Relation Between FT and LT

Fourier vs. Laplace

If $h(t)$ is causal, then $h(t) = 0$ for $t < 0$, and in addition:

$$\int_0^\infty |h(t)| < \infty,$$

then the Fourier Transform of $h(t)$ exists and is equal to:

$$\mathcal{F}\{h(t)\} = H(\omega) = \int_0^\infty h(t) e^{-j\omega t} dt.$$

A comparison with the Laplace Transform is then:

$$\mathcal{F}\{h(t)\} = \mathcal{L}\{h(t)\}|_{s=j\omega}.$$

RELATIONSHIP BETWEEN LAPLACE AND FOURIER TRANSFORM

- The Fourier transform can be obtained from the Laplace transform by making the substitution $s = jw = j2\pi f$.

- A transfer function is

$$H(s) = \frac{1}{\left(s^2 + s + (2\pi 100)^2\right)}$$

By making the substitution $s = jw = j2\pi f$, the Fourier transform of the transfer function is

$$H(f) = \frac{1}{\left(-\left(2\pi f\right)^2 + j2\pi f + \left(2\pi 100\right)^2\right)} = \frac{1}{\left(\left(\left(2\pi 100\right)^2 - \left(2\pi f\right)^2\right) + j2\pi f\right)}$$

RELATIONSHIP BETWEEN LAPLACE AND FOURIER TRANSFORM

- The Fourier transform when defined in terms of the magnitude and phase is

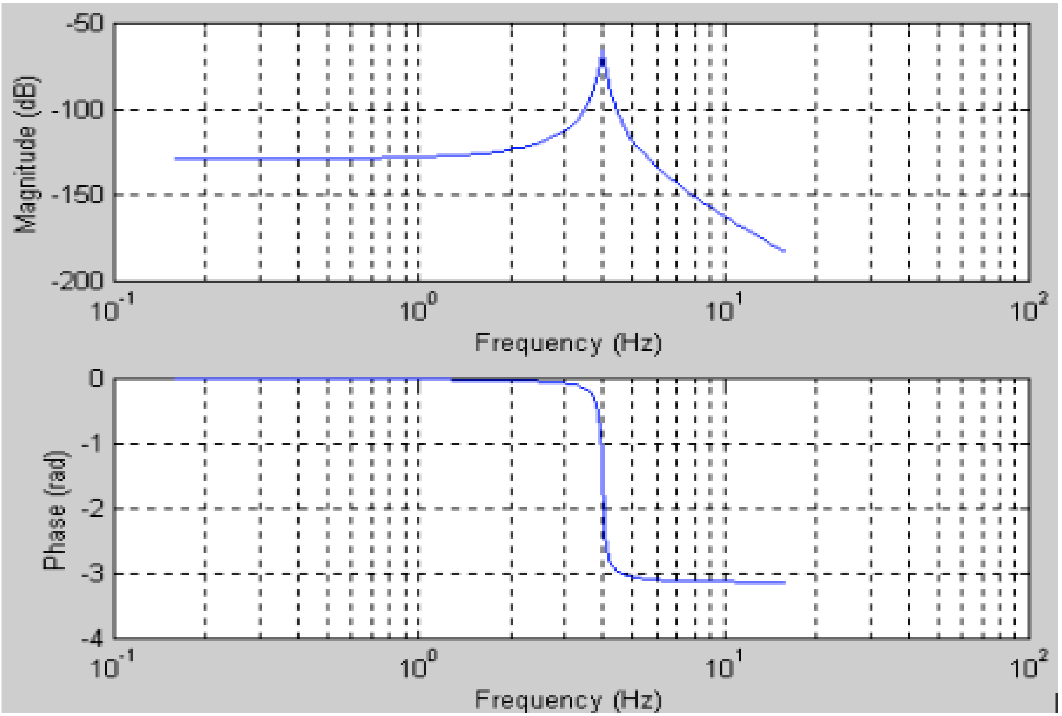
$$|H(f)| = \frac{1}{\sqrt{\left((2\pi 100)^2 - (2\pi f)^2\right)^2 + (2\pi f)^2}}$$

Magnitude

$$\phi(f) = -\tan^{-1}\left(\frac{2\pi f}{(2\pi 100)^2 - (2\pi f)^2}\right)$$

Phase

FREQUENCY RESPONSE OF TRANSFER FUNCTION



Magnitude and phase plot of transfer function