ANGLE MODULATION

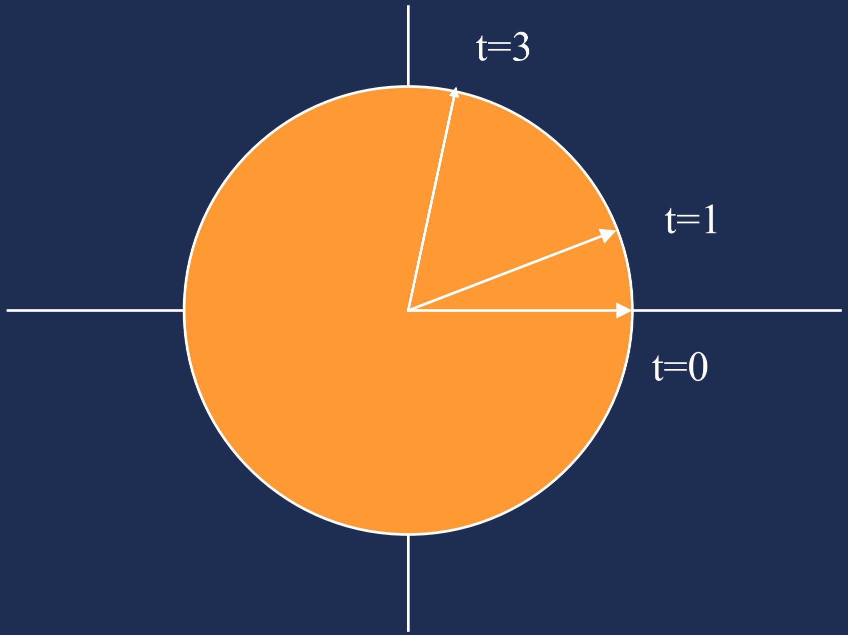
What is Angle Modulation?

- In angle modulation, information is embedded in the *angle* of the carrier.
- We define the angle of a modulated carrier by the argument of...

$$s(t) = A_c \cos(\theta(t))$$

Phasor Form

- **In the complex plane we have**



Phasor rotates with nonuniform speed

Angular Velocity

- Since phase changes nonuniformly vs. time, we can define a rate of change

$$\omega_i = \frac{d\theta_i(t)}{dt}$$

- This is what we know as frequency

$$s(t) = A_c \cos\left(\underbrace{2\pi f_c t + \phi_c}_{\theta_i(t)}\right) \Rightarrow \frac{d\theta_i}{dt} = 2\pi f_c$$

Instantaneous Frequency

- **We are used to signals with constant carrier frequency. There are cases where carrier frequency itself changes with time.**
- **We can therefor talk about *instantaneous frequency* defined as**

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

Examples of Inst. Freq.

- Consider an AM signal

$$s(t) = [1 + km(t)]\cos\left(\underbrace{2\pi f_c t + \phi_c}_{\theta_i(t)}\right) \Rightarrow \frac{d\theta_i}{dt} = 2\pi f_c$$

- Here, the instantaneous frequency is the frequency itself, which is constant

Impressing a message on the angle of carrier

- **There are two ways to form a an angle modulated signal.**
  – Embed it in the phase of the carrier
  Phase Modulation(PM)
  – Embed it in the frequency of the carrier
  Frequency Modulation(FM)

Phase Modulation(PM)

- **In PM, carrier angle changes linearly with the message**

$$s(t) = A_c \cos(\theta_i(t)) = A_c \cos(2\pi f_c t + k_p m(t))$$

- **Where**
  - $2\pi f_c$=angle of unmodulated carrier
  - k_p=phase sensitivity in radians/volt

Frequency Modulation

- In FM, it is the instantaneous frequency that varies linearly with message amplitude, i.e.

$$f_i(t)=f_c+k_f m(t)$$

FM Signal

- We saw that I.F. is the derivative of the phase

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

- Therefore,

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)$$

$$\Rightarrow \quad s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right]$$

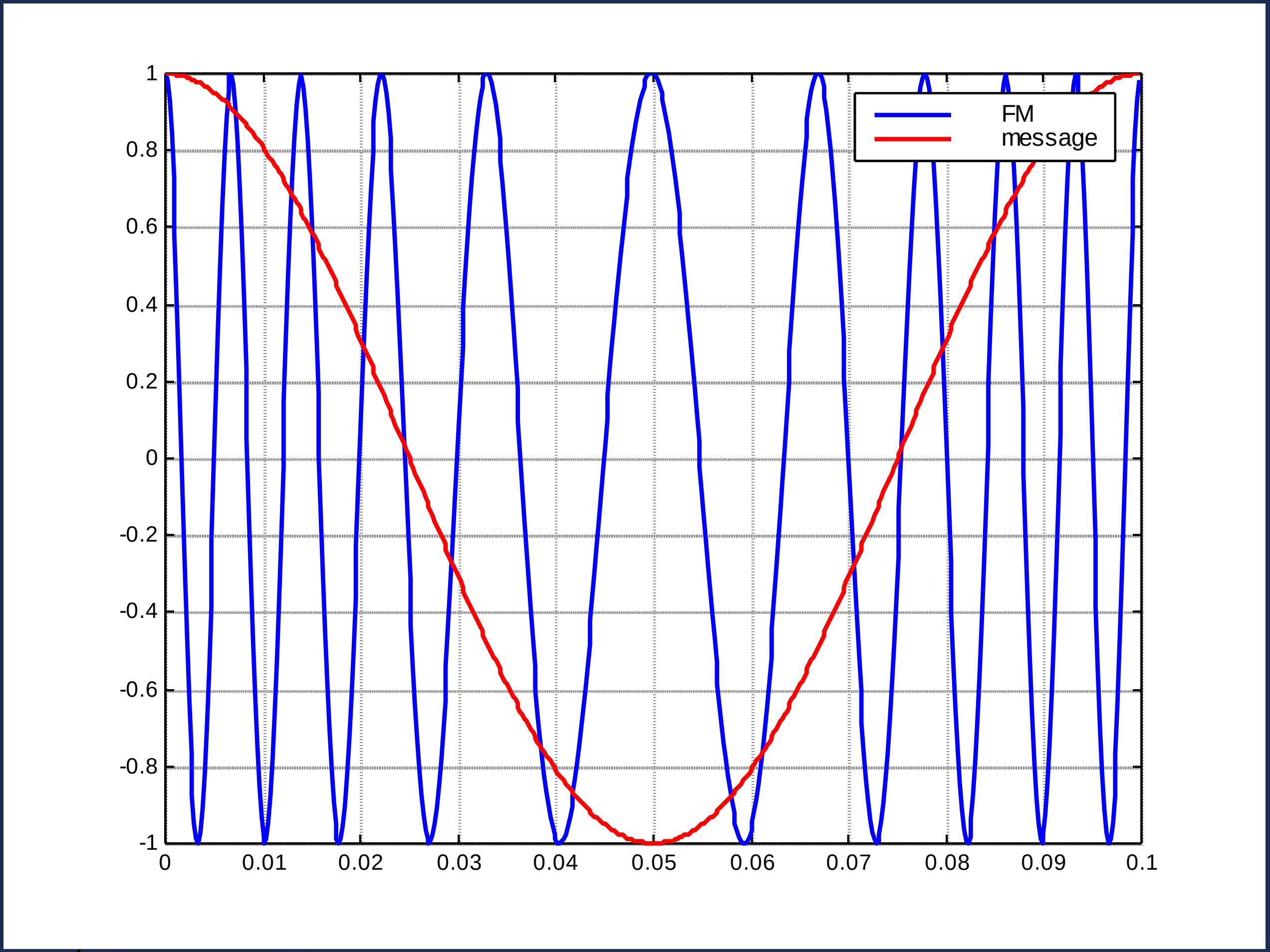
FM for Tone Signals

- Consider a sinusoidal message $m(t) = A_m \cos(2\pi f_m t)$
- The instantaneous frequency corresponding to its FM version is

$$f_i(t) = f_c + k_f m(t)$$

$$= \underbrace{f_c}_{resting\ frequency} + k_f A_m \cos(2\pi f_m t)$$

Illustrating FM



Inst.frequency
Moves with the
Message amplitude

Frequency Deviation

- **Inst. frequency has upper and lower bounds given by**

$$f_i(t) = f_c + \Delta f \cos(2\pi f_m t)$$

$$where$$

$$\Delta f = frequency\, deviation = k_f A_m$$

$$then$$

$$f_i\big|_{\max} = f_c + \Delta f$$

$$f_i\big|_{\min} = f_c - \Delta f$$

FM Modulation index

- **The equivalent of AM modulation index is β which is also called *deviation ratio*. It quantifies how much carrier frequency swings relative to message bandwidth**

$$\beta = \frac{\Delta f}{\underbrace{W}_{baseband}} \; or \; \frac{\Delta f}{\underbrace{f_m}_{tone}}$$

Example:carrier swing

- **A 100 MHz FM carrier is modulated by an audio tone causing 20 KHz frequency deviation. Determine the carrier siwng and highest and lowest carrier frequencies**

$$\Delta f = 20KHz$$

$$frequency\,swing = 2\Delta f = 40KHz$$

$$frequency\,range:$$

$$f_{high} = 100MHz + 20KHz = 100.02MHz$$

$$f_{low} = 100MHz - 20KHz = 99.98MHz$$

Example: deviation ratio

- What is the modulation index (or deviation ratio) of an FM signal with carrier swing of 150 KHz when the modulating signal is 15 KHz?

$$\Delta f = \frac{150}{2} = 75KHz$$

$$\beta = \frac{\Delta f}{f_m} = \frac{75}{15} = 5$$

Myth of FM

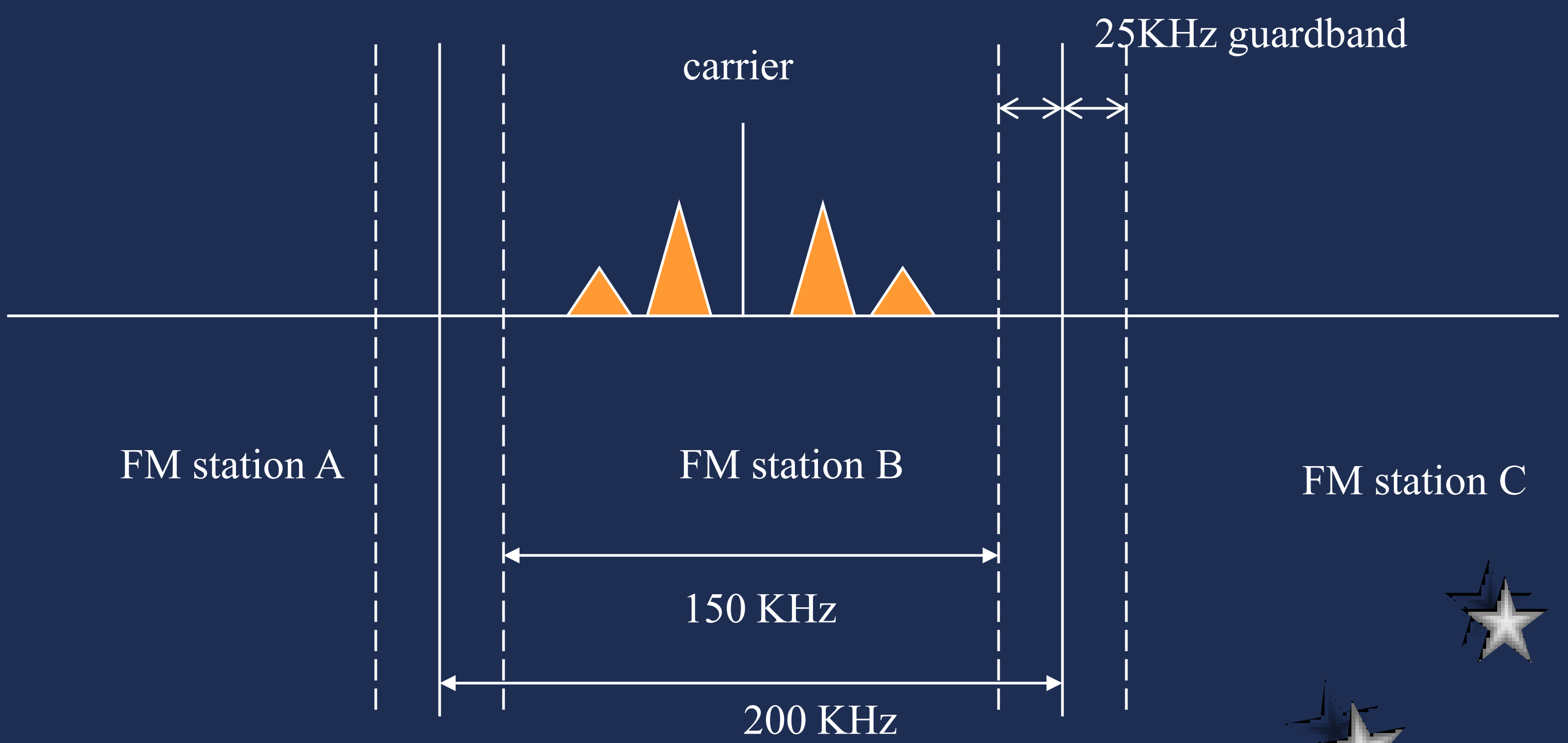
- Deriving FM bandwidth is a lot more involved than AM
- FM was initially thought to be a bandwidth efficient communication because it was thought that FM bandwidth is simply $2\Delta f$
- By keeping frequency deviation low, we can use arbitrary small bandwidth

FM bandwidth

- Deriving FM bandwidth is a lot more involved than AM and it can barely be derived for sinusoidal message
- There is a graphical way to illustrate FM bandwidth

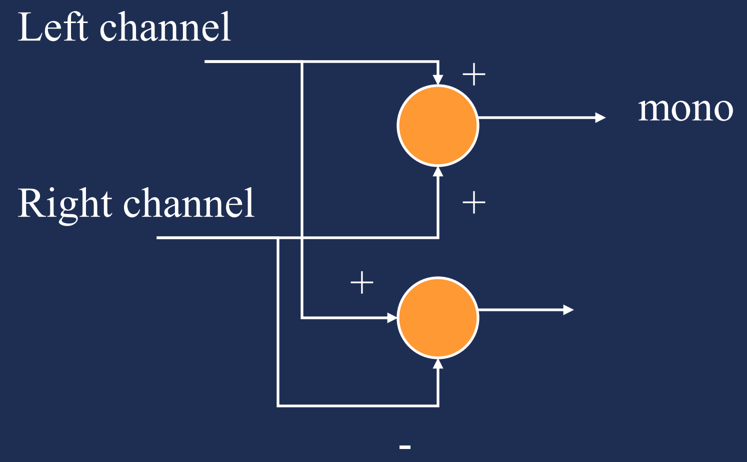
Commercial FM spectrum

- **The FM landscape looks like this**



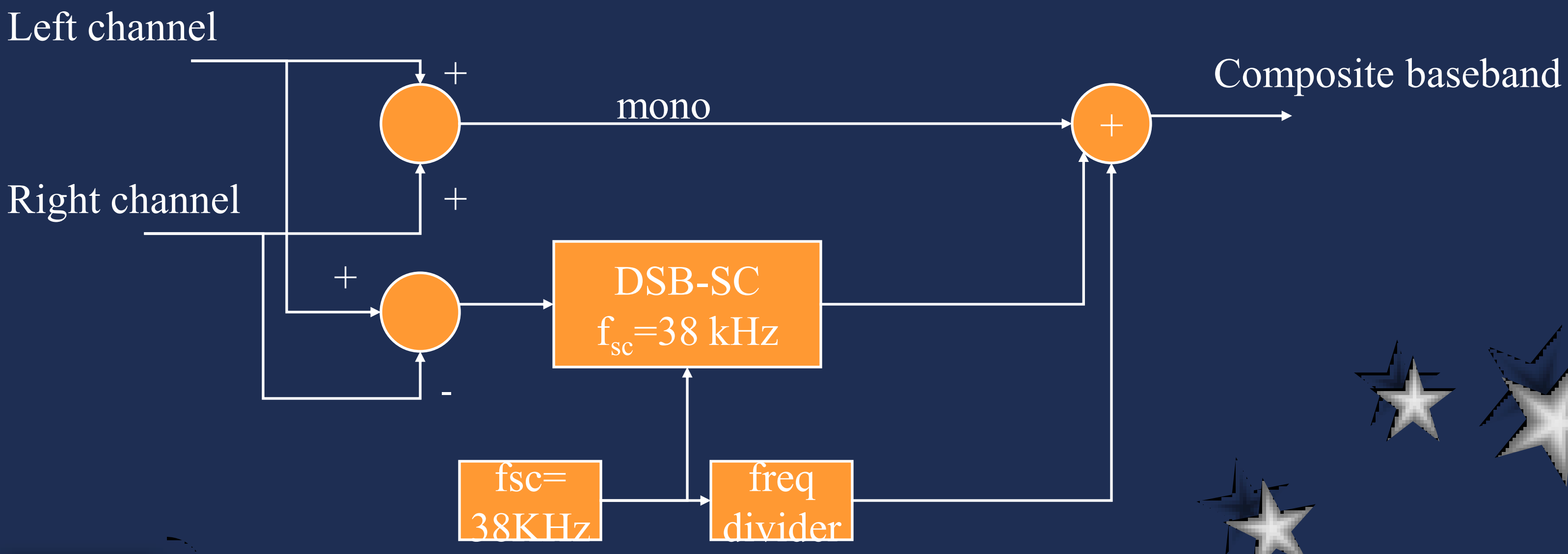
FM stereo:multiplexing

- First, two channels are created; (left+right) and (left-right)
- Left+right is useable by monaural receivers



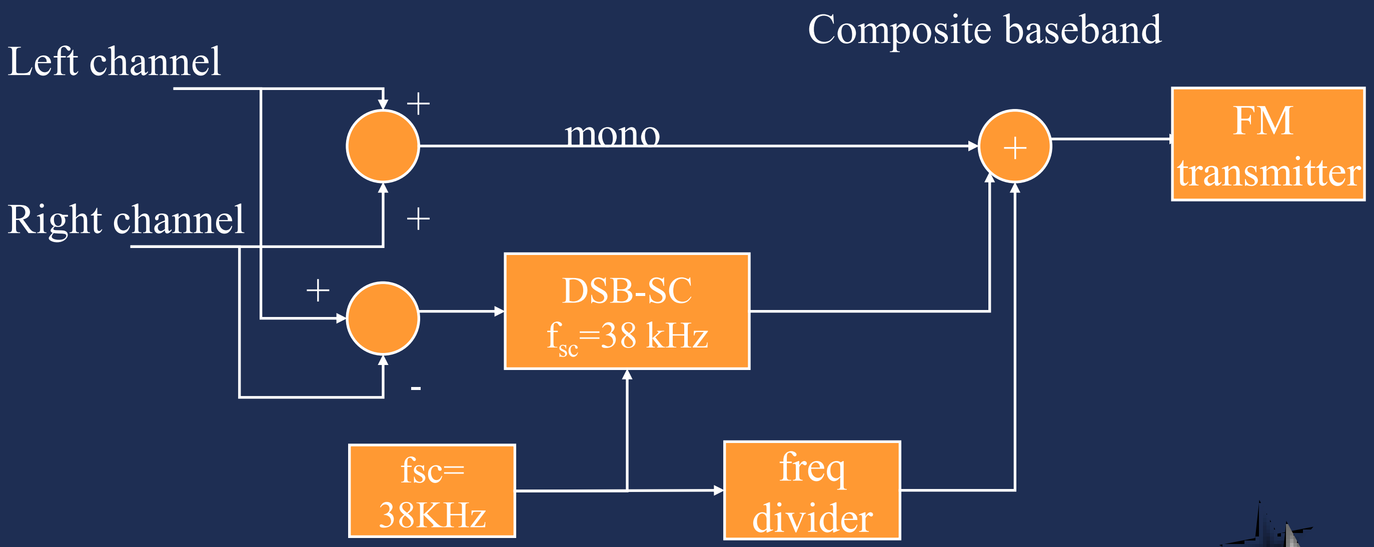
Subcarrier modulation

- **The mono signal is left alone but the difference channel is amplitude modulated with a 38 KHz carrier**



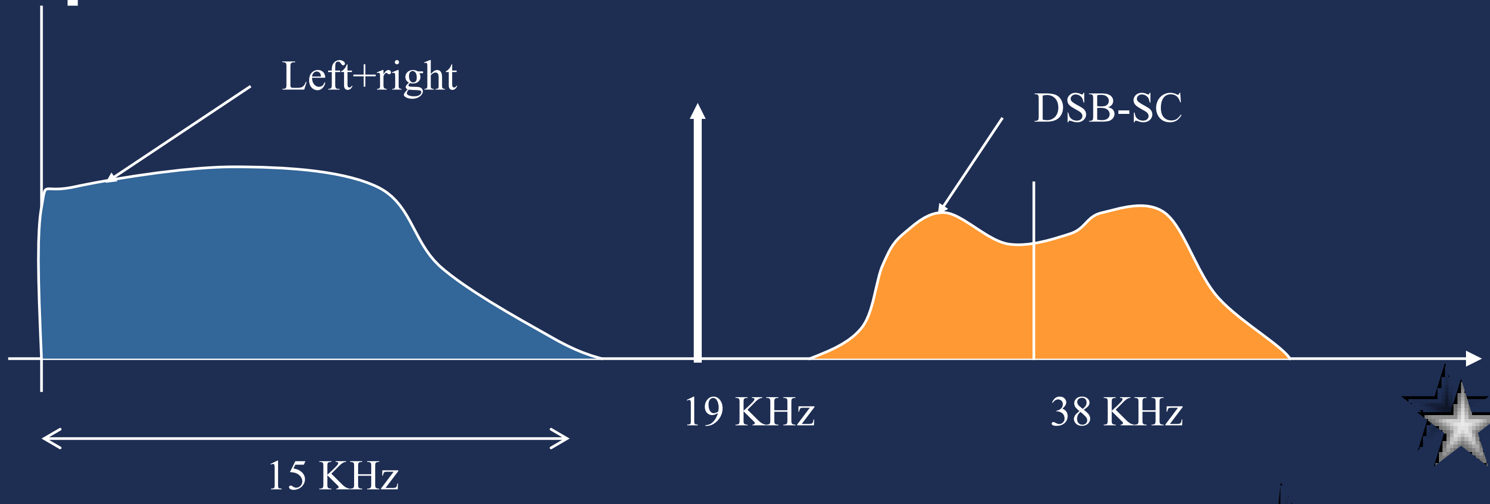
Stereo signal

- **Composite baseband signal is then frequency modulated**



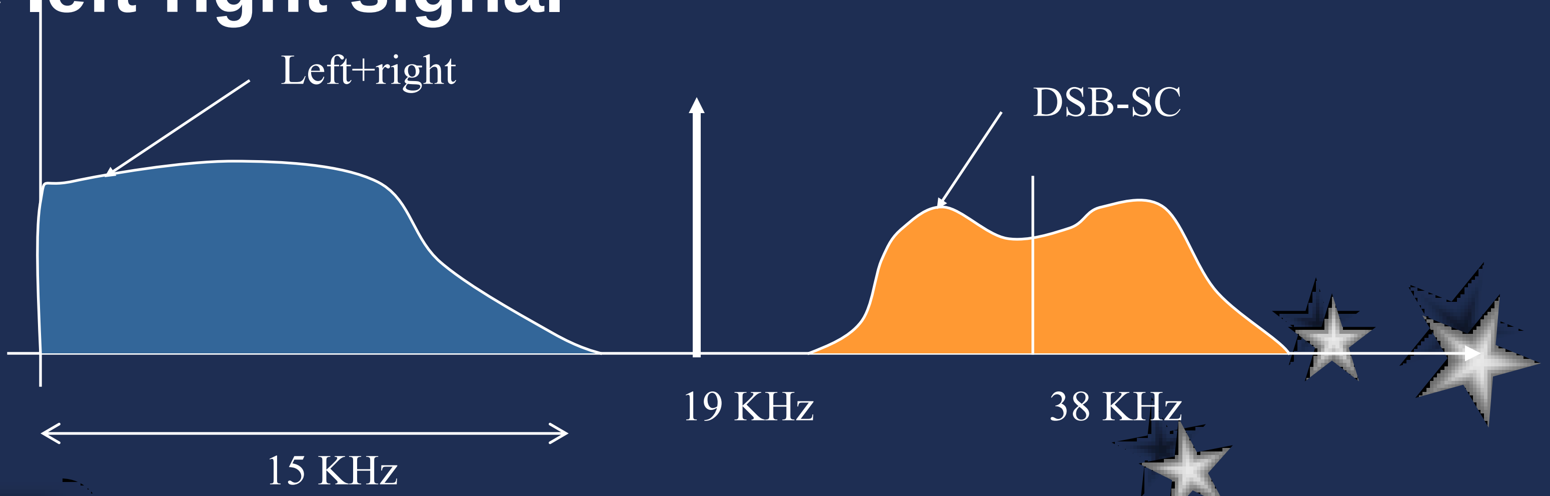
Stereo spectrum

- **Baseband spectrum holds all the information. It consists of composite baseband, pilot tone and DSB-SC spectrum**

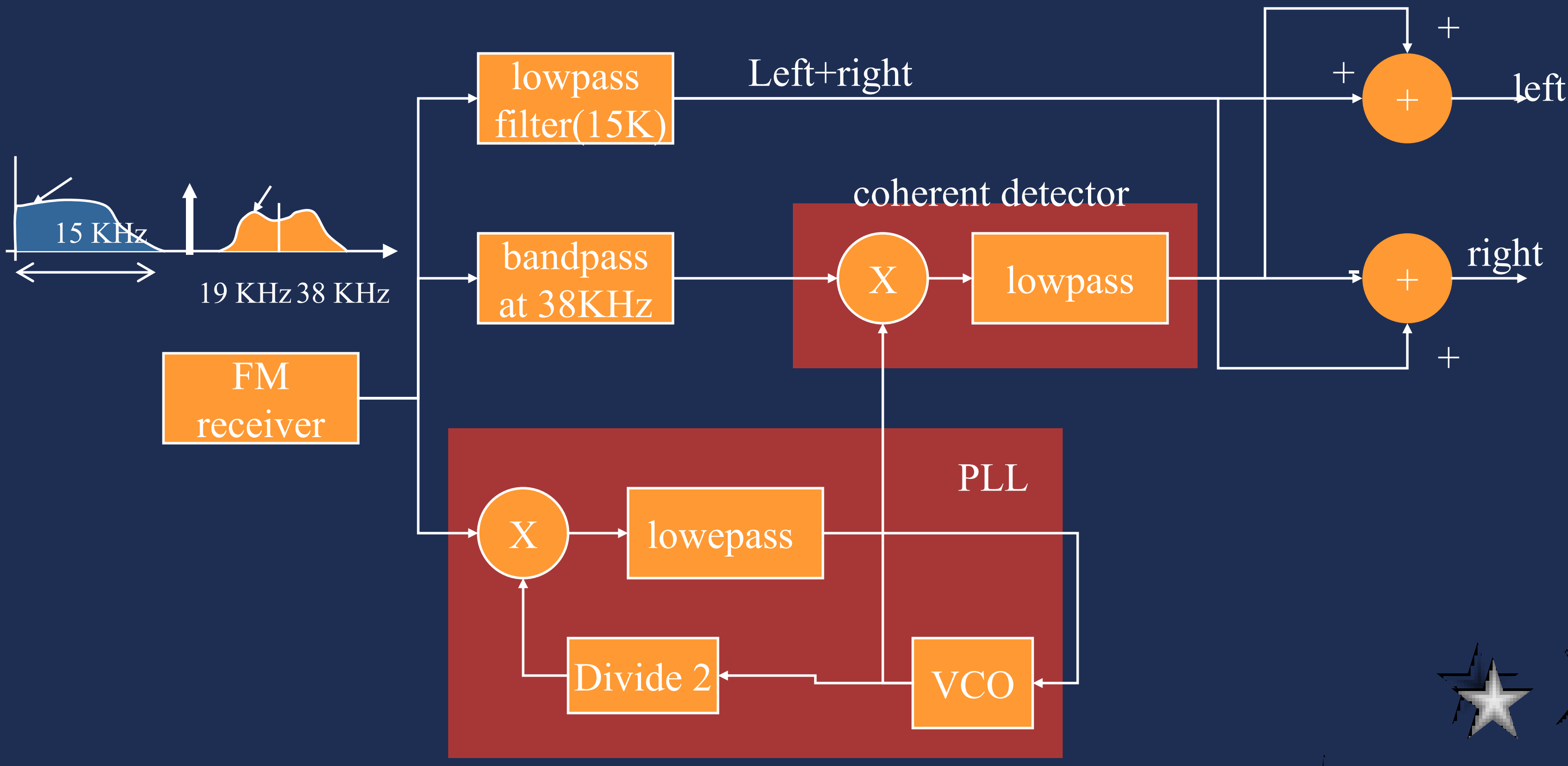


Stereo receiver

- First, FM is stripped, i.e. demodulated
- Second, composite baseband is lowpass filtered to recover the left+right and in parallel amplitude demodulated to recover the left-right signal



Receiver diagram



Subsidiary communication authorization(SCA)

- It is possible to transmit "special programming" ,e.g. commercial-free music for banks, department stores etc. embedded in the regular FM programming
- Such programming is frequency multiplexed on the FM signal with a 67 KHz carrier and ±7.5 KHz deviation

SCA spectrum