

DISCRETE TIME SIGNALS AND SYSTEMS

2 Discrete-Time Signals and Systems

2.1 Discrete-Time Signals

2.2 Discrete-Time Systems

2.3 Analysis of Discrete-Time LTI Systems

2.4 Discrete-Time Systems Described by Difference Equations

2.5 Implementation of Discrete-Time Systems

2.6 Correlation of Discrete-Time Signals

2.2 Discrete-Time Systems

2.2.1 Input-Output Description of Systems

Ex. Determine the response of the following systems to

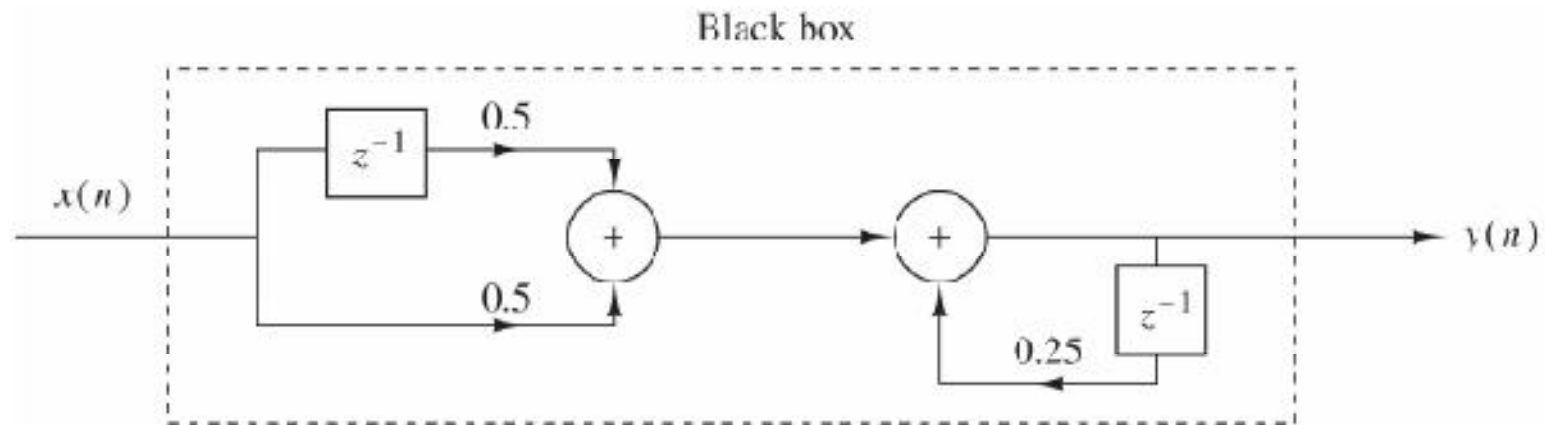
$$x(n) = \{ \dots, 0, 0, -3, -2, -1, \underset{\uparrow}{0}, 1, 2, 3, 0, 0, \dots \}$$

1. $y(n) = x(n)$
2. $y(n) = x(n - 1)$
3. $y(n) = x(n + 1)$
4. $y(n) = [x(n + 1) + x(n) + x(n - 1)]/3$
5. $y(n) = \text{median} [x(n + 1), x(n), x(n - 1)]$
6. $y(n) = x(n) + x(n - 1) + x(n - 2) + \dots$
 $= y(n - 1) + x(n)$

2.2 Discrete-Time Systems

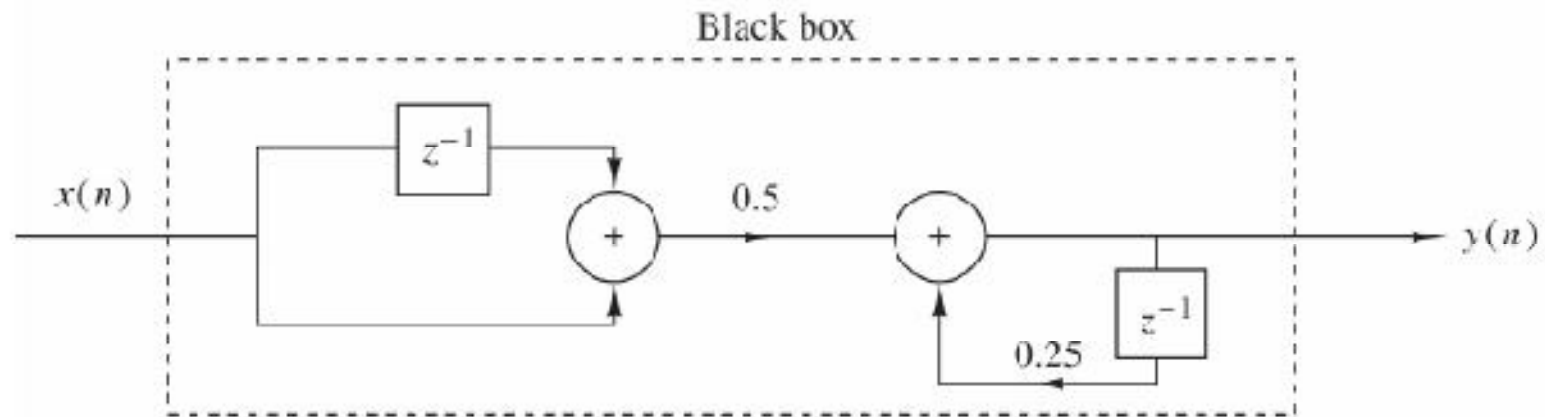
2.2.2 Block Diagram Representation of Discrete-Time Systems

Ex.



$$y(n) = 0.25 y(n-1) + 0.5 x(n) + x(n-1) \quad (a)$$

Initial condition?



(b)

2.2 Discrete-Time Systems

2.2.3 Classification of Discrete-Time Systems

1. Static (memoryless) versus dynamic: $y(n) = T[x(n), n]$
2. Time-invariant versus time-variant: $x(n - k) \rightarrow y(n - k)$
3. Linear versus nonlinear: $T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$
4. Causal versus noncausal: $y(n) = F[x(n), x(n - 1), x(n - 2), \dots]$
5. Stable versus unstable: $|x(n)| \leq M_x < \infty \rightarrow |y(n)| \leq M_y < \infty$

2.2 Discrete-Time Systems

2.2.3 Classification of Discrete-Time Systems

Ex. Time-invariant versus time-variant: $x(n - k) \rightarrow y(n, k) \stackrel{?}{=} y(n - k)$

1. $y(n) = x(n) + x(n - 1)$

$$y(n, k) = x(n - k) + x(n - k - 1) = y(n - k)$$

2. $y(n) = nx(n)$

$$y(n, k) = n x(n - k) \neq y(n - k)$$

3. $y(n) = x(-n)$

$$y(n, k) = x(-n - k) \neq y(n - k)$$

4. $y(n) = x(n) \cos \omega_0 n$

$$y(n, k) \neq y(n - k)$$

2.2 Discrete-Time Systems

2.2.3 Classification of Discrete-Time Systems

Ex. Linear versus nonlinear: $T[a_1x_1(n) + a_2x_2(n)] \stackrel{?}{=} a_1T[x_1(n)] + a_2T[x_2(n)]$

1. $y(n) = nx(n)$

2. $y(n) = x(n^2)$

3. $y(n) = x^2(n)$

4. $y(n) = Ax(n) + B$

5. $y(n) = e^{x(n)}$

Let $x(n) = a_1x_1(n) + a_2x_2(n)$ and

Check $T[x(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$

2.2 Discrete-Time Systems

2.2.3 Classification of Discrete-Time Systems

Ex. Causal versus noncausal: $y(n) \stackrel{?}{=} F[x(n), x(n-1), x(n-2), \dots]$

1. $y(n) = x(n) + x(n-1)$

2. $y(n) = \sum_{k=-\infty}^n x(k)$

3. $y(n) = ax(n)$

4. $y(n) = x(n) + 3x(n+4)$

5. $y(n) = x(n^2)$

6. $y(n) = 2x(n)$

7. $y(n) = x(-n)$

2.2 Discrete-Time Systems

2.2.3 Classification of Discrete-Time Systems

Ex. Stable versus unstable: $|x(n)| \leq M_x < \infty \stackrel{?}{\rightarrow} |y(n)| \leq M_y < \infty$

$$y(n) = y^2(n-1) + x(n)$$

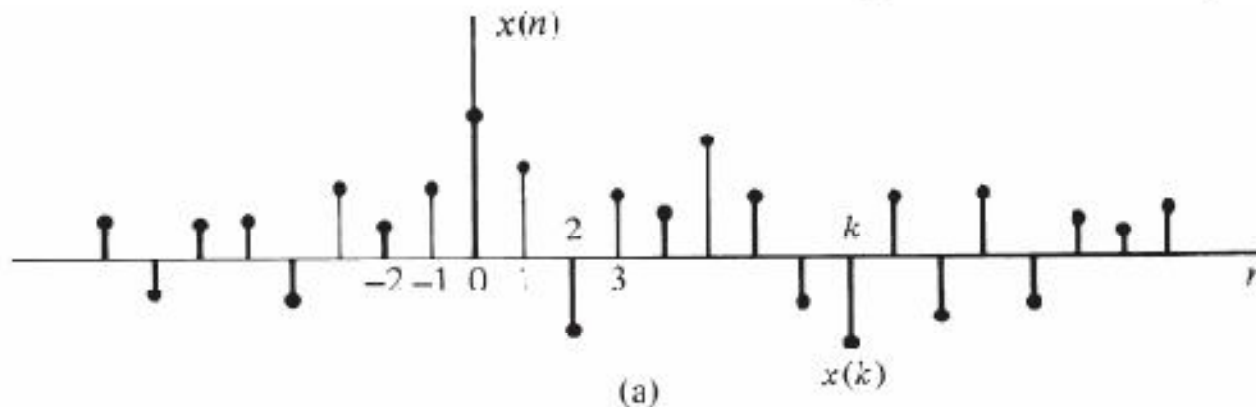
$$x(n) = \{ \dots 0, 0, 2, 0, 0, \dots \}$$



$$y(n) = \{ \dots 0, 0, 2, 2^2, 2^4, 2^8, \dots \}$$

2.3 Analysis of Discrete-Time LTI Systems

2.3.2 Resolution of a Discrete-Time Signal into Impulses (1/2)



$$x(n) = \sum_{k=-\infty}^{\infty} x(k) \delta(n - k)$$

