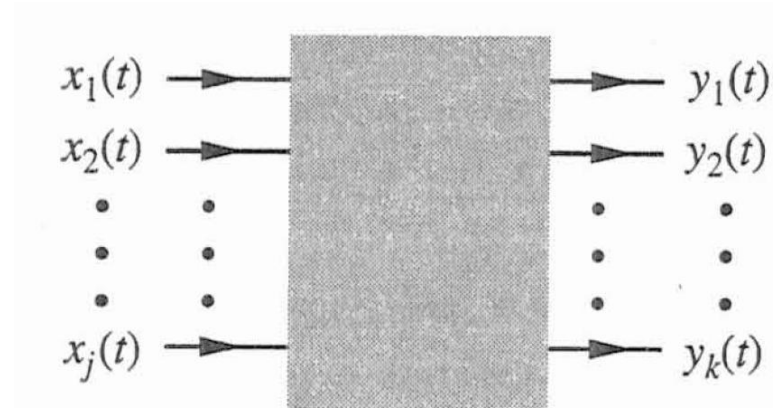


Systems

What are Systems?

- Systems are used to **process signals to modify or extract information**
- Physical system – characterized by their **input-output relationships**
- E.g. electrical systems are characterized by voltage-current relationships
- From this, we derive a **mathematical model of the system**
- **“Black box” model of a system:**



Classification of Systems

Systems may be classified into:

1. Linear and non-linear systems
2. Constant parameter and time-varying-parameter systems
3. Instantaneous (memoryless) and dynamic (with memory) systems
4. Causal and non-causal systems
5. Continuous-time and discrete-time systems
6. Analogue and digital systems
7. Invertible and noninvertible systems
8. Stable and unstable systems

Linear Systems (1)

- A linear system exhibits the **additivity property**:

$$\text{if } x_1 \longrightarrow y_1 \text{ and } x_2 \longrightarrow y_2 \text{ then } x_1 + x_2 \longrightarrow y_1 + y_2$$

- It also must satisfy the **homogeneity or scaling property**:

$$\text{if } x \longrightarrow y \text{ then } kx \longrightarrow ky$$

- These can be combined into the property of **superposition**:

$$\text{if } x_1 \longrightarrow y_1 \text{ and } x_2 \longrightarrow y_2 \text{ then } k_1x_1 + k_2x_2 \longrightarrow k_1y_1 + k_2y_2$$

- A non-linear system is one that is NOT linear (i.e. does not obey the principle of superposition)

Linear Systems (2)

- ◆ Consider the following simple RC circuit:

- ◆ Output $y(t)$ relates to $x(t)$ by:

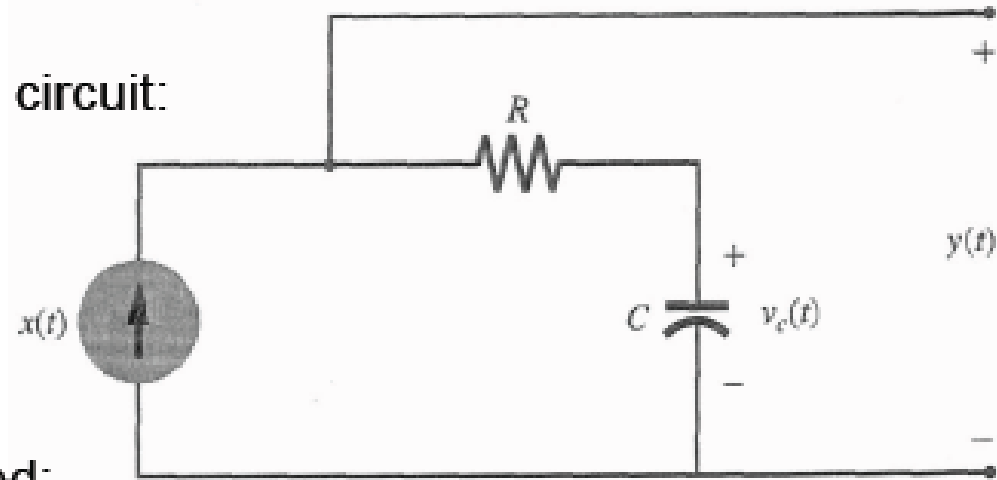
$$y(t) = Rx(t) + \frac{1}{C} \int_{-\infty}^t x(\tau) d\tau$$

- ◆ The second term can be expanded:

$$y(t) = Rx(t) + \frac{1}{C} \int_{-\infty}^0 x(\tau) d\tau + \frac{1}{C} \int_0^t x(\tau) d\tau$$

$$y(t) = v_C(0) + Rx(t) + \frac{1}{C} \int_0^t x(\tau) d\tau \quad t \geq 0$$

- ◆ This is a **single-input, single-output** (SISO) system. In general, a system can be multiple-input, multiple-output (**MIMO**).

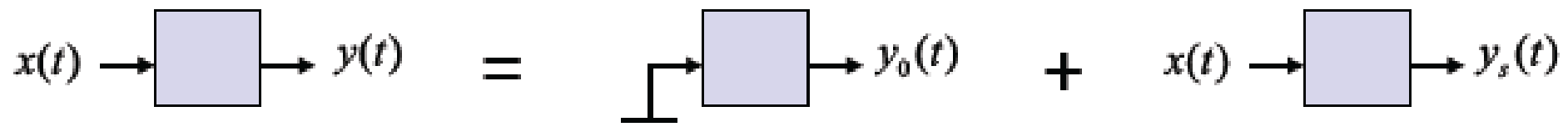


Linear Systems (3)

- ◆ A system's output for $t \geq 0$ is result of 2 independent causes:
 1. Initial conditions when $t = 0$ (**zero-input response**)
 2. Input $x(t)$ for $t \geq 0$ (**zero-state response**)
- ◆ Decomposition property:

Total response = zero-input response + zero-state response

$$y(t) = \underbrace{v_C(0)}_{\text{zero-input response}} + \underbrace{Rx(t) + \frac{1}{C} \int_0^t x(\tau) d\tau}_{\text{zero-state response}} \quad t \geq 0$$



Linear Systems (4)

- ◆ Show that the system described by the equation $\frac{dy}{dt} + 3y(t) = x(t)$ is linear.
- ◆ Let $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$, then

$$\frac{dy_1}{dt} + 3y_1(t) = x_1(t) \quad \text{and} \quad \frac{dy_2}{dt} + 3y_2(t) = x_2(t)$$

- ◆ Multiple 1st equation by k_1 , and 2nd equation by k_2 , and adding them yields:

$$\frac{d}{dt}[k_1y_1(t) + k_2y_2(t)] + 3[k_1y_1(t) + k_2y_2(t)] = k_1x_1(t) + k_2x_2(t)$$

- ◆ This equation is the system equation with

$$x(t) = k_1x_1(t) + k_2x_2(t)$$

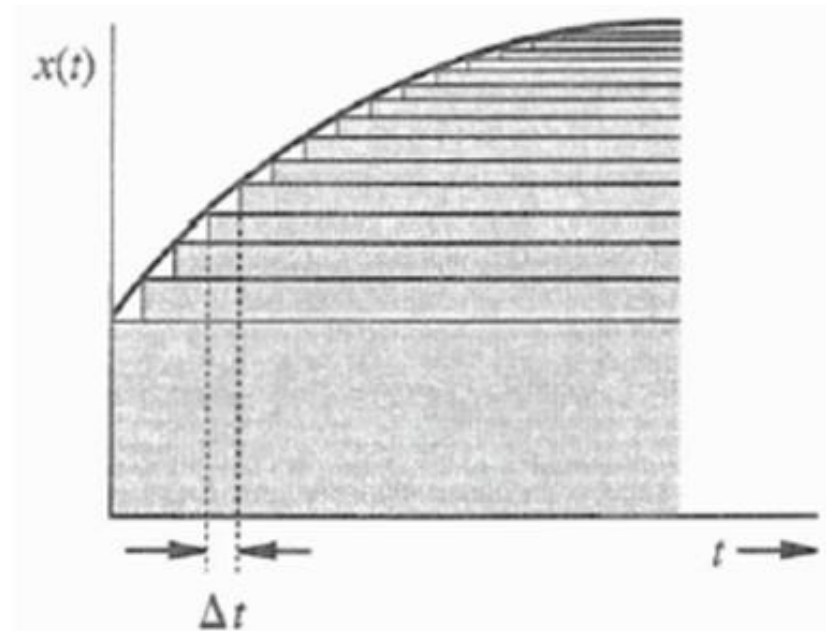
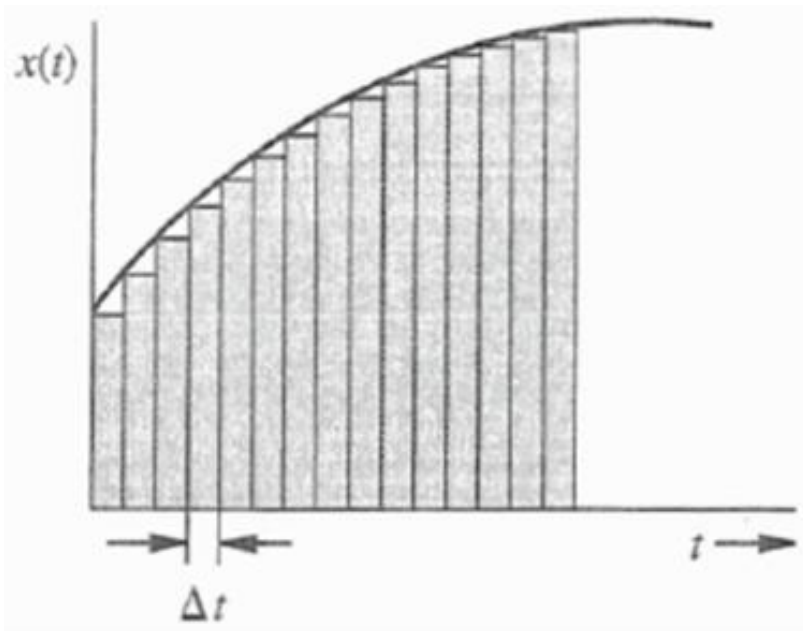
and

$$y(t) = k_1y_1(t) + k_2y_2(t)$$

Is the system $y = x^2$ linear?

Linear Systems (5)

A complex input can be represented as a sum of simpler inputs (pulse, step, sinusoidal), and then use linearity to find the response to this simple inputs to find the system output to the complex input.

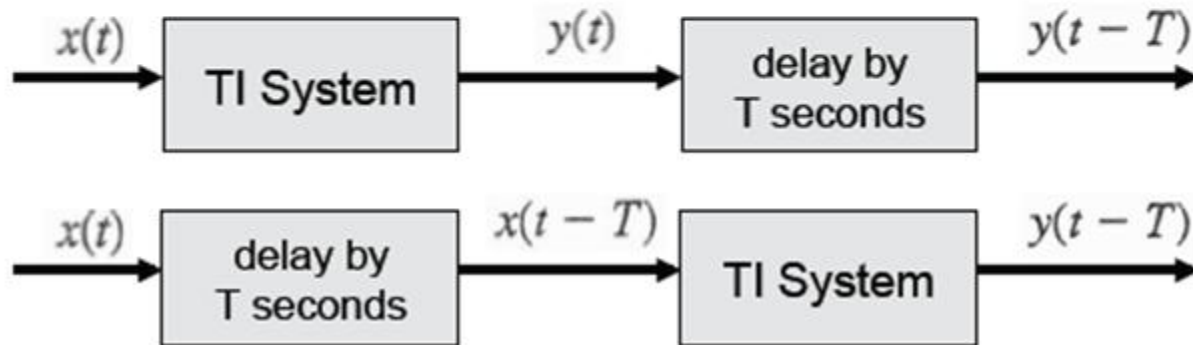
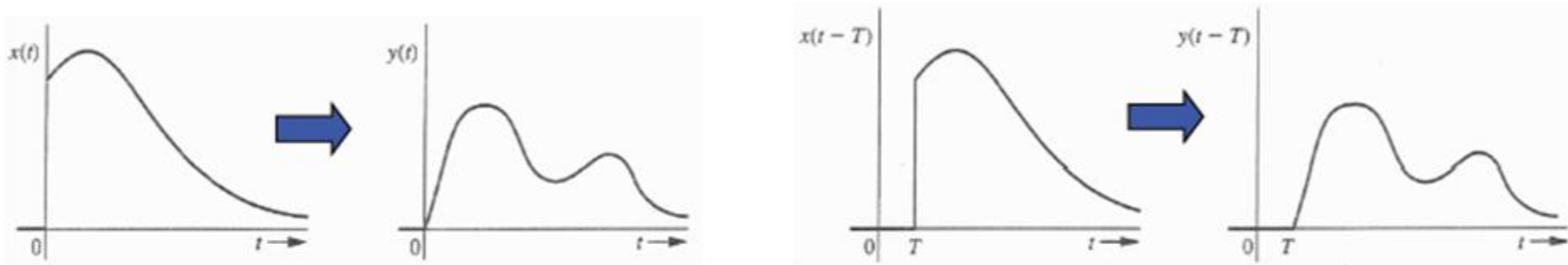


$$x(t) = a_1x_1(t) + a_2x_2(t) + \cdots + a_mx_m(t)$$

$$y(t) = a_1y_1(t) + a_2y_2(t) + \cdots + a_my_m(t)$$

Time-Invariant System

- ◆ **Time-invariant system** is one whose parameters do not change with time:



Which of the system is time-invariant?

- (a) $y(t) = 3x(t)$ (b) $y(t) = t x(t)$

Instantaneous and Dynamic Systems

- ◆ In general, a system's output at time t **depends** on the entire **past input**. Such a system is a **dynamic** (with memory) **system**
 - Analogous to a state machine in a digital system
- ◆ If the system's **past history is irrelevant** in determining the response, it is an **instantaneous** or **memoryless** systems
 - Analogous to a combinatorial circuit in a digital system

Causal and Noncausal Systems

- ◆ **Causal** system – output at t_0 depends only on $x(t)$ for $t \leq t_0$
- ◆ I.e. present output depends only on the past and present inputs, **not on future inputs**
- ◆ Any practical **REAL TIME system must be causal.**
- ◆ **Noncausal** systems are important because:
 1. Realizable when the independent variable is something other than “time” (e.g. space)
 2. Even for temporal systems, can prerecord the data (non-real time), mimic a non-causal system
 3. Study upper bound on the performance of a causal system

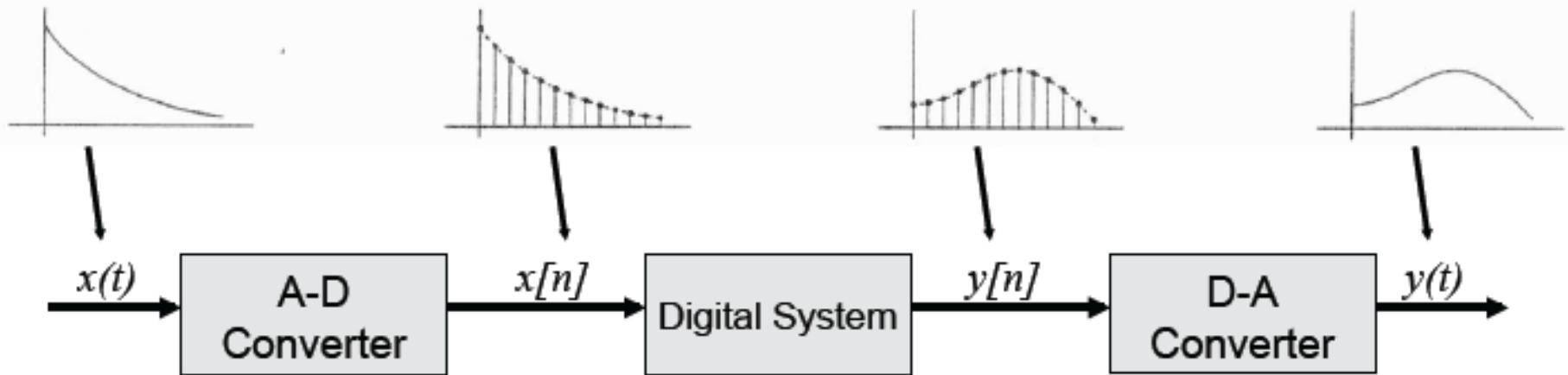
Which of the two systems is causal?

a) $y(t) = 3x(t) + x(t-2)$

b) $y(t) = 3x(t) + x(t+2)$

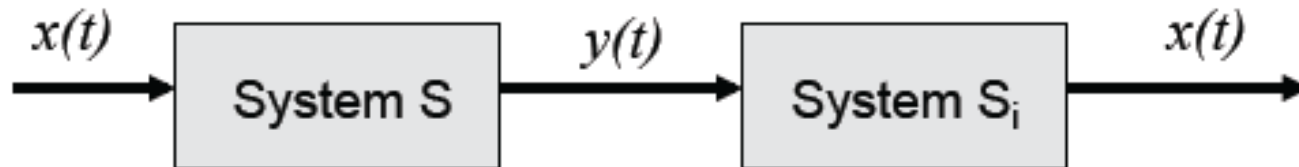
Analogue and Digital Systems

- ◆ Previously the samples are discrete in time, but are continuous in amplitude
- ◆ Most modern systems are DIGITAL DISCRETE-TIME systems, e.g. internal circuits of the MP3 player



Invertible and Noninvertible

- ◆ Let a system S produces $y(t)$ with input $x(t)$, if there exists another system S_i , which produces $x(t)$ from $y(t)$, then S is invertible
- ◆ Essential that there is **one-to-one mapping** between input and output
- ◆ For example if S is an amplifier with gain G , it is invertible and S_i is an attenuator with gain $1/G$
- ◆ Apply S_i following S gives an **identity system** (i.e. input $x(t)$ is not changed)



Which of the two systems is invertible?

a) $y(t) = x^2$

b) $y = 2x$