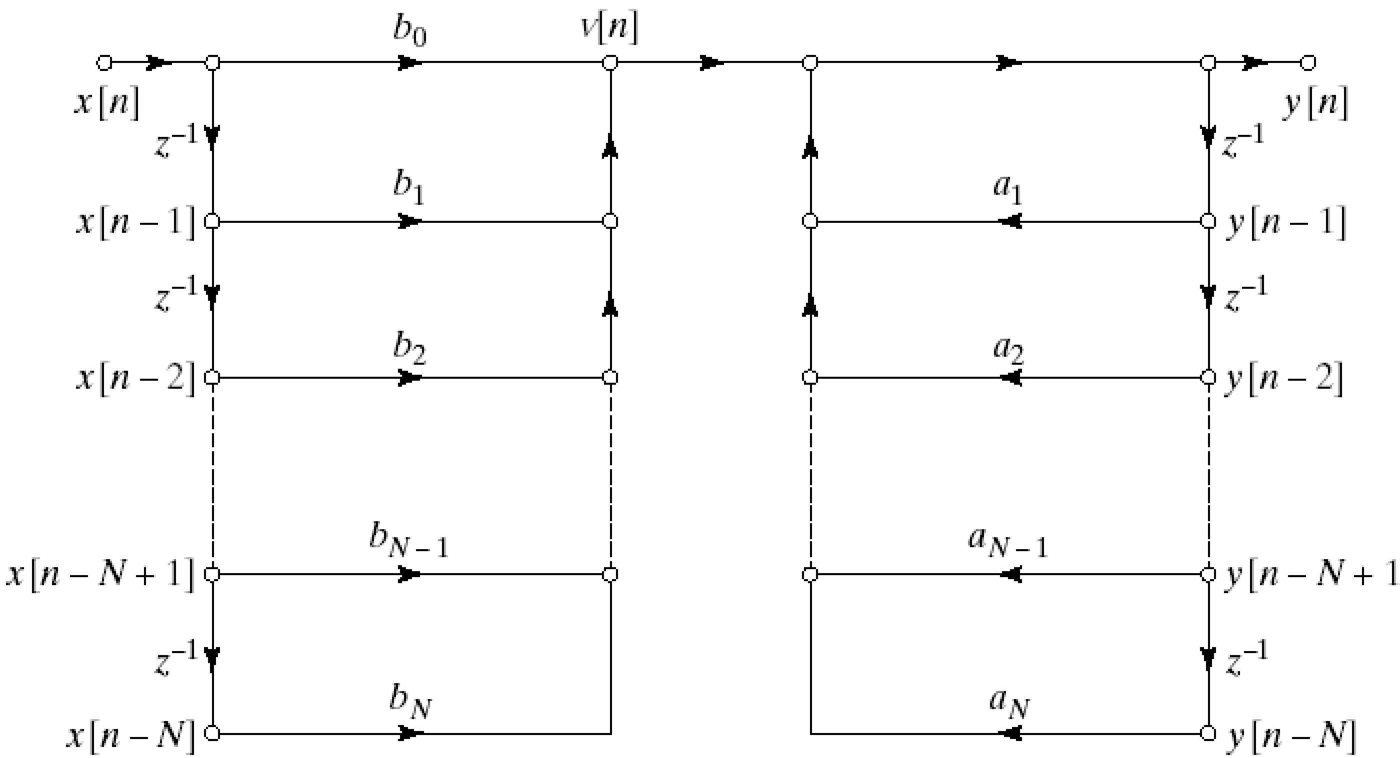
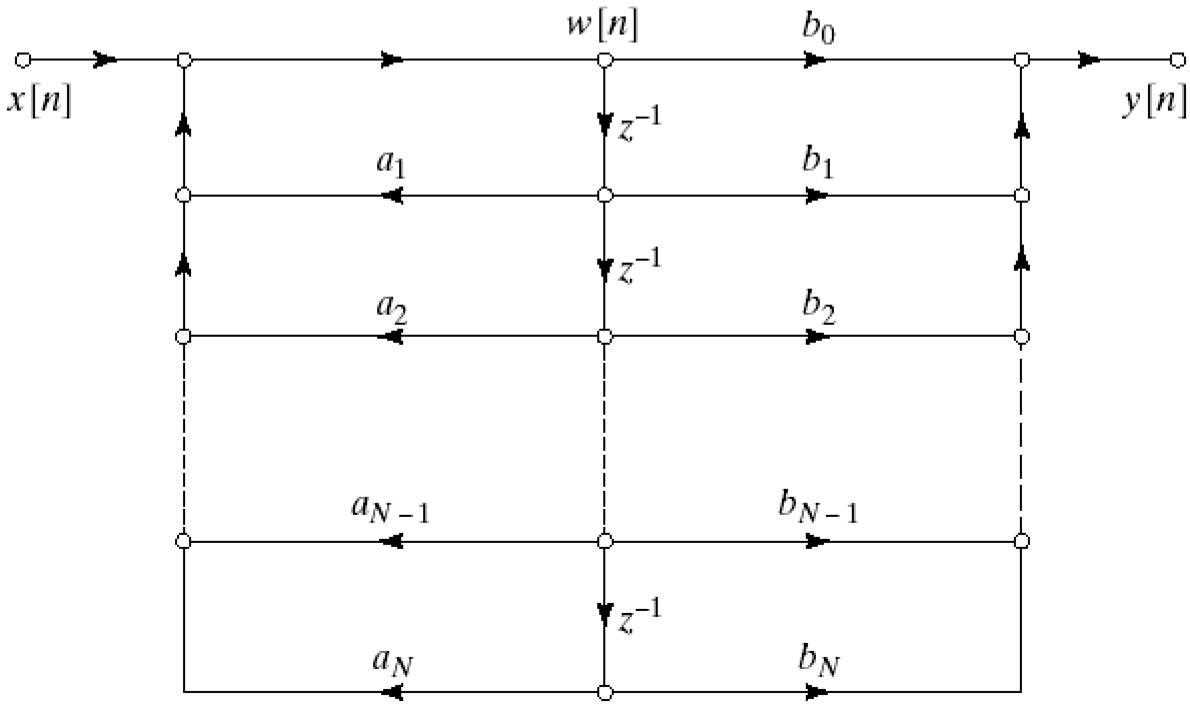
IIR SYSTEMS

Basic Structures for IIR Systems: Direct Form I



Basic Structures for IIR Systems: Direct Form II



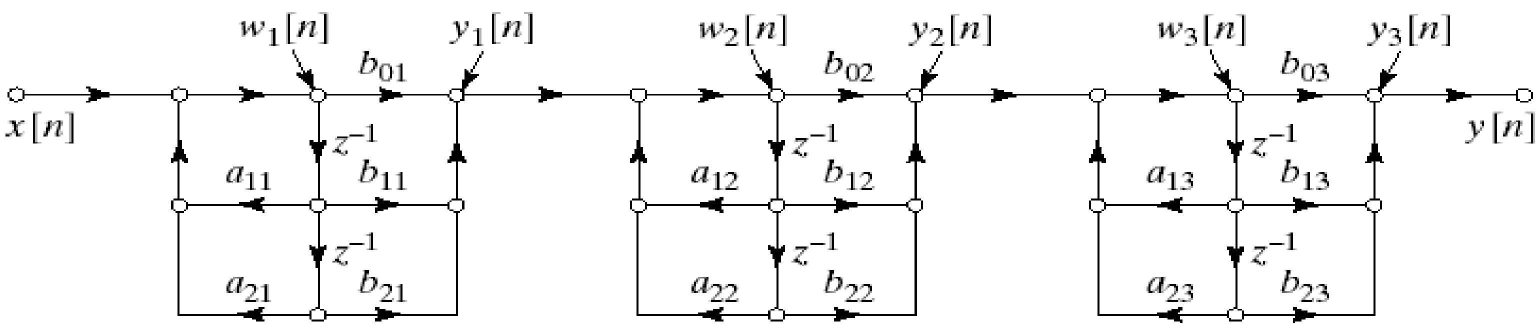
Basic Structures for IIR Systems: Cascade Form

- General form for cascade implementation

$$H(z) = A \frac{\prod_{k=1}^{M_1} \left(1 - f_k z^{-1}\right) \prod_{k=1}^{M_2} \left(1 - g_k z^{-1}\right)\left(1 - g_k^* z^{-1}\right)}{\prod_{k=1}^{N_1} \left(1 - c_k z^{-1}\right) \prod_{k=1}^{N_2} \left(1 - d_k z^{-1}\right)\left(1 - d_k^* z^{-1}\right)}$$

$$H(z) = \prod_{k=1}^{M_1} \frac{b_{0k} + b_{1k} z^{-1} - b_{2k} z^{-2}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

- More practical form in 2nd order systems

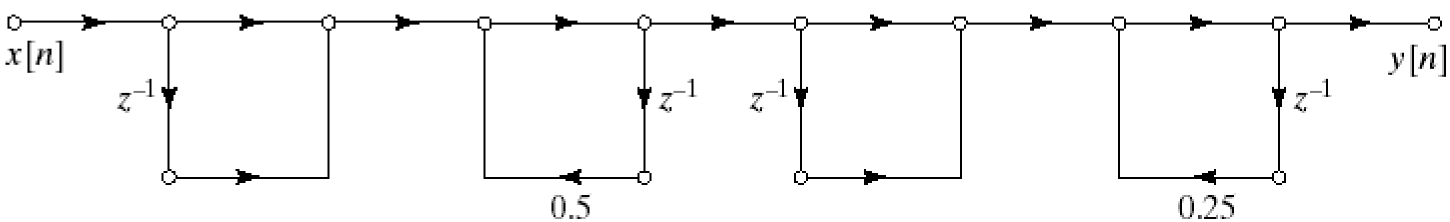


Example

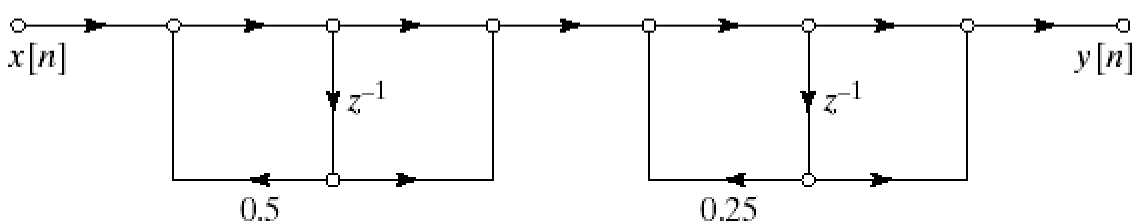
$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}} = \frac{(1 + z^{-1})(1 + z^{-1})}{(1 - 0.5z^{-1})(1 - 0.25z^{-1})}$$

$$= \frac{(1 + z^{-1})}{(1 - 0.5z^{-1})} \frac{(1 + z^{-1})}{(1 - 0.25z^{-1})}$$

- Cascade of Direct Form I subsections



- Cascade of Direct Form II subsections



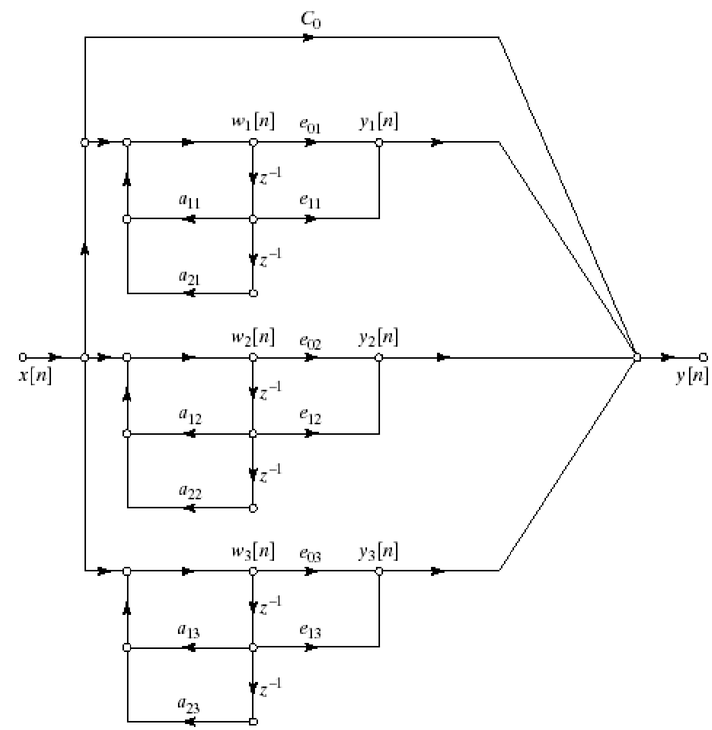
Basic Structures for IIR Systems: Parallel Form

- Represent system function using partial fraction expansion

$$H(z) = \sum_{k=0}^{N_P} C_k z^{-k} + \sum_{k=1}^{N_P} \frac{A_k}{1 - c_k z^{-1}} + \sum_{k=1}^{N_P} \frac{B_k\left(1 - e_k z^{-1}\right)}{\left(1 - d_k z^{-1}\right)\left(1 - d_k^* z^{-1}\right)}$$

- Or by pairing the real poles

$$H(z) = \sum_{k=0}^{N_P} C_k z^{-k} + \sum_{k=1}^{N_S} \frac{e_{0k} + e_{1k} z^{-1}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$



Example

- Partial Fraction Expansion

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}} = 8 + \frac{18}{\left(1 - 0.5z^{-1}\right)} - \frac{25}{\left(1 - 0.25z^{-1}\right)}$$

$$H(z) = 8 + \frac{-7 + 8z^{-1}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

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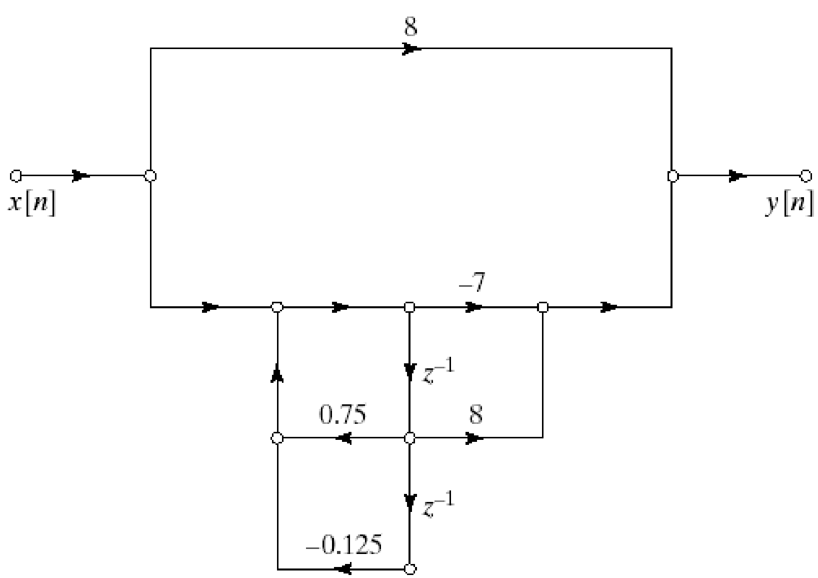
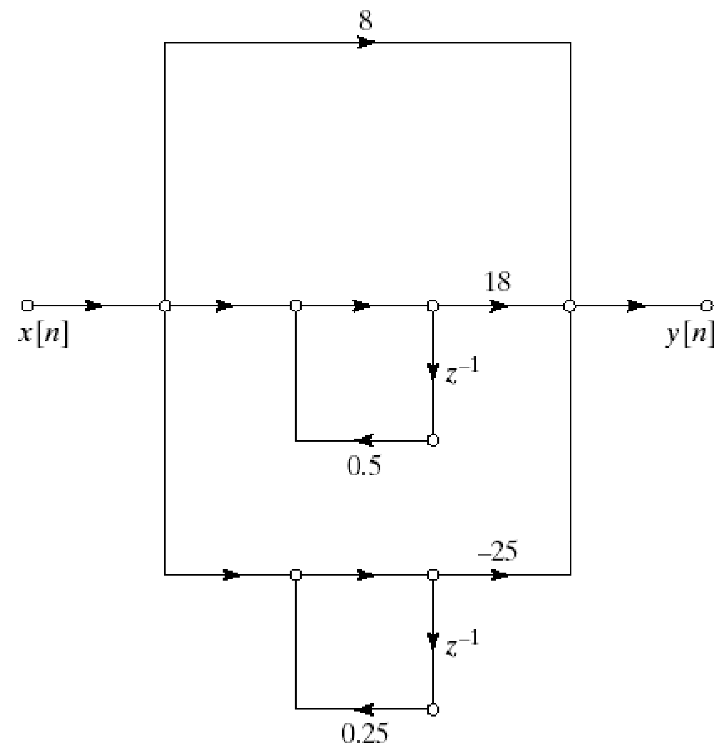
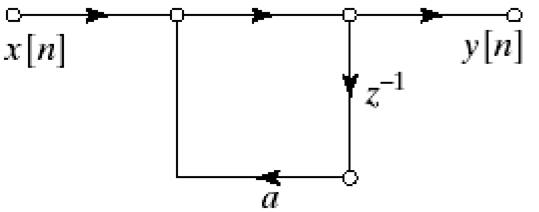




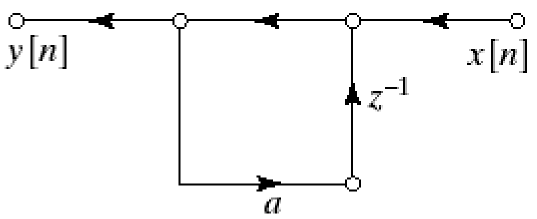
Transposed Forms

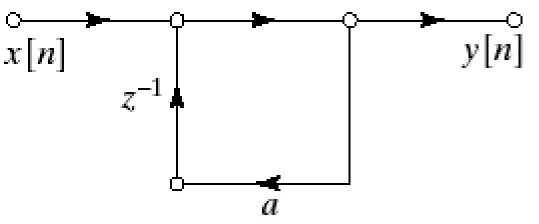
- Linear signal flow graph property:
  - Transposing doesn't change the input-output relation
- Transposing:
  - Reverse directions of all branches
  - Interchange input and output nodes



- Example:

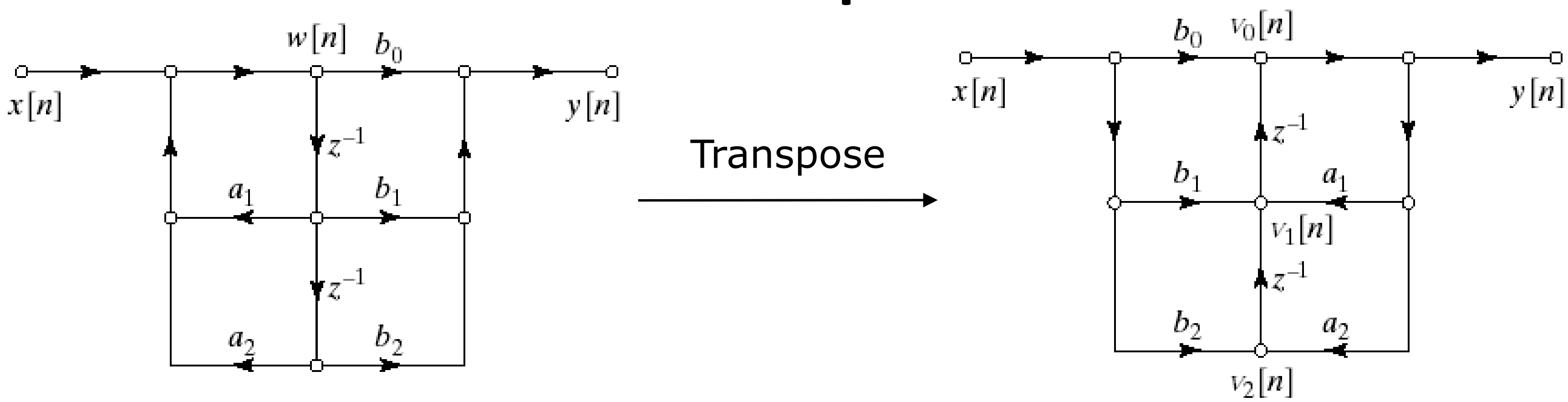
$$H(z) = \frac{1}{1 - az^{-1}}$$





  - Reverse directions of branches and interchange input and output

Example



$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

- Both have the same system function or difference equation