation, register footprint, special hardware, etc.

DIFFERENCE BETWEEN IIR FILTER AND FIR FILTER

FIR Digital Filter

- Let {h[n]}: impulse response
  {x(n)}: input,
  {y(n)}: output
- Finite impulse response (FIR) filter:

$$y(n) = \sum_{j=0}^{J-1} h(j)x(n-j)$$

IIR Digital Filter

- Infinite impulse response (IIR) filter

$$y(n) = \sum_{i=1}^{P} a(i)y(n-i) + \sum_{k=0}^{Q} b(k)x(n-k)$$

$$\frac{Y(z)}{X(z)} = \frac{\sum_{m=0}^{Q} b(m)z^{-m}}{1+\sum_{k=1}^{P} a(k)z^{-k}} = \frac{B(z)V(z)}{A(z)V(z)}$$

FIR Digital Filter

- Impulse input:

  if $x(n)=\delta(n)$, $y(n)=h(n)$ is the impulse response that has finite extent.

- Computation is the same as convolution.

IIR Digital Filter

- The length of $\{y(n)\}$ may be infinite!
- Stability concerns:
  - The magnitude of $y(n)$ may become infinity even if all $x(n)$ are finite!
  - coefficient values,
  - quantization error

FIR Digital Filter

- FIR filter can be implemented using direct form or fast convolution methods like FFT ,hence STABLE.
- Realized by Non-Recursive methods.

IIR Digital Filter

- IIR filters are often factored into products (cascade realization) or sum (parallel realization) of 1st order or 2nd order sections due to numerical concerns(Manual Calculation only possible)
- Realised by Recursive(Feedback) methods.

FIR Digital Filter

- They have LINEAR PHASE.
- Less susceptible to Noise.
- To design we have

a)Park Mc Clellan's method.

b)Fourier Series method.

c)Frequency Sampling OR Inverse Fourier Transform method.

d)Window technique.

E.g. Rectangular,Hamming,Hanning,Bartlett,Blackmann,Kaiser Windows.

e)Minimax or Optimal Filter Design.

IIR Digital Filter

- They don't have linear phase & hence are used at places where phase distortion is tolerable.
- More susceptible to Noise.
- To design we have

a)Impulse Invarience method.

b)Bilinear Transformation method.

c)Backward difference method.

FIR Digital Filter

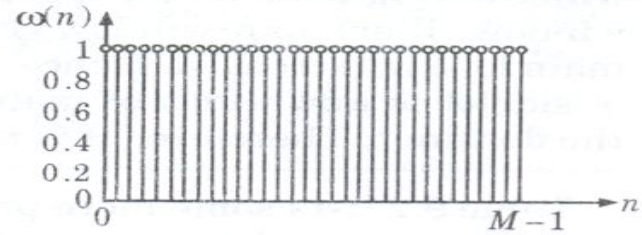
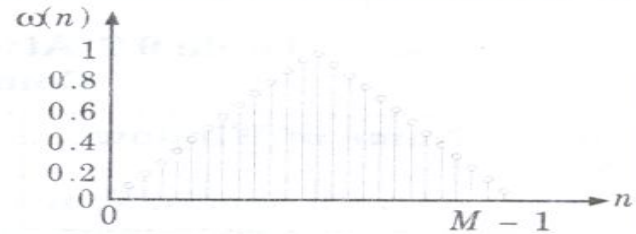
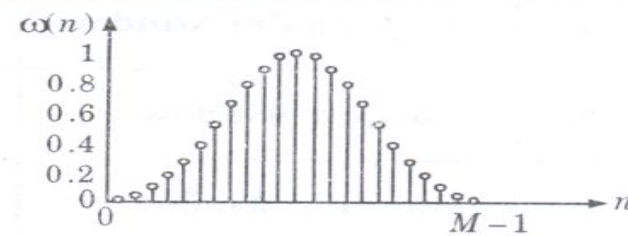
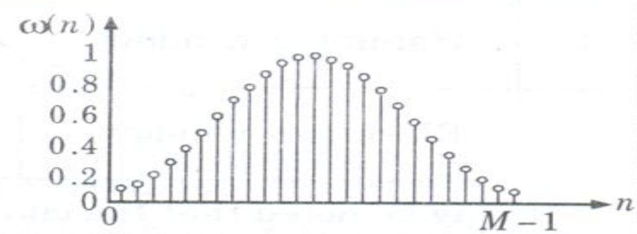
- Storage Requirements & Arithmetic operation is more here.
- Greater Flexibility to control the shape of their Magnitude response & Realization Efficiency.

IIR Digital Filter

- Storage Requirements & Arithmetic operation is less.
- Less Flexibility to control the shape of their Magnitude response.
- Often derived from analog filters

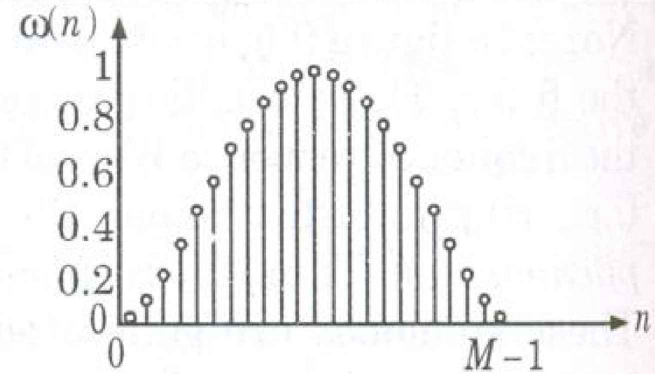
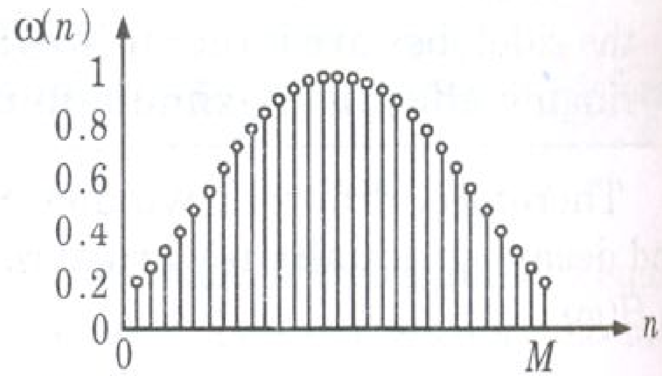
Various other window functions

Table 9.1 Various window functions and their corresponding shapes

Sr. No.	Name of Window	Time-domain sequence, $\omega(n)$, $0 \le n \le M-1$	Shape of window function
1.	Rectangular	1	
2.	Bartlett (triangular)	$1-\dfrac{2\left\lvert n-\dfrac{M-1}{2}\right\rvert}{M-1}$	
3.	Blackman	$0.42\text{–}05\cos\dfrac{2\pi n}{M-1}+0.08\cos\dfrac{4\pi n}{M-1}$	
4.	Hanning	$0.54-0.46\cos\dfrac{2\pi n}{M-1}$	

(Contd..)

Various other window functions

5.	Hanning	$\frac{1}{2}\left(1-\cos\frac{2\pi n}{M-1}\right)$	
6.	Kaiser	$\frac{I_0\left\{\beta\left[1-\left(\frac{n-\alpha}{\alpha}\right)^2\right]^{\frac{1}{2}}\right\}}{I_0(\beta)}$	

Comparative Study for Trade Off between Attenuation of Sidelobes & Transition Width of main Lobe.

Table 5.2 Attenuation and Transition Widths of Commonly Used Windows

S. No.	Name of Window	Transition width of the main lobe	Minimum stopband attenuation	Relative amplitude of sidelobe
1.	Rectangular window	$\frac{4\pi}{M+1}$	– 21 dB	– 13 dB
2.	Bartlett window	$\frac{8\pi}{M}$	– 25 dB	– 25 dB
3.	Hanning window	$\frac{8\pi}{M}$	– 44 dB	– 31 dB
4.	Hamming window	$\frac{8\pi}{M}$	– 53 dB	– 41 dB
5.	Blackman window	$\frac{12\pi}{M}$	– 74 dB	– 57 dB

It may be noted that the characteristics of Kaiser window have not been mentioned

FIR Filter Design: Rectangular Window

- Let w(n)=Rectangular Window Function,
- Where
- $w(n)=1 \qquad\qquad 0 \le n \le M-1$

hd(n)=Infinite Input Sequence(Arbitrary),&

h(n)=Finite Truncated Impulse Response.

Then

$h(n)=hd(n) \times w(n)$

Fig. 9.4 *Rectangular window.*

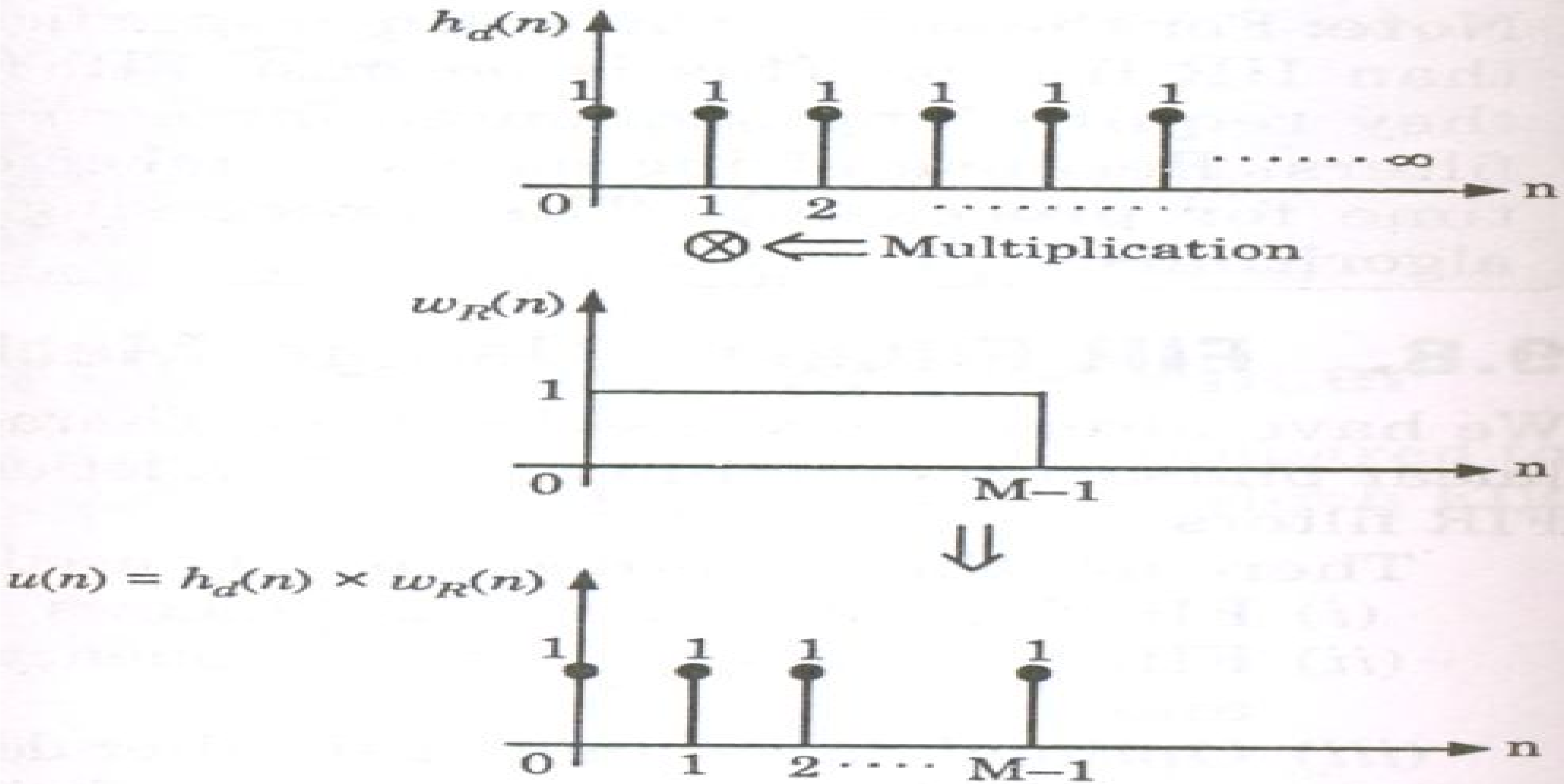


Fig. 9.5. *Truncation process*

for $\quad n = 0, 1, \ldots, M-1$

Gibbs Phenomenon:Ringing Effect/Oscillatory Behaviour due to Sidelobes(generated owing to the sharp cut-off/abrupt discontinuity) in the Frequency Response of the window Function

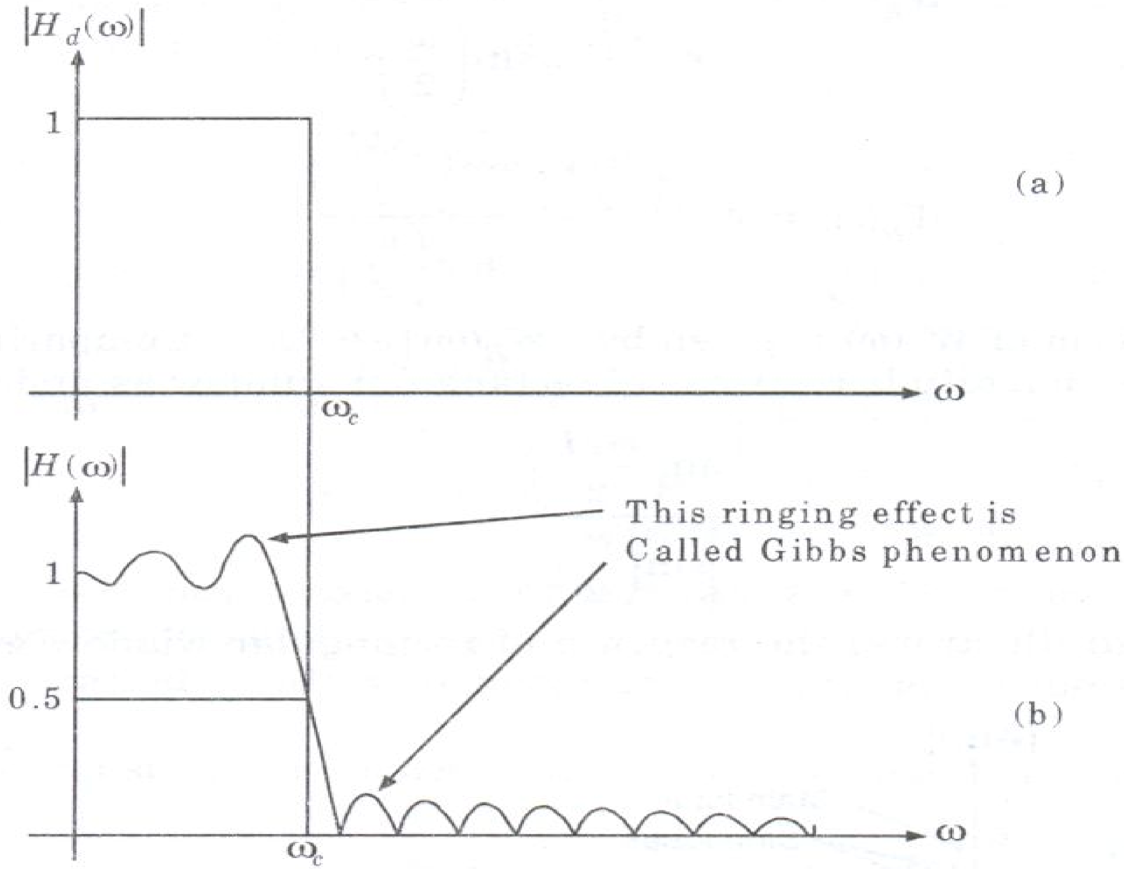


Fig. 9.6. *(a) The desired frequency response $H_d(\omega)$ (b) The frequency response of FIR filter obtained by windowing. It has smoothing and ringing effect because of windowing.*