Response of First and Second order Systems

Response of First-Order Systems

- **Initial and final value formulae**
  - increasing or decreasing exponential waveforms (for either voltage or current) are given by:

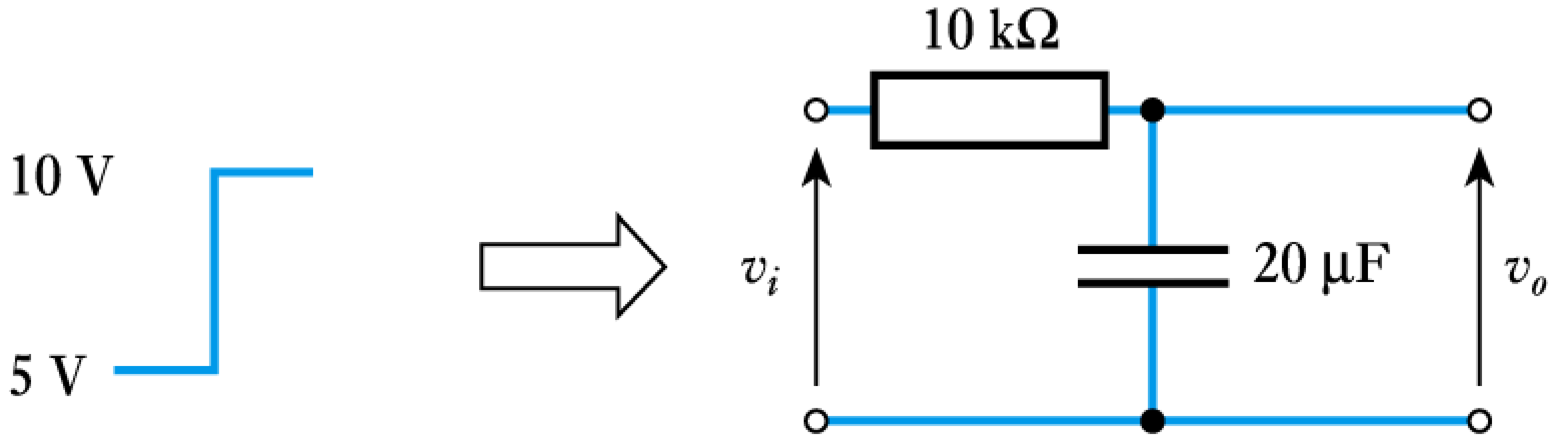
$$v = V_f + (V_i - V_f)e^{-t/T}$$

$$i = I_f + (I_i - I_f)e^{-t/T}$$

  - where V_i and I_i are the *initial* values of the voltage and current
  - where V_f and I_f are the *final* values of the voltage and current
  - the first term in each case is the **steady-state response**
  - the second term represents the **transient response**
  - the combination gives the **total response** of the arrangement

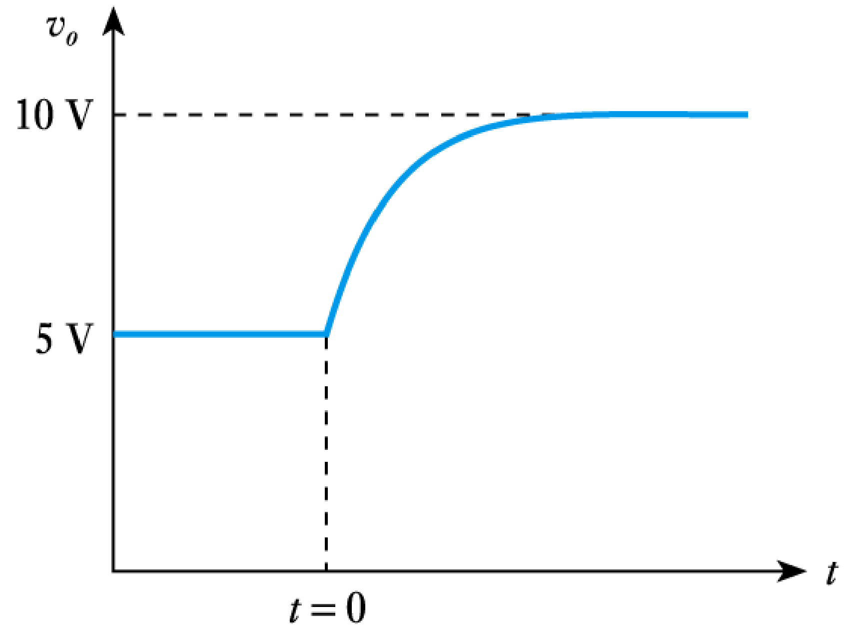
Example – see Example 18.3 from course text

The input voltage to the following *CR* network undergoes a step change from 5 V to 10 V at time t = 0. Derive an expression for the resulting output voltage.

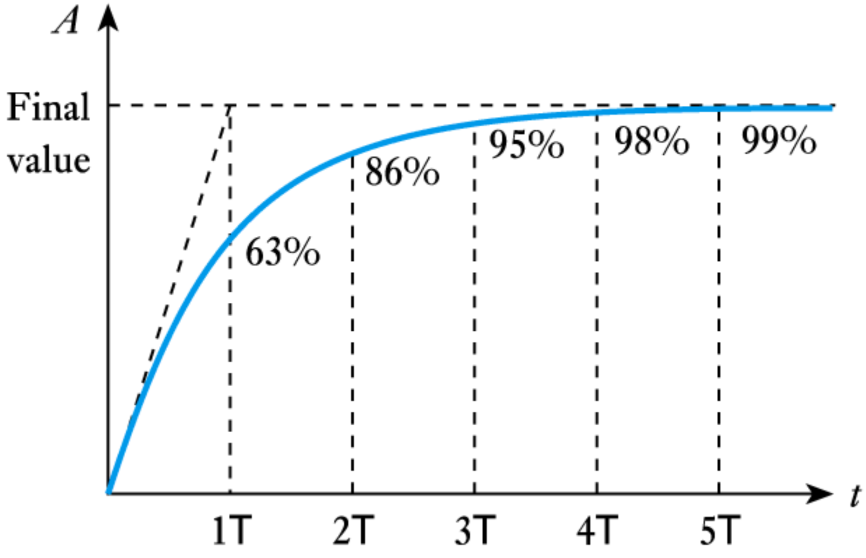


Here the initial value is 5 V and the final value is 10 V. The time constant of the circuit equals $CR = 10 \times 10^3 \times 20 \times 10^{-6} = 0.2$s. Therefore, from above, for $t \geq 0$

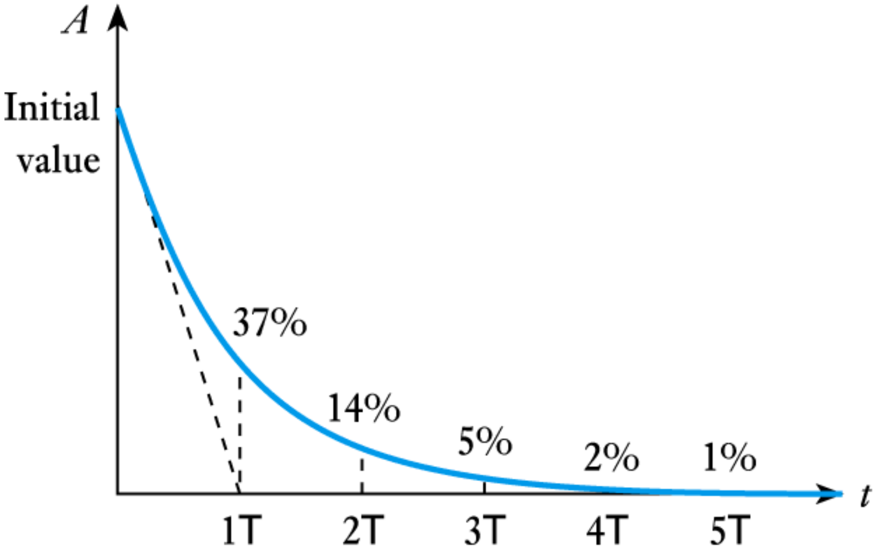
$$\begin{aligned} v &= V_f + (V_i - V_f)e^{-t/T} \\ &= 10 + (5 - 10)e^{-t/0.2} \\ &= 10 - 5e^{-t/0.2} \text{ volts} \end{aligned}$$



The nature of exponential curves



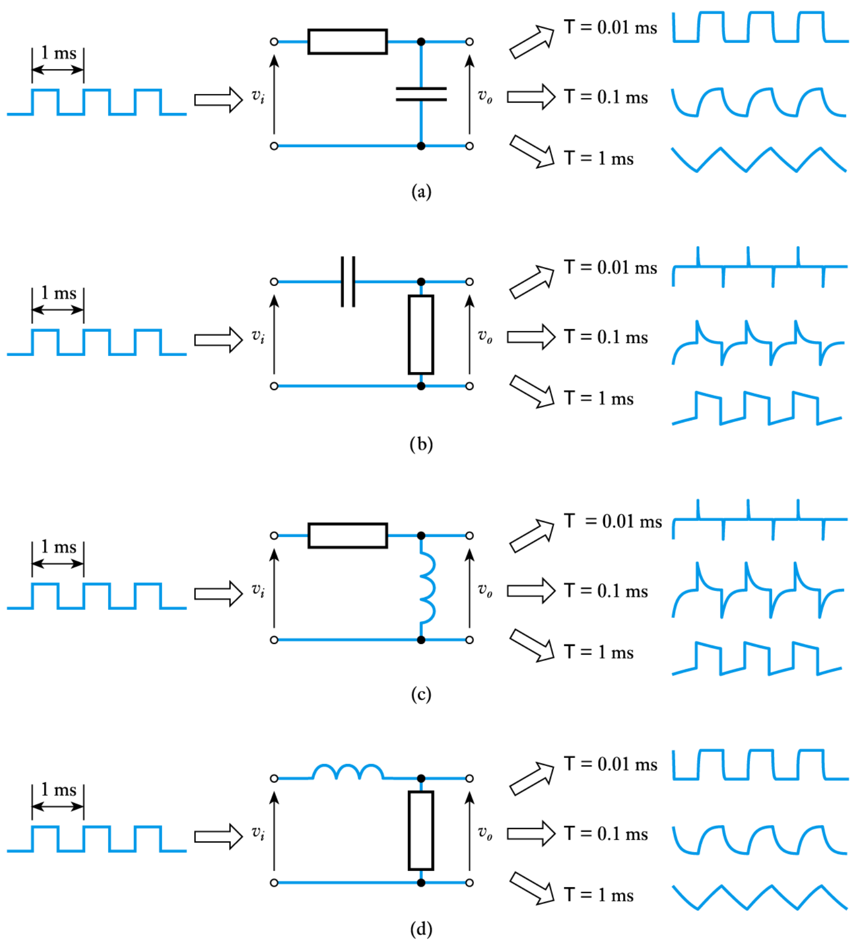
(a) $A(1 - e^{-t/T})$



(b) $A\,e^{-t/T}$

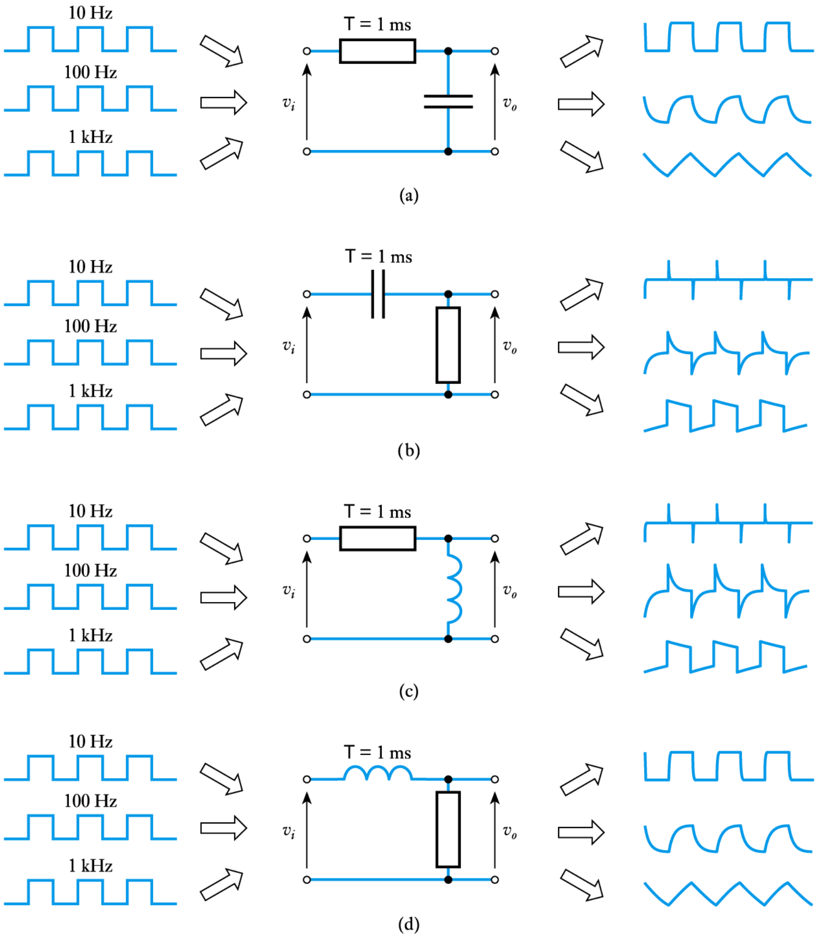
Response of first-order systems to a square waveform

– see **Section 18.4.3**



Response of first-order systems to a square waveform of different frequencies

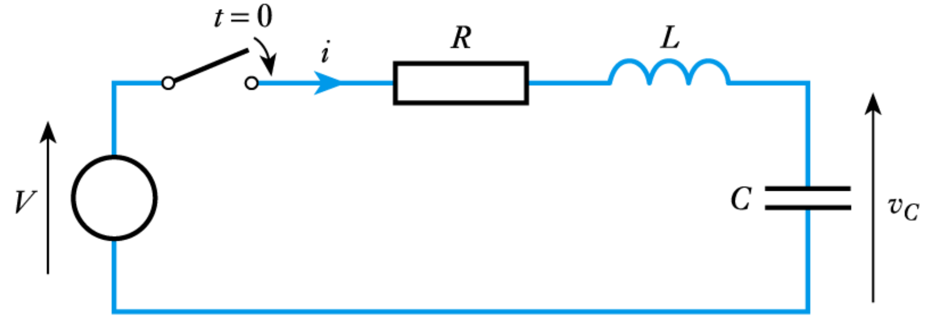
– see **Section 18.4.3**



Second-Order Systems

- Circuits containing both capacitance and inductance are normally described by second-order differential equations. These are termed **second-order systems**
  - for example, this circu

$$LC\frac{d^2v_C}{dt^2}+RC\frac{dv_C}{dt}+v_C=V$$



- When a step input is applied to a second-order system, the form of the resultant transient depends on the relative magnitudes of the coefficients of its differential equation. The general form of the response is

$$\frac{1}{\omega_n{}^2}\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}+\frac{2\zeta}{\omega_n}\frac{\mathrm{d}y}{\mathrm{d}t}+y=x$$

  - where ω_n is the **undamped natural frequency** in rad/s and ζ (Greek Zeta) is the **damping factor**

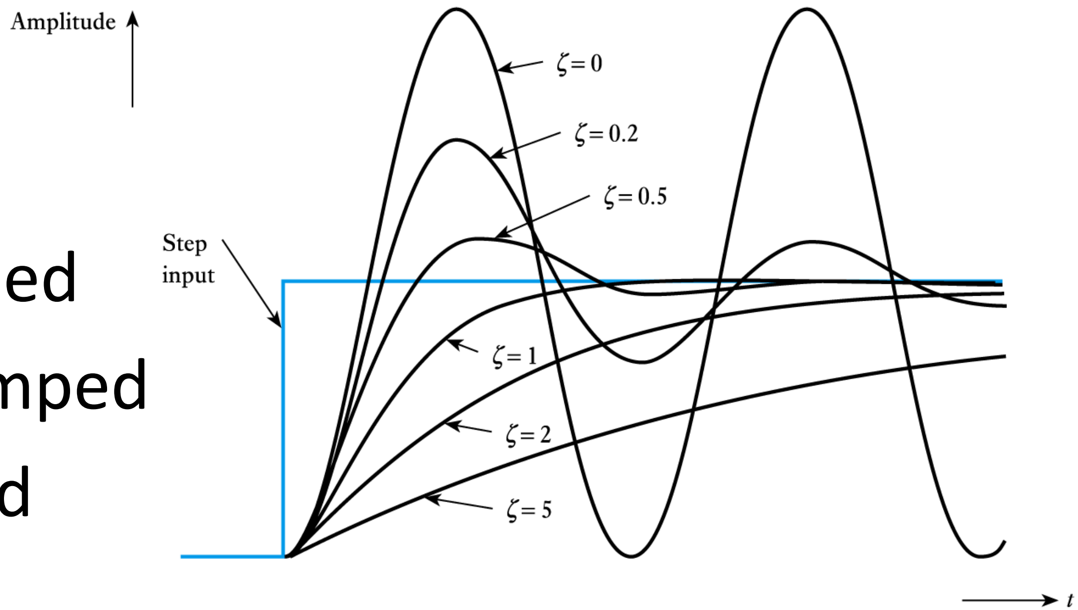
Response of second-order systems

ζ =0 undamped

ζ <1 under damped

ζ =1 critically damped

ζ >1 over damped



Higher-Order Systems

- Higher-order systems are those that are described by third-order or higher-order equations
- These often have a transient response similar to that of the second-order systems described earlier
- Because of the complexity of the mathematics involved, they will not be discussed further here

Key Points

- The charging or discharging of a capacitor, and the energising and de-energising of an inductor, are each associated with exponential voltage and current waveforms
- Circuits that contain resistance, and either capacitance or inductance, are termed first-order systems
- The increasing or decreasing exponential waveforms of first-order systems can be described by the initial and final value formulae
- Circuits that contain both capacitance and inductance are usually second-order systems. These are characterised by their undamped natural frequency and their damping factor