# SECTION-C

# Time and Frequency Domain Analysis of Systems

# Introduction

- So far we have looked at the behaviour of systems in response to:
  - fixed DC signals
  - constant AC signals
- We now turn our attention to the operation of circuits before they reach steady-state conditions
  - this is referred to as the **transient response**
- We will begin by looking at simple *RC* and *RL* circuits

# Charging Capacitors and Energising Inductors

## Capacitor Charging

- Consider the circuit shown he
  - Applying Kirchhoff's voltage lav

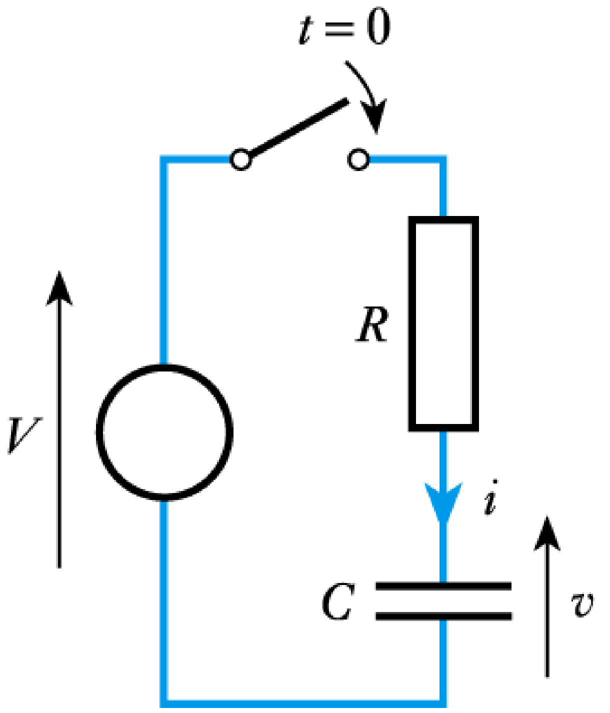
$$iR + v = V$$

  - Now, in a capacitor

$$i = C\frac{dv}{dt}$$

  - which substituting gives

$$CR\frac{dv}{dt} + v = V$$



- The above is a first-order differential equation with constant coefficients
- Assuming $V_C$ = 0 at $t$ = 0, this can be solved to give

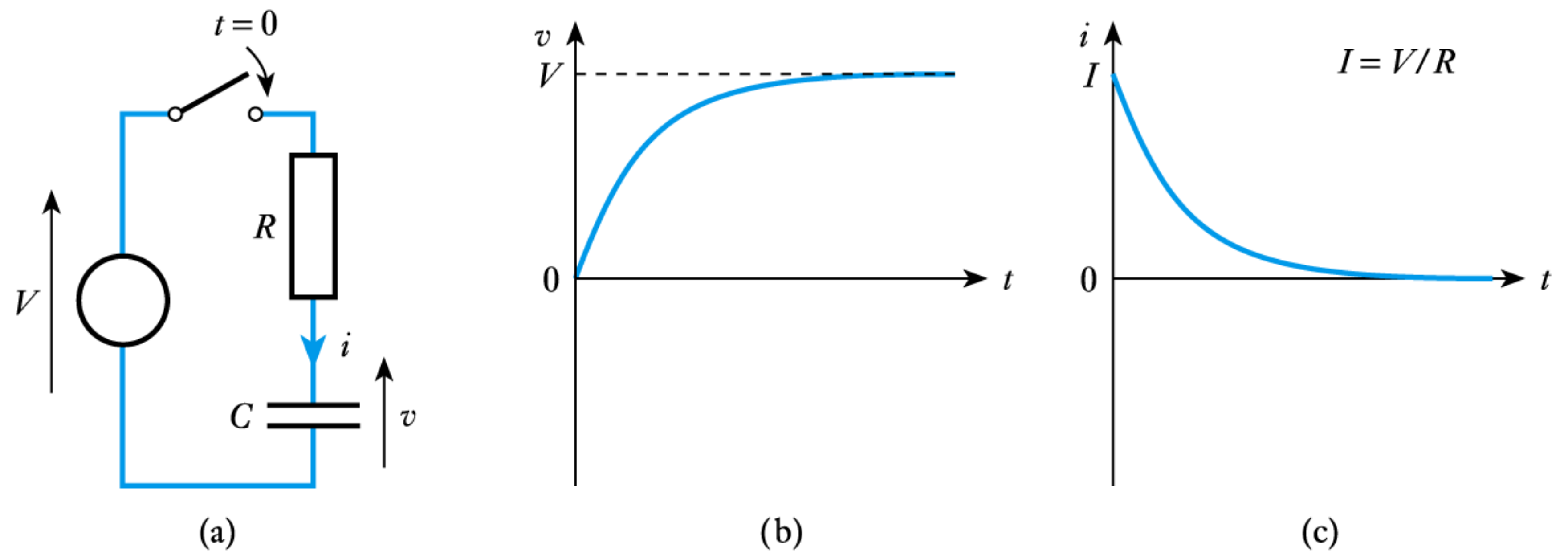
$$v = V(1 - e^{-\frac{t}{CR}}) = V(1 - e^{-\frac{t}{T}})$$

  - see **Section 18.2.1** of the course text for this analysis
- Since $i = C\mathrm{d}v/\mathrm{d}t$ this gives (assuming $V_C$ = 0 at $t$ = 0)

$$i = Ie^{-\frac{t}{CR}} = Ie^{-\frac{t}{T}}$$

  - where $I = V/R$

- Thus both the voltage and current have an exponential form

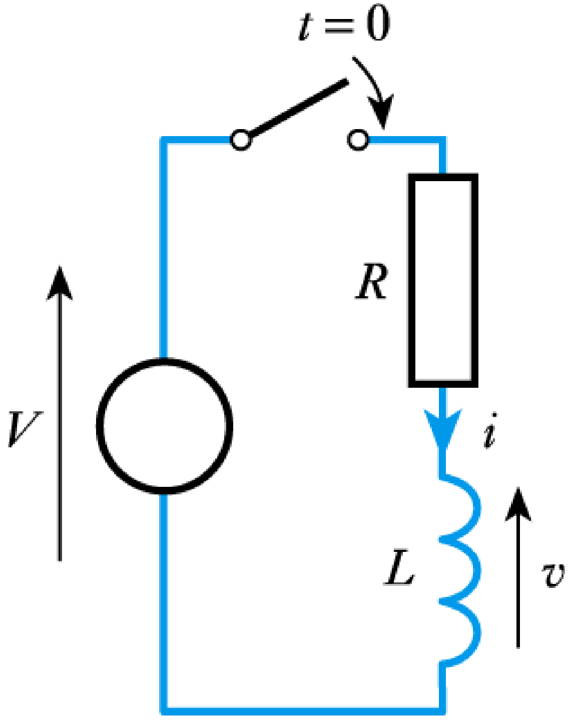


# Inductor energising

- A similar analysis of this circuit gives

$$v = Ve^{-\frac{Rt}{L}} = Ve^{-\frac{t}{T}}$$

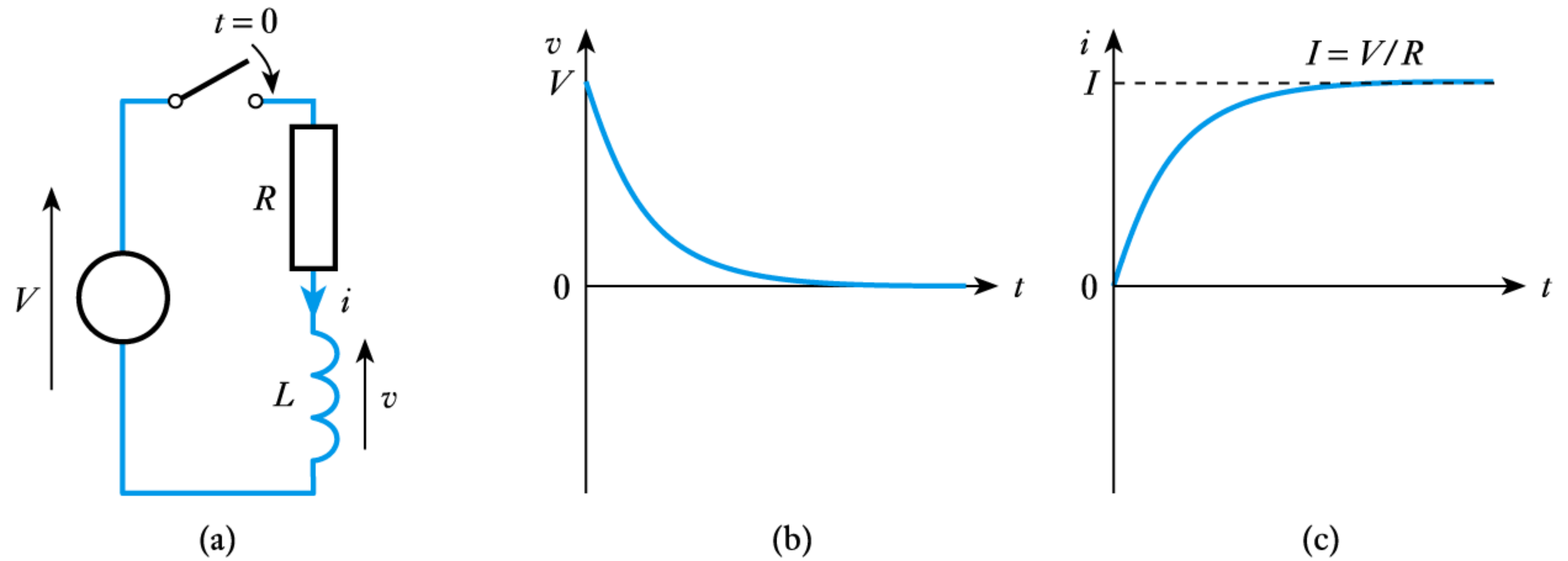
$$i = I(1 - e^{-\frac{Rt}{L}}) I(1 - e^{-\frac{t}{T}})$$




where *I* = *V/R*

– see **Section 18.2.2** for this analysis

- Thus, again, both the voltage and current have an exponential form



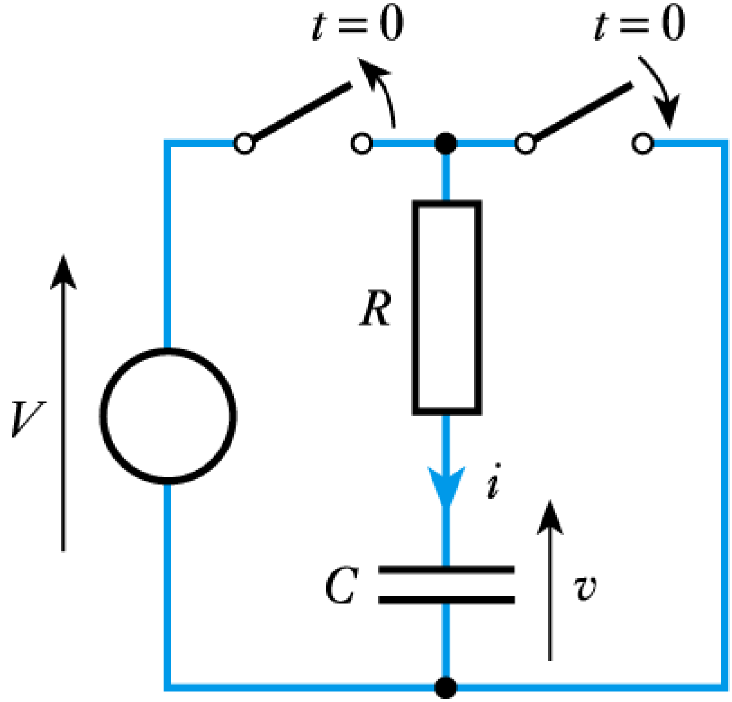
# Discharging Capacitors and De-energising Inductors

## Capacitor discharging

- Consider this circuit for discharging a capacitor
  - At $t = 0$, $V_C = V$
  - From Kirchhoff's voltage law

$$iR + v = 0$$

  - giving $$CR\frac{\mathrm{d}v}{\mathrm{d}t} + v = 0$$



- Solving this as before gives

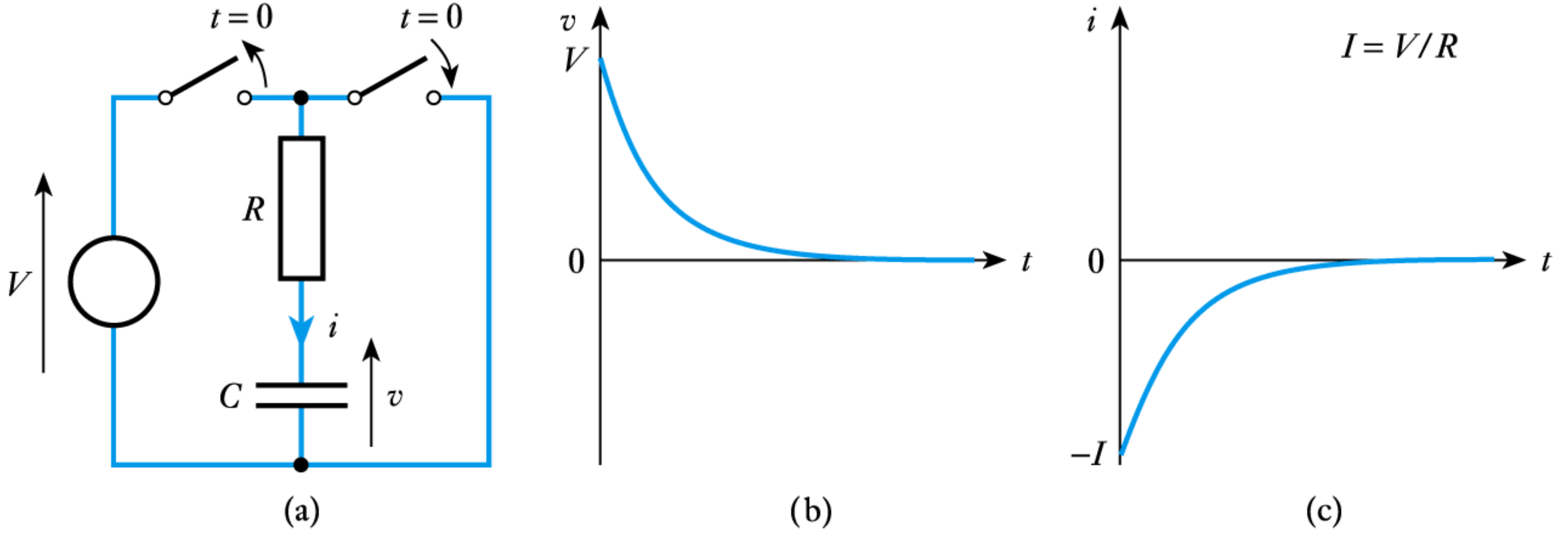
$$v = Ve^{-\frac{t}{CR}} = Ve^{-\frac{t}{T}}$$

$$i = -Ie^{-\frac{t}{CR}} = -Ie^{-\frac{t}{T}}$$

  – where $I = V/R$

  – see **Section 18.3.1** for this analysis

- In this case, both the voltage and the current take the form of decaying exponentials



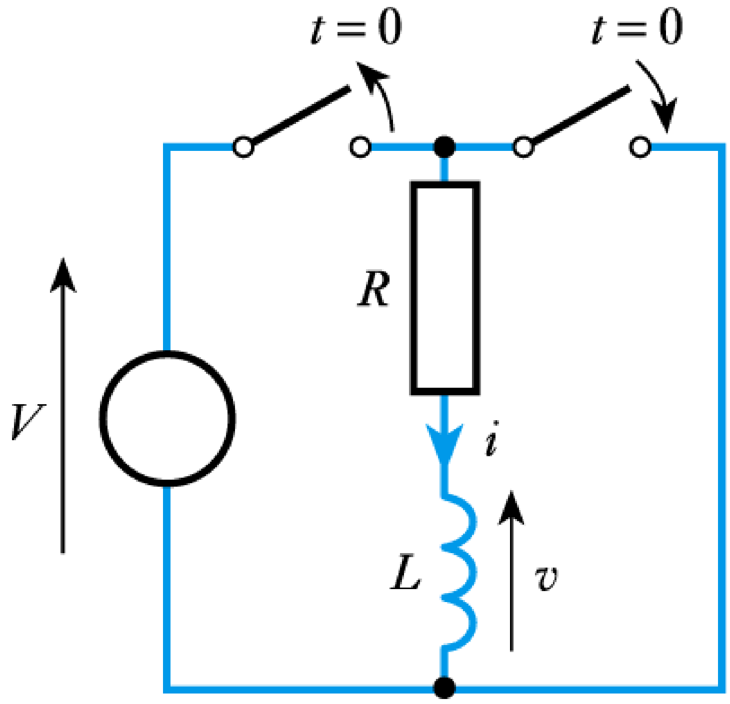
# Inductor de-energising

- A similar analysis of this circuit gives

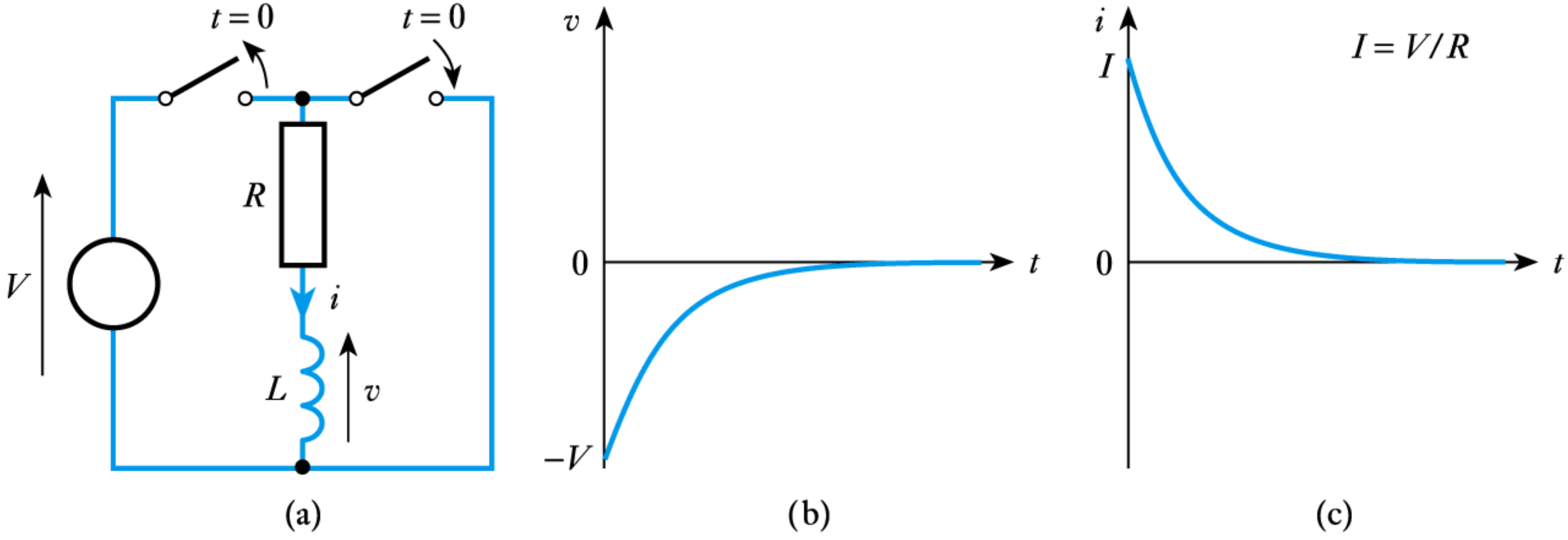
$$v = -Ve^{-\frac{Rt}{L}} = -Ve^{-\frac{t}{T}}$$

$$i = Ie^{-\frac{Rt}{L}} = Ie^{-\frac{t}{T}}$$

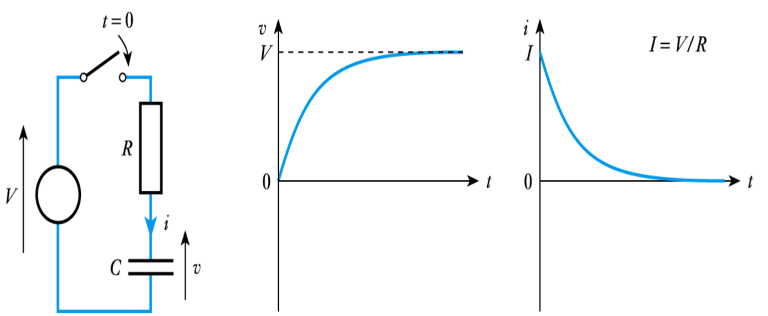
  - where $I = V/R$
  - see **Section 18.3.1** for this analysis



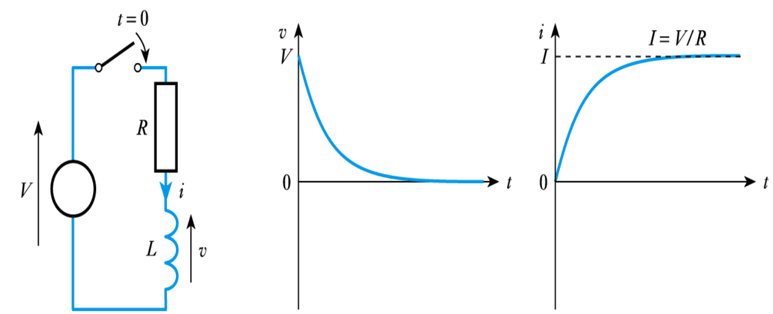
- And once again, both the voltage and the current take the form of decaying exponentials



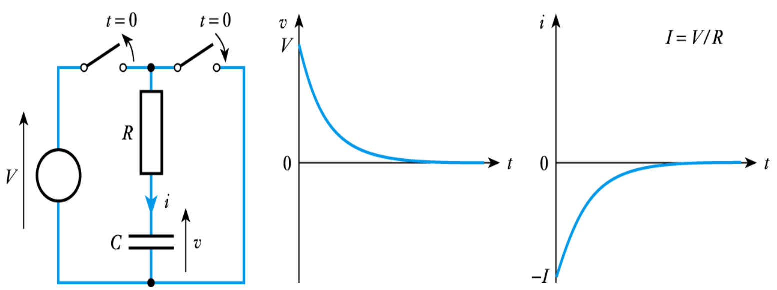
# A comparison of the four circuits



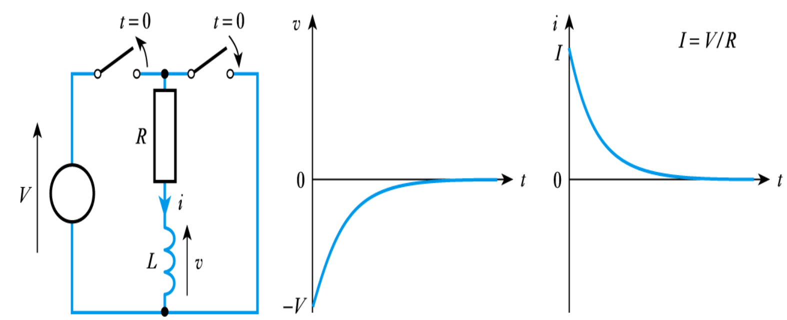

(a) (b) (c)




(a) (b) (c)




(a) (b) (c)




(a) (b) (c)