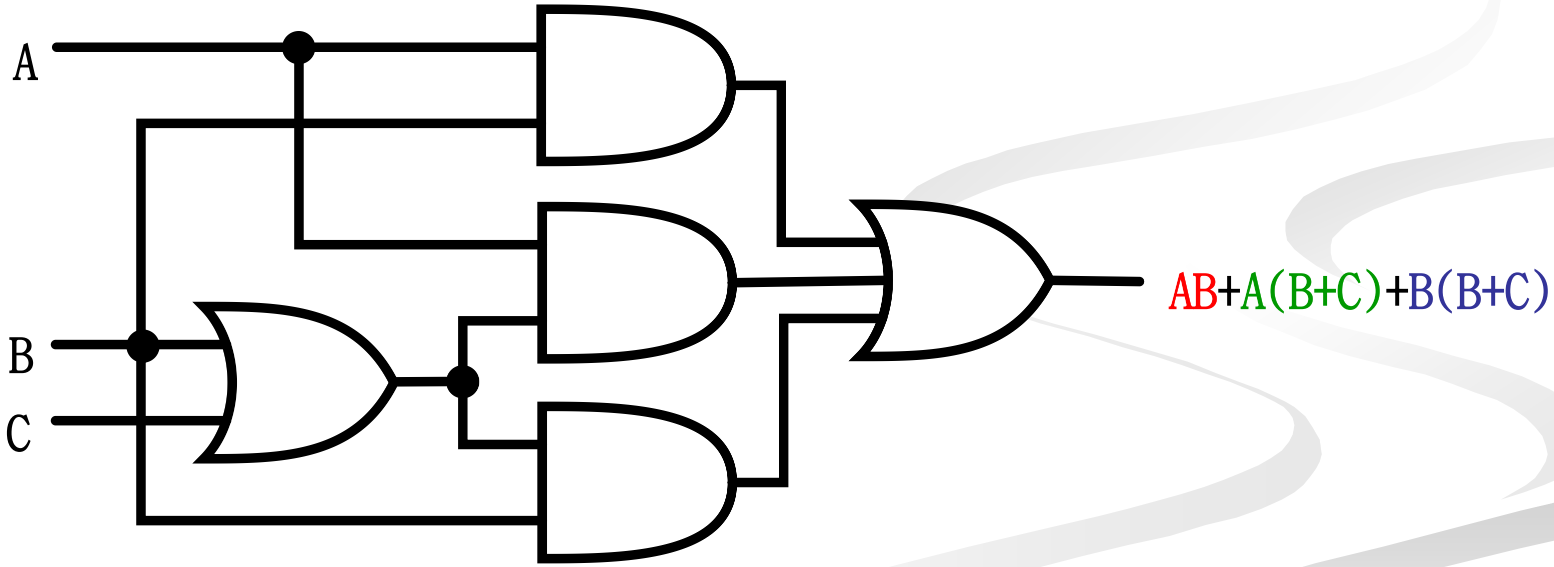
Lecture 7
Logic Simplification

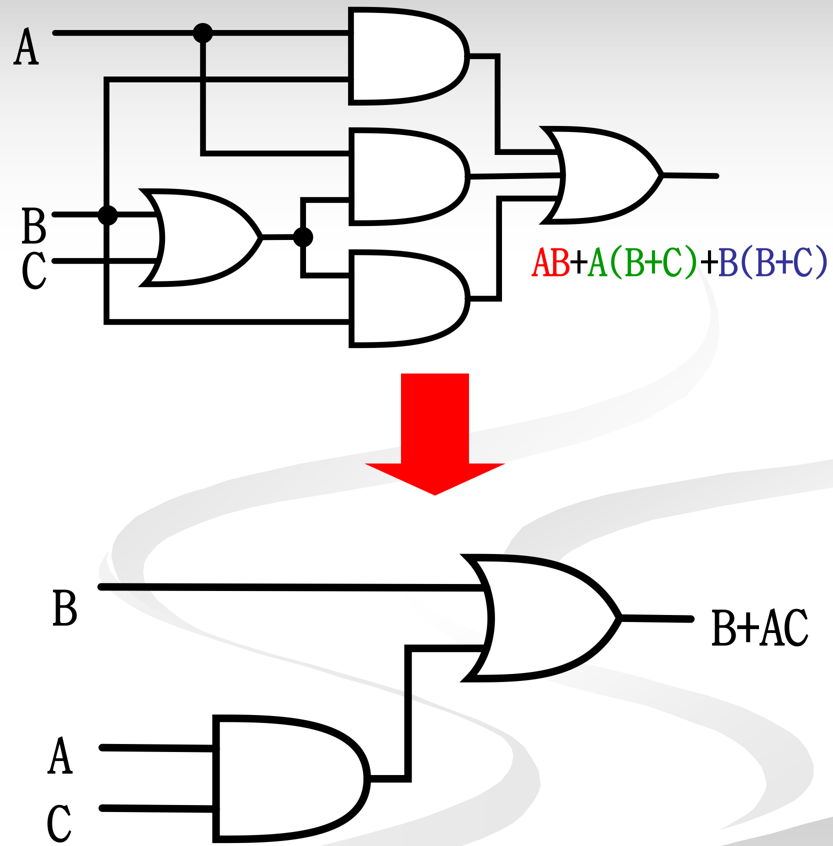
Simplification Using Boolean Algebra

- A simplified Boolean expression uses the fewest gates possible to implement a given expression.



Simplification Using Boolean Algebra

- AB+A(B+C)+B(B+C)
  - (distributive law)
    - AB+AB+AC+BB+BC
  - (rule 7; BB=B)
    - AB+AB+AC+**B**+BC
  - (rule 5; AB+AB=AB)
    - **AB**+AC+B+BC
  - (rule 10; B+BC=B)
    - AB+AC+**B**
  - (rule 10; AB+B=B)
    - **B**+AC



Simplification Using Boolean Algebra

Assignment

$$[A\overline{B}(C+BD)+\overline{A}\,\overline{B}]C$$

$$\overline{A}BC+A\overline{B}\,\overline{C}+\overline{A}\,\overline{B}\,\overline{C}+A\overline{B}C+ABC$$

$$\overline{AB+AC}+\overline{A}\,\overline{B}C$$

Standard Forms of Boolean Expressions

- All Boolean expressions, regardless of their form, can be converted into either of two standard forms:
  - The sum-of-products (SOP) form
  - The product-of-sums (POS) form
- Standardization makes the evaluation, simplification, and implementation of Boolean expressions much more systematic and easier.

Sum-of-Products (SOP)

The Sum-of-Products (SOP) Form

- An SOP expression → when two or more product terms are summed by Boolean addition.
  - Examples:

$$AB + ABC$$

$$ABC + CDE + \overline{B}C\overline{D}$$

$$\overline{A}B + \overline{A}B\overline{C} + AC$$

  - Also:

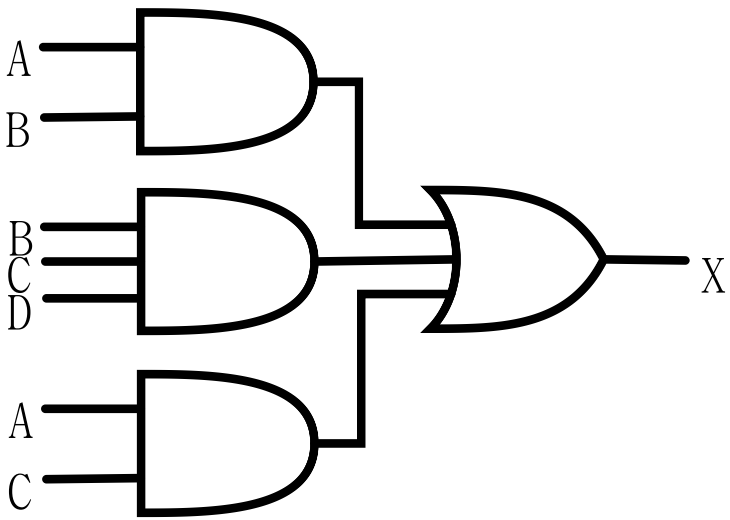
$$A + \overline{A}\overline{B}C + BC\overline{D}$$

- In an SOP form, a single overbar cannot extend over more than one variable; however, more than one variable <u>in a term</u> can have an overbar:
  - example: $\overline{A}\overline{B}\overline{C}$ is OK!
  - **But not:** $\overline{ABC}$

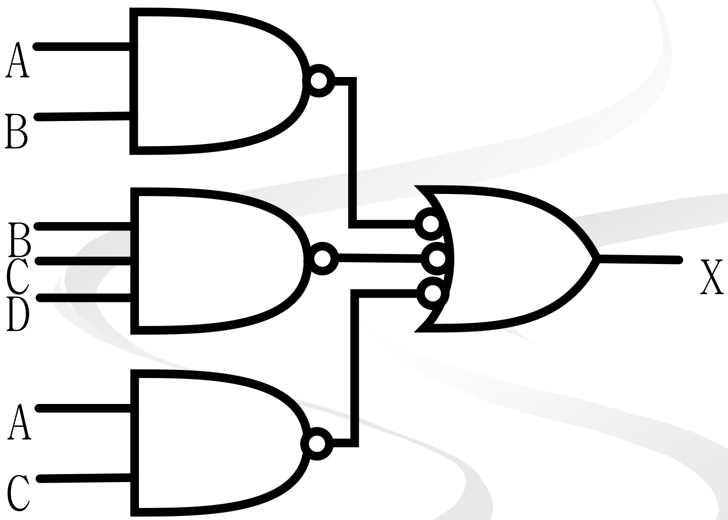
Implementation of an SOP

$X = AB + BCD + AC$

- AND/OR implementation



- NAND/NAND implementation



General Expression → SOP

- Any logic expression can be changed into SOP form by applying Boolean algebra techniques.

  ex:

$$A(B+CD) = \boxed{AB + ACD}$$

$$AB + B(CD+EF) = \boxed{AB + BCD + BEF}$$

$$(A+B)(B+C+D) = \boxed{AB + AC + AD + BB + BC + BD}$$

$$\overline{(\overline{A+B}) + C} = (\overline{\overline{A+B}})\overline{C} = (A+B)\overline{C} = \boxed{A\overline{C} + B\overline{C}}$$

The Standard SOP Form

- A standard SOP expression is one in which *all* the variables in the domain appear in each product term in the expression.
  - Example:

$$A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + AB\overline{C}\,\overline{D}$$

- Standard SOP expressions are important in:
  - Constructing truth tables
  - The Karnaugh map simplification method

Converting Product Terms to Standard SOP

- **Step 1:** Multiply each nonstandard product term by a term made up of the sum of a missing variable and its complement. This results in two product terms.
  - As you know, you can multiply anything by 1 without changing its value.
- **Step 2:** Repeat step 1 until all resulting product term contains all variables in the domain in either complemented or uncomplemented form. In converting a product term to standard form, the number of product terms is doubled for each missing variable.

Converting Product Terms to Standard SOP (example)

- Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

$$A\overline{B}C = A\overline{B}C(D+\overline{D}) = \boxed{A\overline{B}CD + A\overline{B}C\overline{D}}$$

$$\overline{A}\,\overline{B} = \overline{A}\,\overline{B}(C+\overline{C}) = \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$$

$$\overline{A}\,\overline{B}C(D+\overline{D}) + \overline{A}\,\overline{B}\,\overline{C}(D+\overline{D}) = \boxed{\overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}}$$

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D = \boxed{A\overline{B}CD + A\overline{B}C\overline{D}} + \boxed{\overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}} + AB\overline{C}D$$

Binary Representation of a Standard Product Term

- A standard product term is equal to 1 for only one combination of variable values.
  - Example: $A\overline{B}C\overline{D}$ is equal to 1 when A=1, B=0, C=1, and D=0 as shown below

$$A\overline{B}C\overline{D} = 1 \bullet \overline{0} \bullet 1 \bullet \overline{0} = 1 \bullet 1 \bullet 1 \bullet 1 = 1$$

  - And this term is 0 for all other combinations of values for the variables.

Product-of-Sums (POS)

The Product-of-Sums (POS) Form

- When two or more sum terms are multiplied, the result expression is a product-of-sums (POS):
  - Examples:

$$(\overline{A}+B)(A+\overline{B}+C)$$

$$(\overline{A}+\overline{B}+\overline{C})(C+\overline{D}+E)(\overline{B}+C+D)$$

$$(A+B)(A+\overline{B}+C)(\overline{A}+C)$$

  - Also:

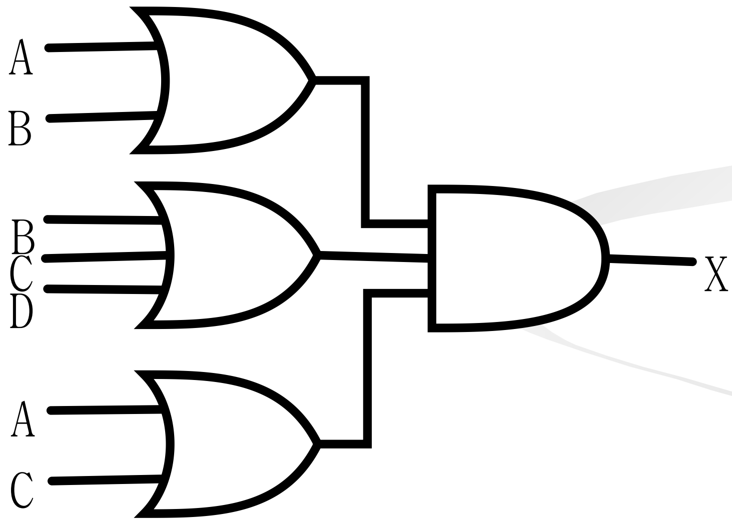
$$\overline{A}(\overline{A}+\overline{B}+C)(B+C+\overline{D})$$

- In a POS form, a single overbar cannot extend over more than one variable; however, more than one variable <u>in a term</u> can have an overbar:
  - example: $\overline{A}+\overline{B}+\overline{C}$ is OK!
  - **But not:** $\overline{A+B+C}$

Implementation of a POS

$X=(A+B)(B+C+D)(A+C)$

- OR/AND implementation



The Standard POS Form

- A standard POS expression is one in which *all* the variables in the domain appear in each sum term in the expression.
  - Example: $(\overline{A}+\overline{B}+\overline{C}+\overline{D})(A+\overline{B}+C+D)(A+B+\overline{C}+D)$

- Standard POS expressions are important in:
  - Constructing truth tables
  - The Karnaugh map simplification method

Converting a Sum Term to Standard POS

- **Step 1:** Add to each nonstandard product term a term made up of the product of the missing variable and its complement. This results in two sum terms.
  - As you know, you can add 0 to anything without changing its value.
- **Step 2:** Apply rule 12 → A+BC=(A+B)(A+C).
- **Step 3:** Repeat step 1 until all resulting sum terms contain all variable in the domain in either complemented or uncomplemented form.

Converting a Sum Term to Standard POS (example)

- Convert the following Boolean expression into standard POS form:

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$

$$A+\overline{B}+C = A+\overline{B}+C+D\overline{D} = (A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})$$

$$\overline{B}+C+\overline{D} = \overline{B}+C+\overline{D}+A\overline{A} = (A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})$$

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D) =$$
$$(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$

Binary Representation of a Standard Sum Term

- A standard sum term is equal to 0 for only one combination of variable values.
  - Example: $A+\overline{B}+C+\overline{D}$ is equal to 0 when A=0, B=1, C=0, and D=1 as shown below

$$A+\overline{B}+C+\overline{D}=0+\overline{1}+0+\overline{1}=0+0+0+0=0$$

  - And this term is 1 for all other combinations of values for the variables.

SOP/POS

Converting Standard SOP to Standard POS

- The Facts:
  - The binary values of the product terms in a given standard SOP expression are not present in the equivalent standard POS expression.
  - The binary values that <u>are not</u> represented in the SOP expression are present in the equivalent POS expression.

Converting Standard SOP to Standard POS

- What can you use the facts?
  - Convert from standard SOP to standard POS.
- How?
  - **Step 1:** Evaluate each product term in the SOP expression. That is, determine the binary numbers that represent the product terms.
  - **Step 2:** Determine all of the binary numbers not included in the evaluation in Step 1.
  - **Step 3:** Write the equivalent sum term for each binary number from Step 2 and express in POS form.

Converting Standard SOP to Standard POS (example)

- Convert the SOP expression to an equivalent POS expression:

$$\overline{A}\,\overline{B}\,\overline{C}+\overline{A}B\overline{C}+\overline{A}BC+A\overline{B}C+ABC$$

  - The evaluation is as follows:

$$000+010+011+101+111$$

  - There are 8 possible combinations. The SOP expression contains five of these, so the POS must contain the other 3 which are: 001, 100, and 110.

$$(A+B+\overline{C})(\overline{A}+B+C)(\overline{A}+\overline{B}+C)$$

Boolean Expressions & Truth Tables

- All standard Boolean expression can be easily converted into truth table format using binary values for each term in the expression.
- Also, standard SOP or POS expression can be determined from the truth table.

Converting SOP Expressions to Truth Table Format

- Recall the fact:
  - An SOP expression is equal to 1 only if at least one of the product term is equal to 1.
- Constructing a truth table:
  - **Step 1:** List all possible combinations of binary values of the variables in the expression.
  - **Step 2:** Convert the SOP expression to standard form if it is not already.
  - **Step 3:** Place a 1 in the output column (X) for each binary value that makes the <u>standard SOP</u> expression a 1 and place 0 for all the remaining binary values.

Converting SOP Expressions to Truth Table Format (example)

- Develop a truth table for the standard SOP expression

$$\overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + ABC$$

Inputs			Output	Product Term
A	B	C	X	
0	0	0	0	
0	0	1	1	$\overline{A}\,\overline{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\overline{B}\,\overline{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	ABC

Converting POS Expressions to Truth Table Format

- Recall the fact:
  - A POS expression is equal to 0 only if at least one of the product term is equal to 0.
- Constructing a truth table:
  - **Step 1:** List all possible combinations of binary values of the variables in the expression.
  - **Step 2:** Convert the POS expression to standard form if it is not already.
  - **Step 3:** Place a 0 in the output column (X) for each binary value that makes the <u>standard POS</u> expression a 0 and place 1 for all the remaining binary values.

Converting POS Expressions to Truth Table Format (example)

- Develop a truth table for the standard SOP expression

$$(A+B+C)(A+\overline{B}+C)(A+\overline{B}+\overline{C})$$
$$(\overline{A}+B+\overline{C})(\overline{A}+\overline{B}+C)$$

Inputs			Output	Product Term
A	B	C	X	
0	0	0	0	$(A+B+C)$
0	0	1	1	
0	1	0	0	$(A+\overline{B}+C)$
0	1	1	0	$(A+\overline{B}+\overline{C})$
1	0	0	1	
1	0	1	0	$(\overline{A}+B+\overline{C})$
1	1	0	0	$(\overline{A}+\overline{B}+C)$
1	1	1	1	

Determining Standard Expression from a Truth Table

- To determine the standard **SOP expression** represented by a truth table.
- Instructions:
  - **Step 1:** List the binary values of the input variables for which the output is 1.
  - **Step 2:** Convert each binary value to the corresponding product term by replacing:
    - each 1 with the corresponding variable, and
    - each 0 with the corresponding variable complement.
- Example: 1010 → $A\overline{B}C\overline{D}$

Determining Standard Expression from a Truth Table

- To determine the standard **POS expression** represented by a truth table.
- Instructions:
  - **Step 1:** List the binary values of the input variables for which the output is 0.
  - **Step 2:** Convert each binary value to the corresponding product term by replacing:
    - each 1 with the corresponding variable complement, and
    - each 0 with the corresponding variable.
- Example: 1001 → $\overline{A}+B+C+\overline{D}$

Determining Standard Expression from a Truth Table (example)

I/P			O/P
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

SOP

- There are four 1s in the output and the corresponding binary value are 011, 100, 110, and 111.

$$011 \rightarrow \overline{A}BC$$
$$100 \rightarrow A\overline{B}\,\overline{C}$$
$$110 \rightarrow AB\overline{C}$$
$$111 \rightarrow ABC$$

$$X = \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

POS

- There are four 0s in the output and the corresponding binary value are 000, 001, 010, and 101.

$$000 \rightarrow A+B+C$$
$$001 \rightarrow A+B+\overline{C}$$
$$010 \rightarrow A+\overline{B}+C$$
$$101 \rightarrow \overline{A}+B+\overline{C}$$

$$X = (A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(\overline{A}+B+\overline{C})$$

Rules of Boolean Algebra

$$1.\ A + 0 = A$$

$$2.\ A + 1 = 1$$

$$3.\ A \bullet 0 = 0$$

$$4.\ A \bullet 1 = A$$

$$5.\ A + A = A$$

$$6.\ A + \overline{A} = 1$$

$$7.\ A \bullet A = A$$

$$8.\ A \bullet \overline{A} = 0$$

$$9.\ \overline{\overline{A}} = A$$

$$10.\ A + AB = A$$

$$11.\ A + \overline{A}B = A + B$$

$$12.\ (A + B)(A + C) = A + BC$$

A, B, and C can represent a single variable or a combination of variables.