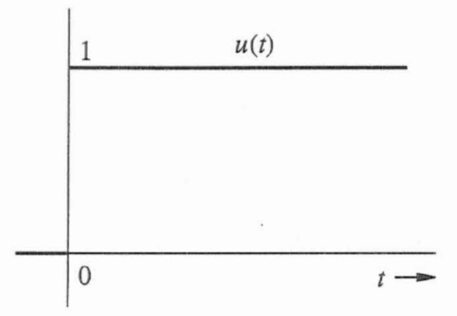
Signal Models

Signal Models – Unit Step Function u(t)

Step function defined by:

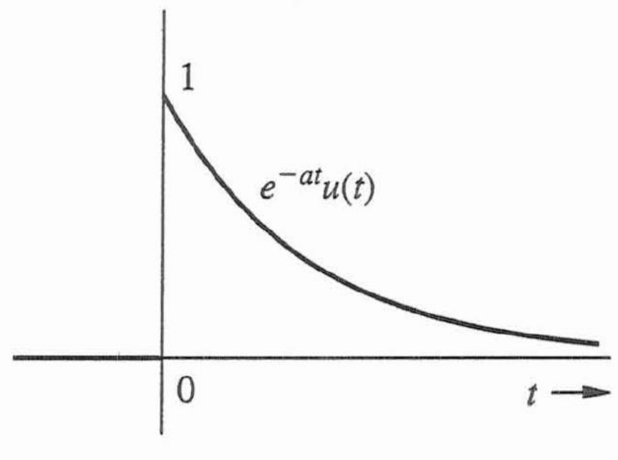
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



Useful to describe a signal that begins at t = 0 (i.e. causal signal).

For example, the signal e^{-at} represents an everlasting exponential that starts at t = -∞.

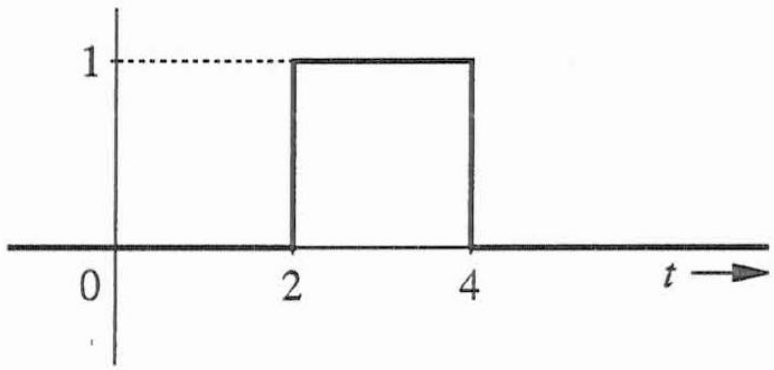
The causal for of this exponential $e^{-at}u(t)$

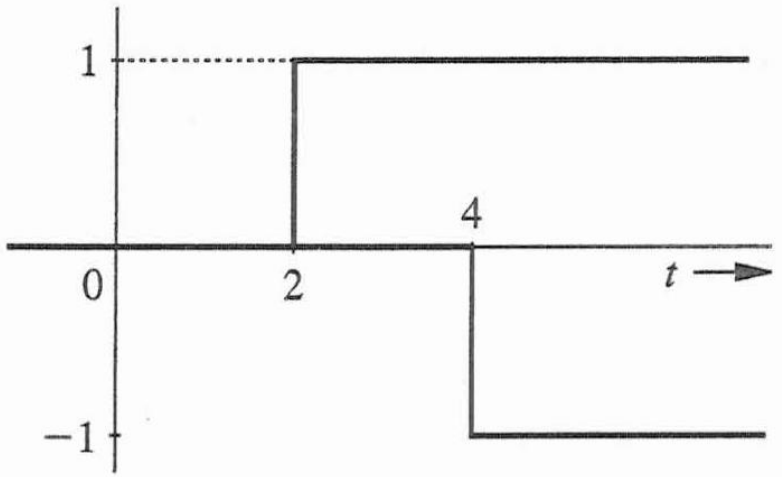


Signal Models – Pulse Signal

A pulse signal can be presented by two step functions:

$x(t) = u(t\text{-}2) - u(t\text{-}4)$



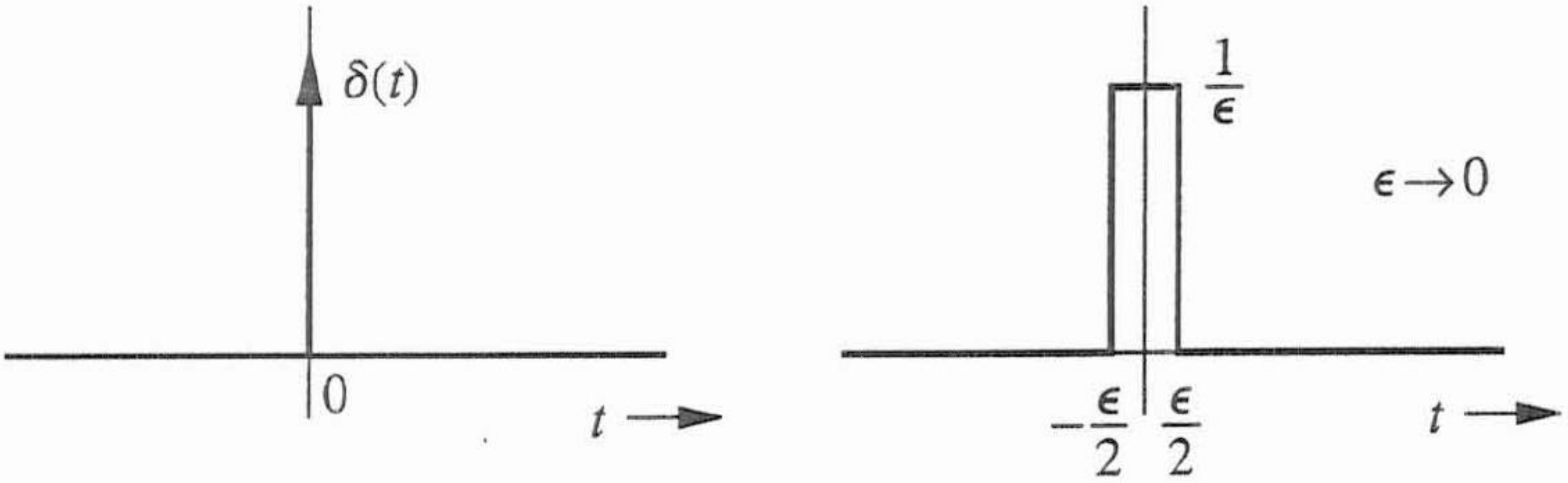


Signal Models – Unit Impulse Function δ(t)

First defined by Dirac as:

$$\delta(t) = 0 \qquad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



Multiplying Function $\phi(t)$ by an Impulse

Since impulse is non-zero only at t = 0, and ϕ(t) at t = 0 is ϕ(0), we get:

$$\phi(t)\delta(t) = \phi(0)\delta(t)$$

We can generalize this for t = T:

$$\phi(t)\delta(t-T) = \phi(T)\delta(t-T)$$

Sampling Property of Unit Impulse Function

Since we have: $\phi(t)\delta(t) = \phi(0)\delta(t)$

It follows that:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt = \phi(0)\int_{-\infty}^{\infty} \delta(t)\,dt = \phi(0)$$

This is the same as “sampling” ϕ(t) at t = 0.

If we want to sample ϕ(t) at t = T, we just multiple ϕ(t) with $\delta(t-T)$

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-T)\,dt = \phi(T)$$

This is called th rty” of the impulse.

Examples

Simplify the following expression

$$\left(\frac{1}{j\omega+2}\right)\delta(\omega+3)$$

Evaluate the following

$$\int_{-\infty}^{\infty}\delta(t+3)e^{-t}dt$$

Find dx/dt for the following signal

$$x(t) = u(t\text{-}2) - 3u(t\text{-}4)$$

The Exponential Function e^{st}

This exponential function is very important in signals & systems, and the parameter *s is a complex variable given by:*

$$s = \sigma + j\omega$$

Therefore

$$e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} e^{j\omega t} = e^{\sigma t}(\cos \omega t + j \sin \omega t)$$

Since $s^* = \sigma - j\omega$ (the conjugate of s), then

$$e^{s^* t} = e^{\sigma - j\omega} = e^{\sigma t} e^{-j\omega t} = e^{\sigma t}(\cos \omega t - j \sin \omega t)$$

and

$$e^{\sigma t} \cos \omega t = \tfrac{1}{2}(e^{st} + e^{s^* t})$$

The Exponential Function e^{st}

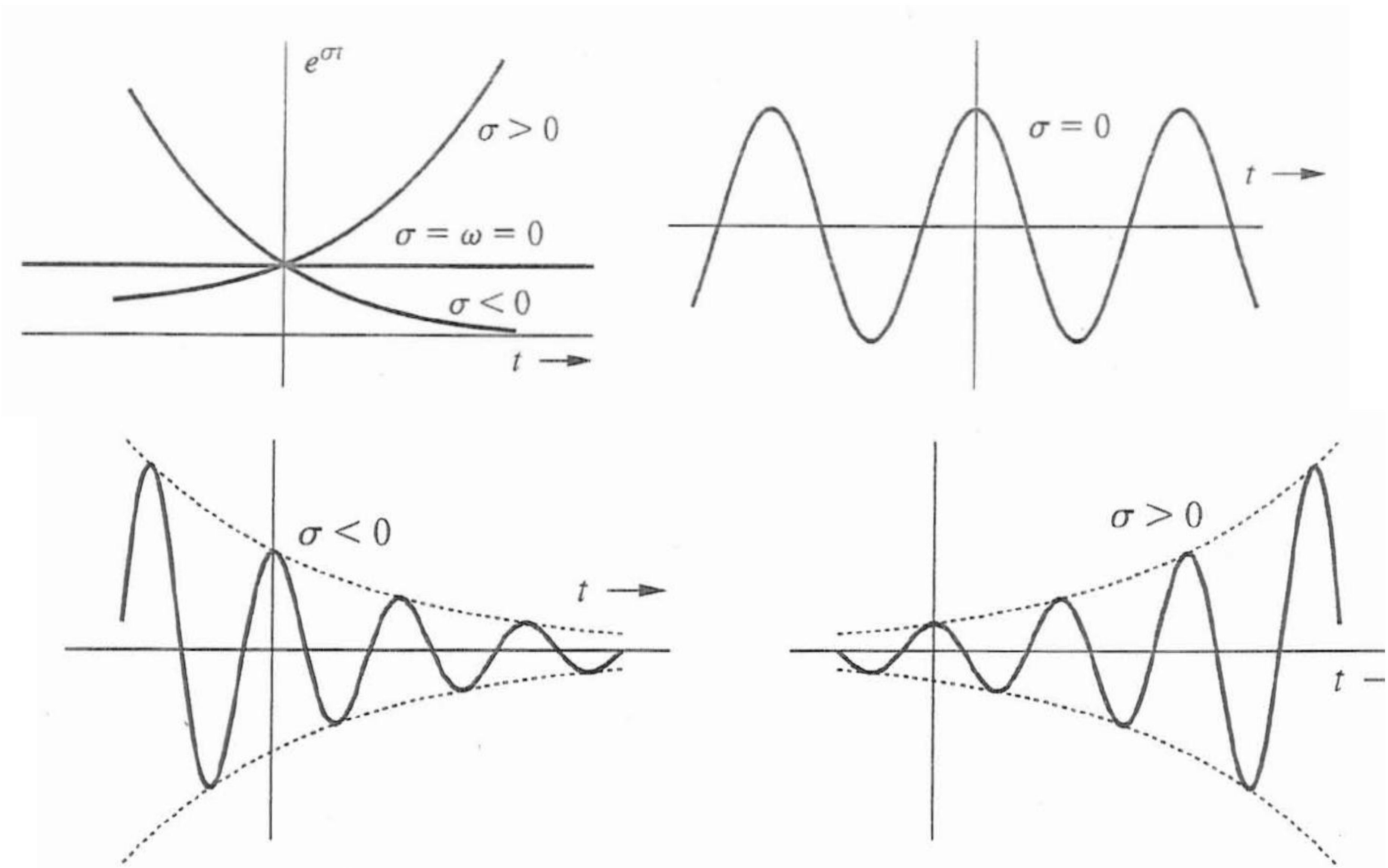
If σ = 0, then we have the function $e^{j\omega t}$, which has a real frequency of ω

Therefore the complex variable *s = σ +jω is the* ***complex frequency***

The function e^{st} *can be used to describe a very large class of* signals and functions. Here are a number of example:

1. A constant $k = ke^{0t}$ $(s = 0)$
2. A monotonic exponential $e^{\sigma t}$ $(\omega = 0,\ s = \sigma)$
3. A sinusoid $\cos \omega t$ $(\sigma = 0,\ s = \pm j\omega)$
4. An exponentially varying sinusoid $e^{\sigma t} \cos \omega t$ $(s = \sigma \pm j\omega)$

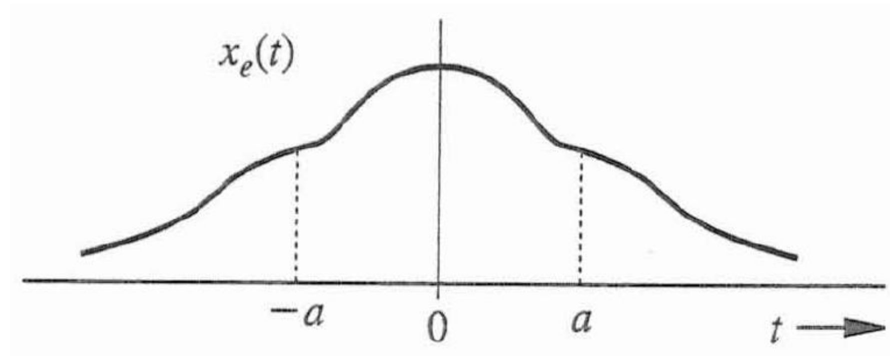
The Exponential Function e^{st}



The Complex Frequency Plane s=σ + jω

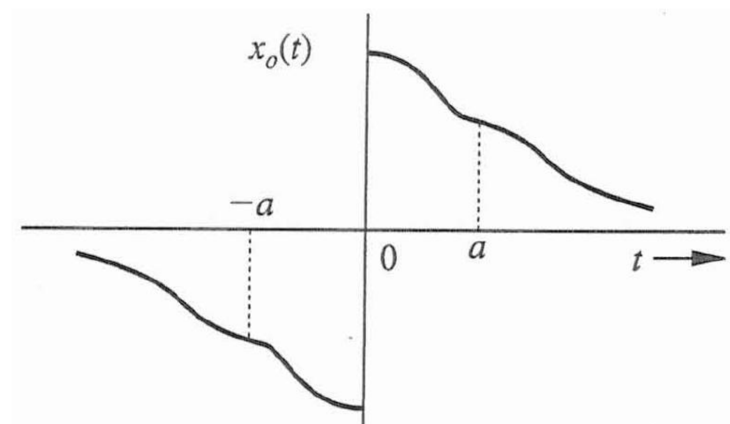
A real function $x_e(t)$ *is said to be an even function of t if*

$$x_e(t) = x_e(-t)$$



A real function $x_o(t)$ *is said to be an odd function of t if*

$$x_o(t) = -x_o(-t)$$



HW1_Ch1: 1.1-3, 1.1-4, 1.2-2(a,b,d), 1.2-5, 1.4-3, 1.4-4, 1.4-5, 1.4-10 (b, f)

Even and Odd Function

Even and odd functions have the following properties:

- Even x Odd = Odd
- Odd x Odd = Even
- Even x Even = Even

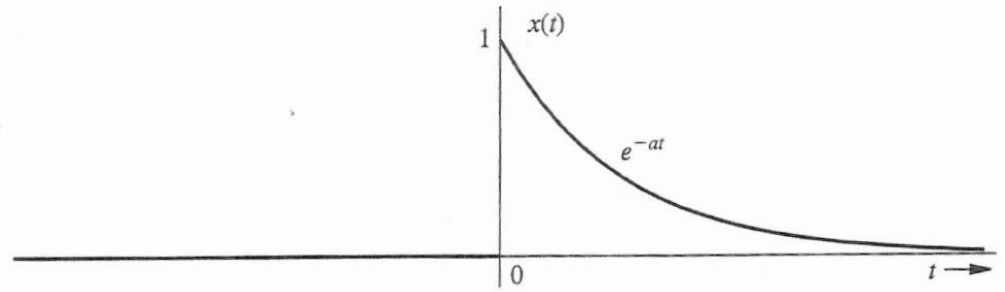
Every signal *x(t) can be expressed as a sum of even and* odd components because:

$$x(t) = \underbrace{\tfrac{1}{2}[x(t) + x(-t)]}_{\text{even}} + \underbrace{\tfrac{1}{2}[x(t) - x(-t)]}_{\text{odd}}$$

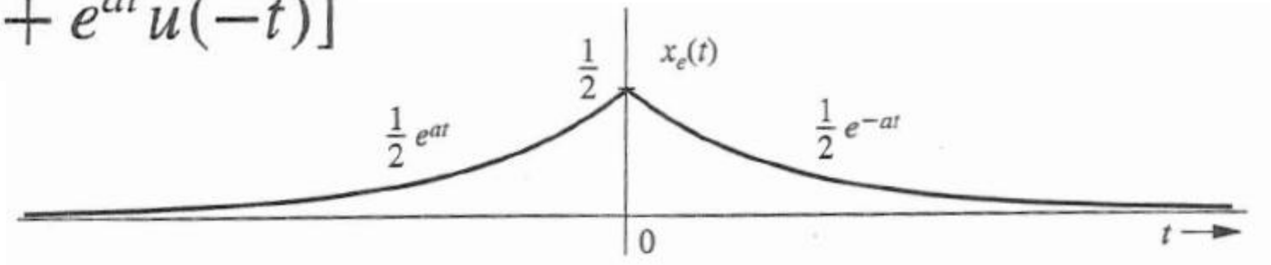
Even and Odd Function

Consider the causal exponential function $x(t) = e^{-at}u(t)$

$$x(t) = x_e(t) + x_o(t)$$



$$x_e(t) = \frac{1}{2}[e^{-at}u(t) + e^{at}u(-t)]$$



$$x_o(t) = \frac{1}{2}[e^{-at}u(t) - e^{at}u(-t)]$$