## Lecture 11 Combinational Circuits

## Combinational Circuits



Fig. 4-1 Block Diagram of Combinational Circuit

## What is Combinational Circuits?

- A Combinational Circuit is a combination of Logic gates, the output depends upon the current value of the inputs.


Fig. 4-2 Logic Diagram for Analysis Example

## Examples of Combinational Circuits

- Addition:
- Half Adder (HA).
- Full Adder (FA).
- BCD(Decimal) Adder.
- Subtraction:
- Half Subtractor.
- Full Subtractor.
- Multiplication:
- Binary Multipliers.
- Comparator:
- Magnitude Comparator.


# Examples of Combinational Circuits 

- Multiplexers
- Demultiplexers
- Encoders
- Decoders
- Converters
- Binary to Gray Code
- Gray to Binary Code
- Binary to BCD Code

Two types of questions come in the exam based on Combinational Circuit:
1.Designing of a combinational Circuit 2.Analysis of Combinational Circuit

## Designing Combinational Circuits

In general we have to do following steps:

1. Problem description
2. Input/output of the circuit
3. Define truth table
4. Simplification for each output
5. Draw the circuit

## Half Adder

- Adding two single-bit binary values, $\mathrm{X}, \mathrm{Y}$ produces a sum $S$ bit and a carry out C-out bit.
- This operation is called half addition and the circuit to realize it is called a half adder.

Half Adder Truth Table

| Inputs |  | Outputs |  |
| :---: | :---: | :---: | :---: |
| $\mathbf{X}$ | $\mathbf{Y}$ | S | C-out |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |



$$
\begin{aligned}
& S(X, Y)=\Sigma(1,2) \\
& S=X^{\prime} Y+X Y \\
& S=X \oplus \mathbf{Y} \\
& \mathbf{C - o u t}(x, y)=\Sigma(3) \\
& C-o u t=X Y
\end{aligned}
$$



$$
\text { (a) } \begin{aligned}
S & =x y^{\prime}+x^{\prime} y \\
C & =x y
\end{aligned}
$$

(b) $S=x \bigoplus y$
$C=x y$

Fig. 4-5 Implementation of Half-Adder

## Full Adder

- Adding two single-bit binary values, $X, Y$ with a carry input bit $C$-in produces a sum bit $S$ and a carry out $C$ out bit.

Full Adder Truth Table

Inputs

| $\mathbf{X}$ | $\mathbf{Y}$ | C-in | $\mathbf{S}$ | C-out |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |



## Full Adder

| $\mathbf{X}$ | $\mathbf{Y}$ | C-in | $\mathbf{S}$ | C-out |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | If $^{\text {puts }}$ | $\mathbf{0}$ | $\mathbf{1}$ | Outguts |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |

Full Adder Truth Table

$$
\begin{aligned}
& S(X, Y, C-i n)=\Sigma(1,2,4,7) \\
& C \text {-out }(x, y, C-i n)=\Sigma(3,5,6,7)
\end{aligned}
$$



$$
\begin{aligned}
& \mathbf{S}=\mathbf{X}^{\prime} \mathbf{Y}^{\prime}(\mathbf{C}-i n)+\mathbf{X Y} Y^{\prime}(\mathbf{C - i n})^{\prime}+X^{\prime}(\mathbf{C}-i n)^{\prime}+\mathbf{X Y}(\mathbf{C}-i n) \\
& \mathbf{S}=\mathbf{X} \oplus \mathbf{Y} \oplus(\mathbf{C}-i n)
\end{aligned}
$$



$$
\mathbf{C - o u t}=\mathbf{X Y}+\mathbf{X}(\mathbf{C}-\mathrm{in})+\mathbf{Y}(\mathbf{C}-\mathrm{in})
$$

## Full Adder Circuit Using AND-OR



## Full Adder Circuit Using Ex-OR



## Full Adder Circuit Using two half - Adders



Fig. 4-8 Implementation of Full Adder with Two Half Adders and an OR Gate

## Binary adder

- Binary adder that produces the arithmetic sum of binary numbers can be constructed with full adders connected in cascade, with the output carry from each full adder is connected to the input carry of the next full adder in the chain
- Note that the input carry $\mathrm{C}_{0}$ in the least significant position must be 0 .


## Binary Adder



Fig. 4-9 4-Bit Adder

## Binary Adder

- For example to add $A=1011$ and $B=0011$ subscript i: $3 \quad 210$ Input carry: $\begin{array}{llllll} & 1 & 1 & 0 & C_{i}\end{array}$ Augend: $1 \begin{array}{lllll}1 & 0 & 1 & 1 & A_{i}\end{array}$ Addend:
$0 \quad 0 \quad 1$ $1 B_{i}$

Sum:
Output carry:


## DECIMAL/BCD ADDER



- ADD 0110 WHEN $\mathrm{C}_{\mathrm{N}}=1$
- ADD 0000 WHEN $\mathrm{C}_{\mathrm{N}}=0$

