Classification of Systems

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Systems may be classified into:

1. Linear and non-linear systems
2. Constant parameter and time-varying-parameter systems
3. Instantaneous (memory less) and dynamic (with memory) systems
4. Causal and non-causal systems
5. Continuous-time and discrete-time systems
6. Analog and digital systems
7. Invertible and noninvertible systems

Linear Systems (1)

- A **linear system exhibits the additive property:**

  if $x_1 \longrightarrow y_1$ and $x_2 \longrightarrow y_2$ then $x_1 + x_2 \longrightarrow y_1 + y_2$

- It also must satisfy the **homogeneity or scaling property:**

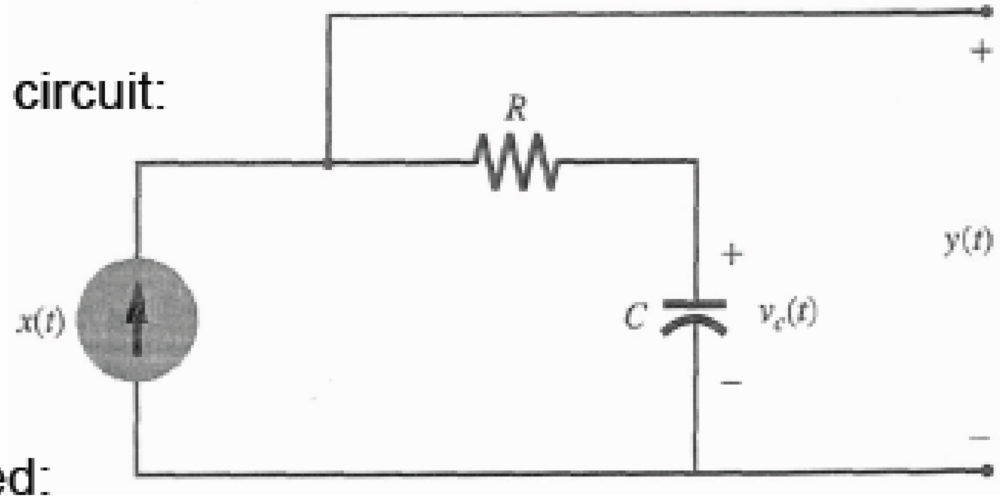
  if $x \longrightarrow y$ then $kx \longrightarrow ky$

- These can be combined into the property of **superposition:**

  if $x_1 \longrightarrow y_1$ and $x_2 \longrightarrow y_2$ then $k_1x_1 + k_2x_2 \longrightarrow k_1y_1 + k_2y_2$

- A non-linear system is one that is NOT linear (i.e. does not obey the principle of superposition)

Linear Systems (2)



- Consider the following simple RC circuit:
- Output *y(t)* relates to *x(t)* by:

$$y(t) = Rx(t) + \frac{1}{C}\int_{-\infty}^{t} x(\tau)\,d\tau$$

- The second term can be expanded:

$$y(t) = Rx(t) + \frac{1}{C}\int_{-\infty}^{0} x(\tau)\,d\tau + \frac{1}{C}\int_{0}^{t} x(\tau)\,d\tau$$

$$y(t) = v_C(0) + Rx(t) + \frac{1}{C}\int_{0}^{t} x(\tau)\,d\tau \qquad t \geq 0$$

- This is a **single-input, single-output** (SISO) system. In general, a system can be multiple-input, multiple-output (**MIMO**).

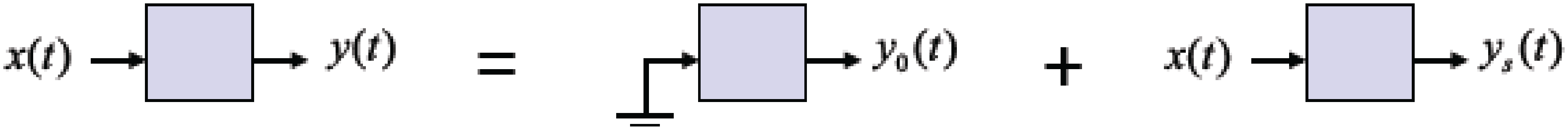
Linear Systems (3)

- A system's output for $t \geq 0$ is result of 2 independent causes:
  1. Initial conditions when $t = 0$ (**zero-input response**)
  2. Input $x(t)$ for $t \geq 0$ (**zero-state response**)

- Decomposition property:

Total response = zero-input response + zero-state response

$$y(t) = \underbrace{v_C(0)}_{\text{zero-input response}} + \underbrace{Rx(t) + \frac{1}{C}\int_0^t x(\tau)\,d\tau}_{\text{zero-state response}} \qquad t \geq 0$$



Linear Systems (4)

- Show that the system described by the equation $\frac{dy}{dt} + 3y(t) = x(t)$ is linear.
- Let $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$, then

$$\frac{dy_1}{dt} + 3y_1(t) = x_1(t) \quad \text{and} \quad \frac{dy_2}{dt} + 3y_2(t) = x_2(t)$$

- Multiple 1st equation by k_1, and 2nd equation by k_2, and adding them yields:

$$\frac{d}{dt}[k_1 y_1(t) + k_2 y_2(t)] + 3[k_1 y_1(t) + k_2 y_2(t)] = k_1 x_1(t) + k_2 x_2(t)$$

- This equation is the system equation with

$$x(t) = k_1 x_1(t) + k_2 x_2(t)$$

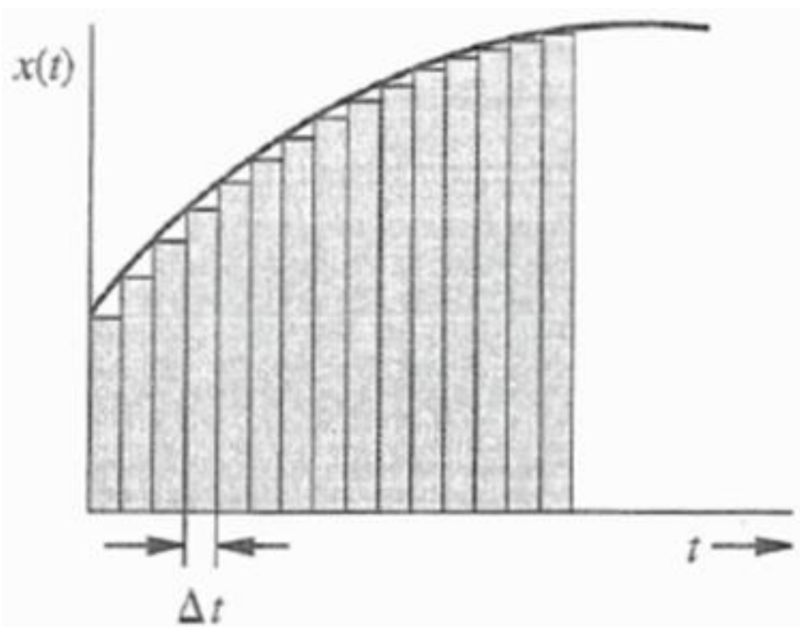
and

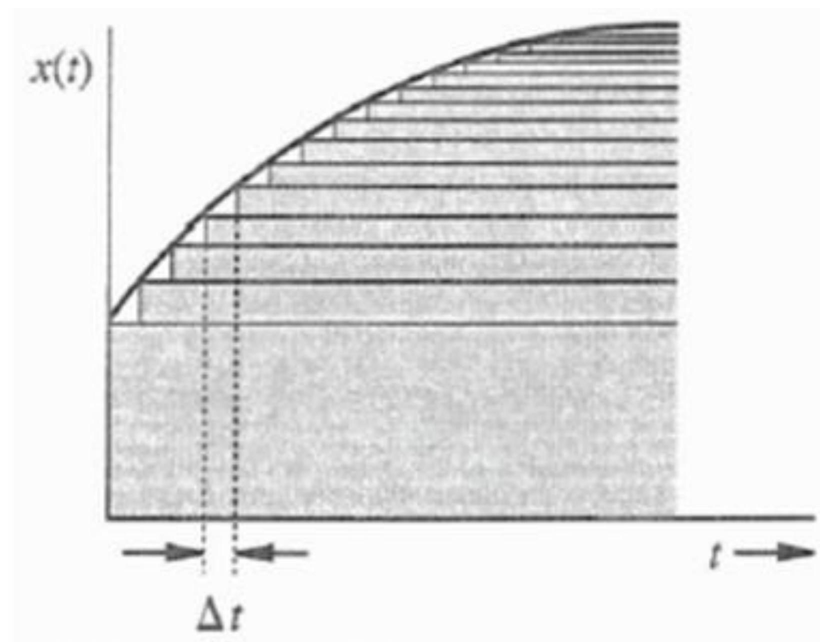
$$y(t) = k_1 y_1(t) + k_2 y_2(t)$$

Is the system $y = x^2$ linear?

Linear Systems (5)

A complex input can be represented as a sum of simpler inputs (pulse, step, sinusoidal), and then use linearity to find the response to this simple inputs to find the system output to the complex input.



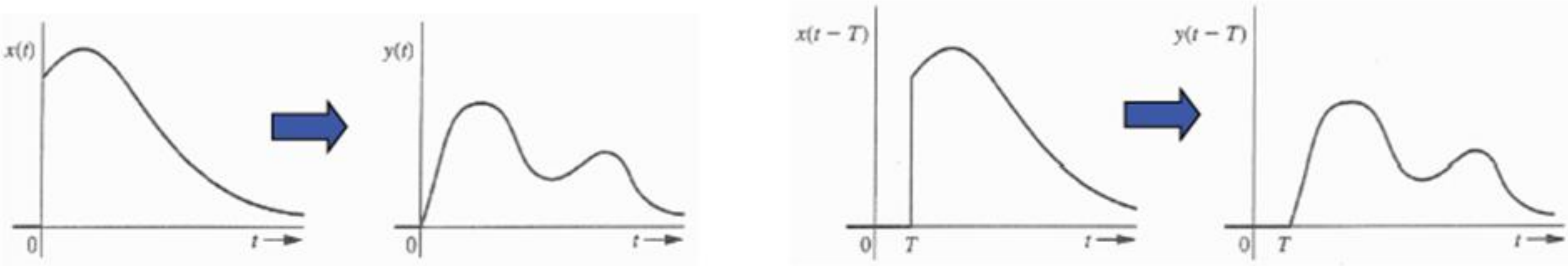


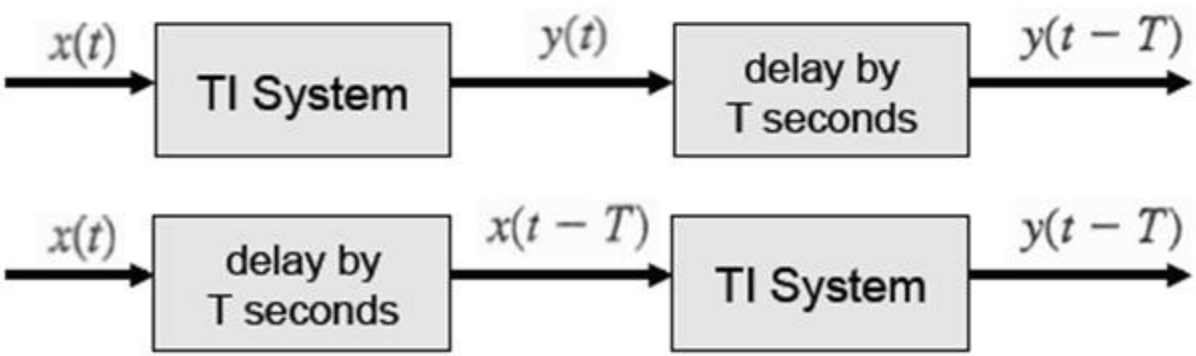
$$x(t) = a_1x_1(t) + a_2x_2(t) + \cdots + a_mx_m(t)$$

$$y(t) = a_1y_1(t) + a_2y_2(t) + \cdots + a_my_m(t)$$

Time-Invariant System

- **Time-invariant system** is one whose parameters do not change with time:





Which of the system is time-invariant?

(a) $y(t) = 3x(t)$ (b) $y(t) = t\,x(t)$

Instantaneous and Dynamic Systems

- In general, a system's output at time t depends on the entire past input. Such a system is a dynamic (with memory) system
  - Analogous to a state machine in a digital system

- If the system's past history is irrelevant in determining the response, it is an instantaneous or memoryless systems
  - Analogous to a combinatorial circuit in a digital system

Causal and Noncausal Systems

- **Causal** system – output at t_0 depends only on x(t) *for* $t \leq t_0$
- I.e. present output depends only on the past and present inputs, **not on future inputs**
- Any practical **REAL TIME system must be causal**.
- **Noncausal** systems are important because:
  1. Realizable when the independent variable is something other than "time" (e.g. space)
  2. Even for temporal systems, can prerecord the data (non-real time), mimic a non -causal system
  3. Study upper bound on the performance of a causal system

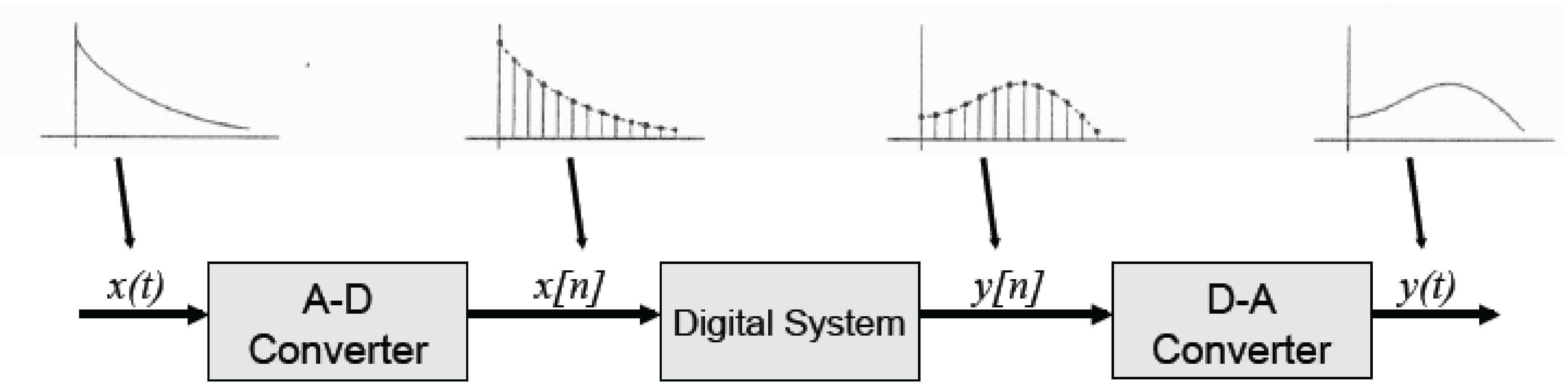
Which of the two systems is causal?

a) y(t) = 3 x(t) + x(t-2)

b) y(t) = 3x(t) + x(t+2)

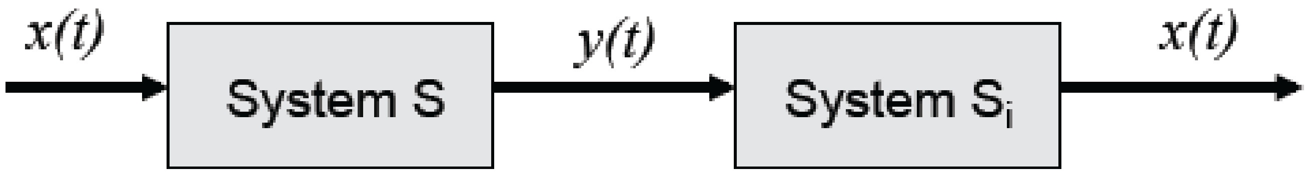
Analog and Digital Systems

- Previously the samples are discrete in time, but are continuous in amplitude
- Most modern systems are DIGITAL DISCRETE-TIME systems, e.g. internal circuits of the MP3 player



Invertible and Noninvertible

- Let a system S produces $y(t)$ with input $x(t)$, if there exists another system S_i, which produces $x(t)$ from $y(t)$, then S is invertible
- Essential that there is **one-to-one mapping** between input and output
- For example if S is an amplifier with gain G, it is invertible and S_i is an attenuator with gain 1/G
- Apply S_i following S gives an **identity system** (i.e. input $x(t)$ is not changed)



Which of the two systems is invertible?

a) $y(t) = x^2$

b) $y = 2x$