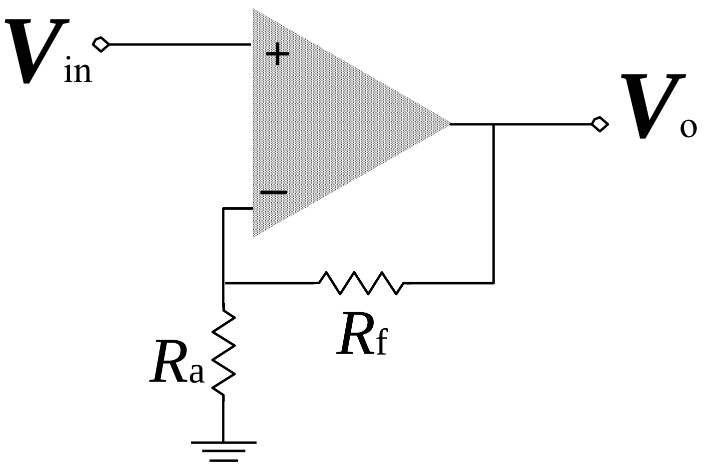
Lecture 25

Op- Amp

Noninverting Amplifier

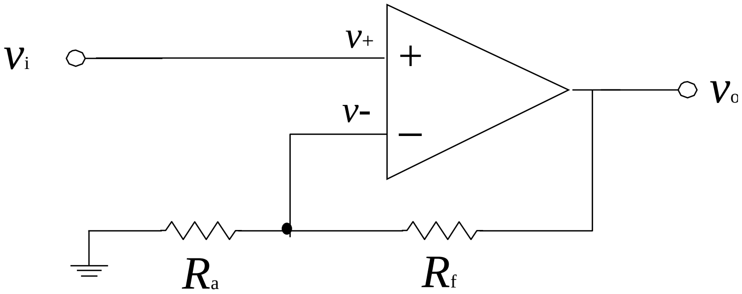


(1) Kirchhoff node equation at V_+ yields, $V_+ = V_i$

(2) Kirchhoff node equation at V_- yields, $\frac{V_- - 0}{R_a} + \frac{V_- - V_o}{R_f} = 0$

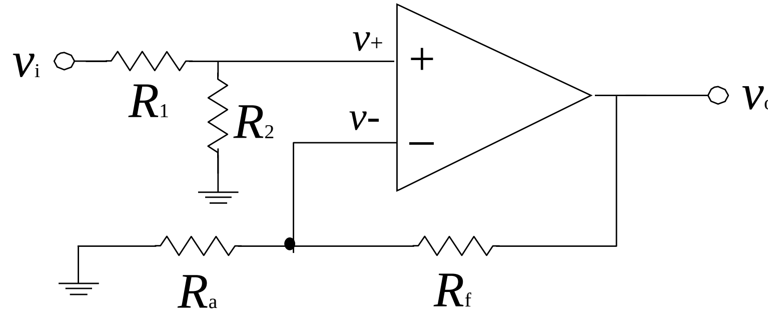
(3) Setting $V_+ = V_-$ yields

$$\frac{V_i}{R_a} + \frac{V_i - V_o}{R_f} = 0 \quad \text{or} \quad \frac{V_o}{V_i} = 1 + \frac{R_f}{R_a}$$



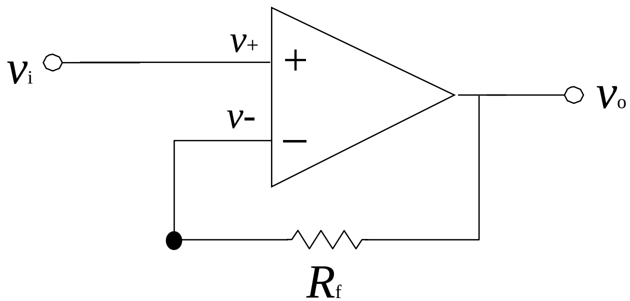
Noninverting amplifier

$$v_o = (1 + \frac{R_f}{R_a}) v_i$$



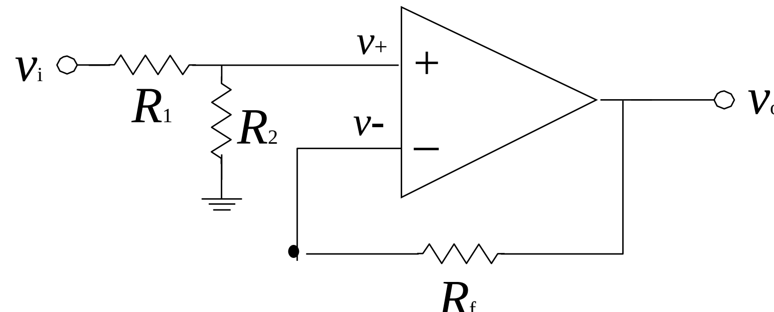
Noninverting input with voltage divider

$$v_o = (1 + \frac{R_f}{R_a})(\frac{R_2}{R_1 + R_2}) v_i$$



Voltage follower

$$v_o = v_i$$



Less than unity gain

$$v_o = \frac{R_2}{R_1 + R_2} v_i$$

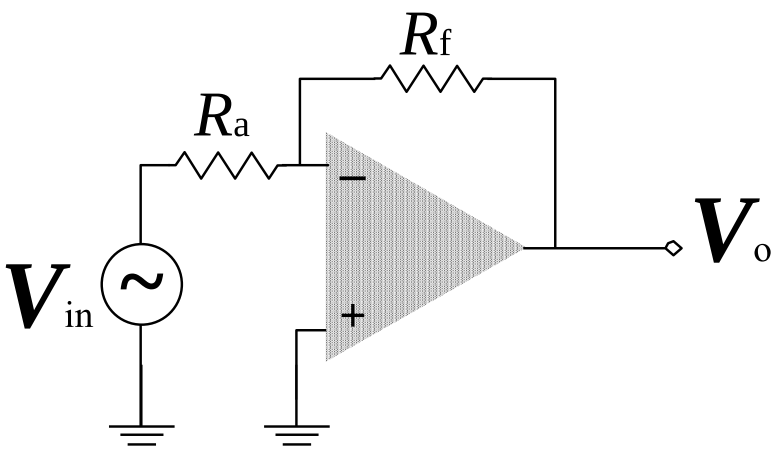
Inverting Amplifier

(1) Kirchhoff node equation at V_+ yields, $V_+ = 0$

(2) Kirchhoff node equation at V_- yields, $\frac{V_{in} - V_-}{R_a} + \frac{V_o - V_-}{R_f} = 0$

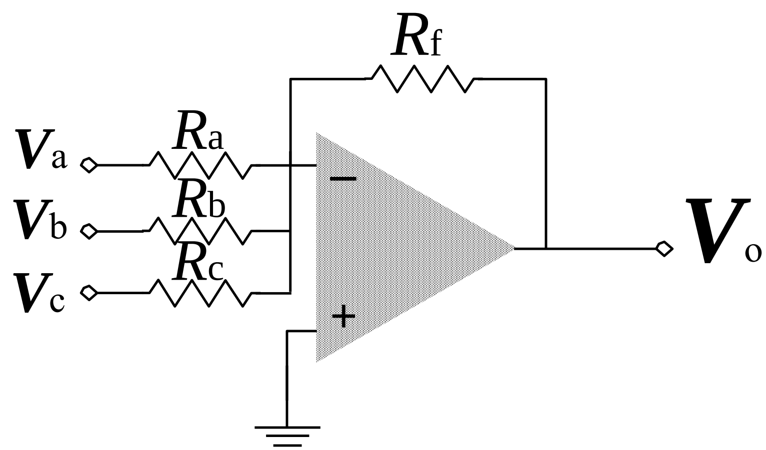
(3) Setting $V_+ = V_-$ yields

$$\frac{V_o}{V_{in}} = \frac{-R_f}{R_a}$$



Notice: The closed-loop gain V_o/V_{in} is dependent upon the ratio of two resistors, and is independent of the open-loop gain. This is caused by the use of feedback output voltage to subtract from the input voltage.

Multiple Inputs



(1) Kirchhoff node equation at V_+ yields, $V_+ = 0$

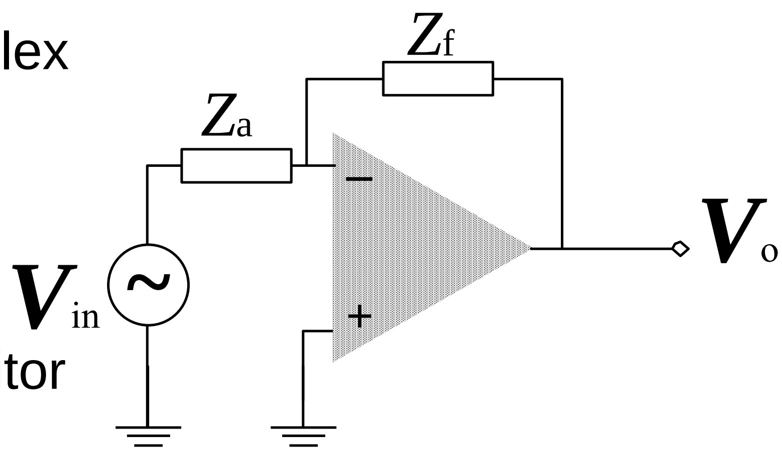
(2) Kirchhoff node equation at V_- yields,

$$\frac{V_- - V_o}{R_f} + \frac{V_- - V_a}{R_a} + \frac{V_- - V_b}{R_b} + \frac{V_- - V_c}{R_c} = 0$$

(3) Setting $V_+ = V_-$ yields

$$V_o = -R_f\left(\frac{V_a}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c}\right) = -R_f \sum_{j=a}^{c} \frac{V_j}{R_j}$$

Inverting Integrator



Now replace resistors R_a and R_f by complex components Z_a and Z_f, respectively, therefore

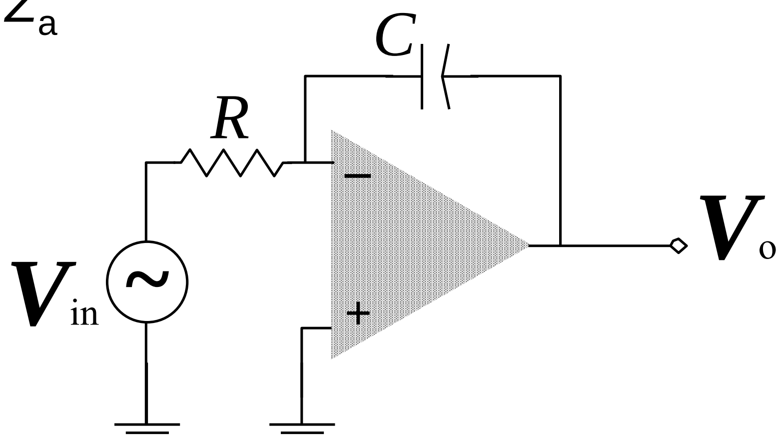
$$V_o = \frac{-Z_f}{Z_a} V_{in}$$

Supposing

(i) The feedback component is a capacitor C, i.e.,

$$Z_f = \frac{1}{j\omega C}$$

(ii) The input component is a resistor R, $Z_a = R$



Therefore, the closed-loop gain (V_o/V_{in}) become:

$$v_o(t) = \frac{-1}{RC} \int v_i(t)\,dt$$

$$v_i(t) = V_i e^{j\omega t}$$

where

What happens if $Z_a = 1/j\omega C$ whereas, $Z_f = R$?

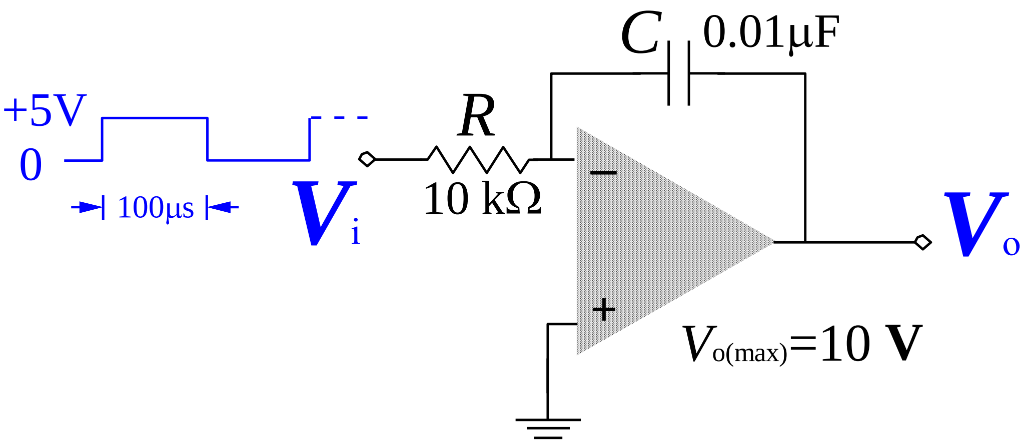
Inverting differentiator

Op-Amp Integrator

Example:

(a) Determine the rate of change of the output voltage.

(b) Draw the output waveform.

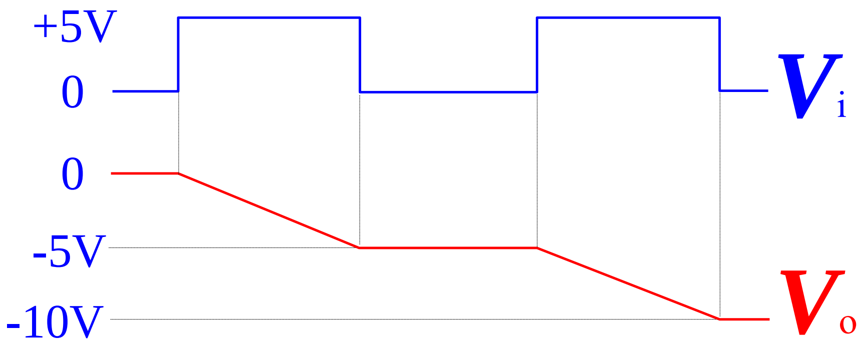


Solution:

(a) Rate of change of the output voltage

$$\frac{\Delta V_o}{\Delta t} = -\frac{V_i}{RC} = \frac{5\,\mathbf{V}}{(10\,\mathbf{k\Omega})(0.01\,\mu\mathbf{F})}$$

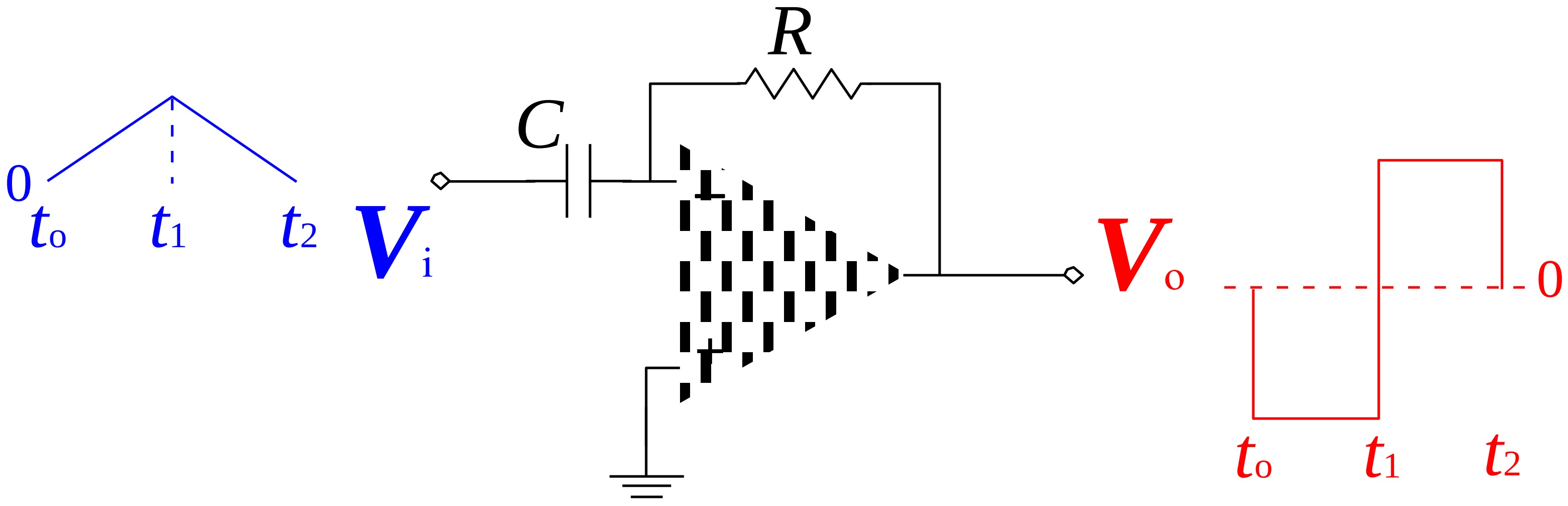
$$= -50\,\mathbf{mV}/\mu\mathbf{s}$$



(b) In 100 μs, the voltage decrease

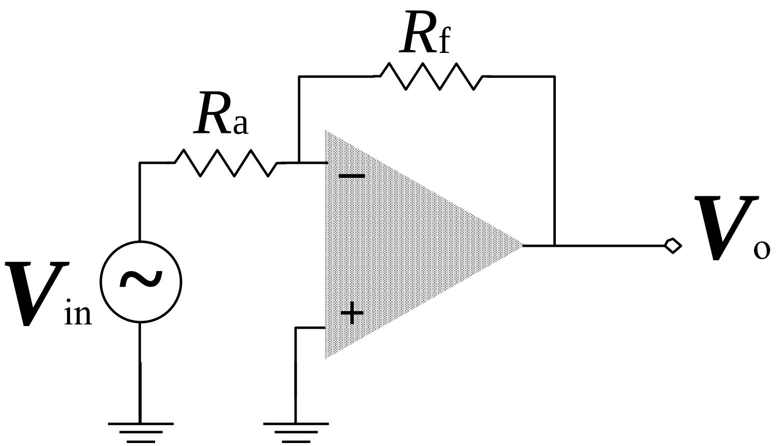
$$\Delta V_o = (-50\,\mathbf{mV}/\mu\mathbf{s})(100\,\mathbf{\mu s}) = -5\mathbf{V}$$

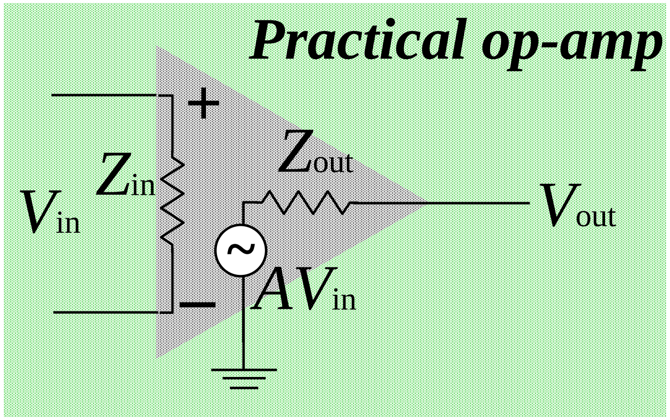
Op-Amp Differentiator



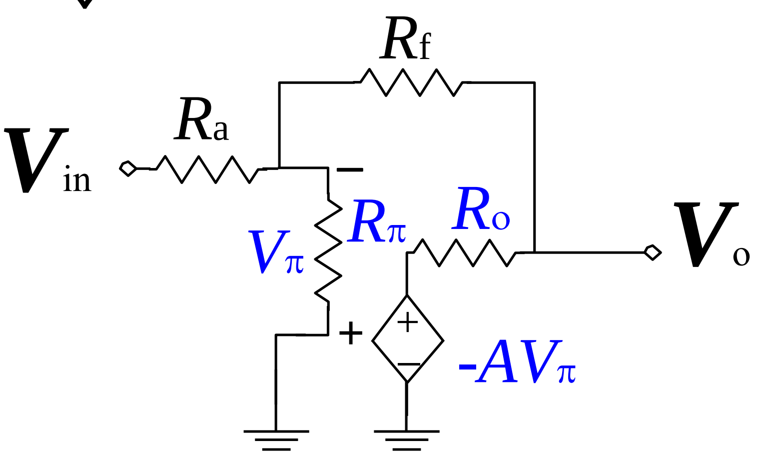
$$v_o = -\left(\frac{dV_i}{dt}\right)RC$$

Non-ideal case (Inverting Amplifier)





⇓ Equivalent Circuit



3 categories are considering

- ❑ Close-Loop Voltage Gain
- ❑ Input impedance
- ❑ Output impedance

Close-Loop Gain

Applied KCL at V– terminal,

$$\frac{V_{in} - V_{\pi}}{R_a} + \frac{-V_{\pi}}{R_{\pi}} + \frac{V_o - V_{\pi}}{R_f} = 0$$

By using the open loop gain,

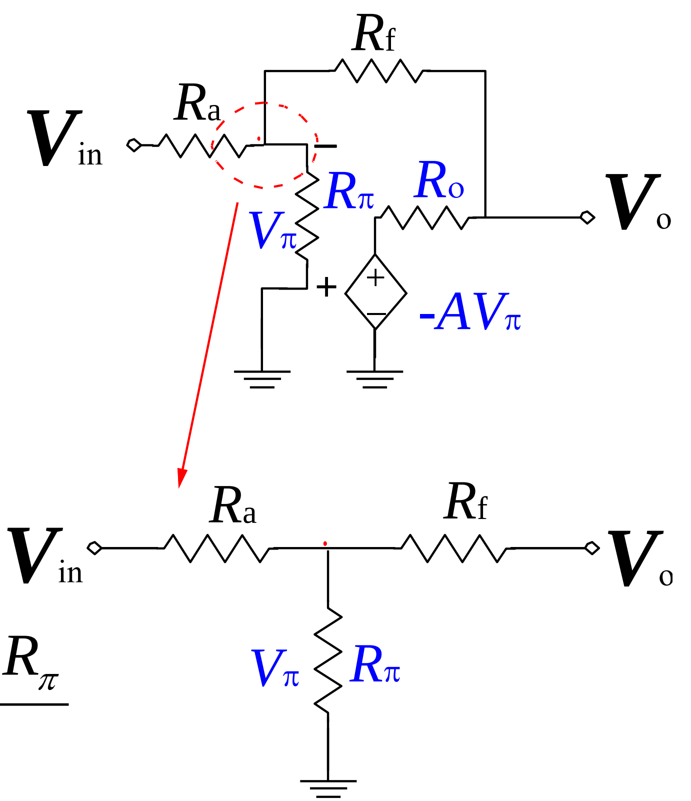
$$V_o = -AV_{\pi}$$

$$\Rightarrow \quad \frac{V_{in}}{R_a} + \frac{V_o}{AR_a} + \frac{V_o}{AR_{\pi}} + \frac{V_o}{R_f} + \frac{V_o}{AR_f} = 0$$

$$\Rightarrow \quad \frac{V_{in}}{R_a} = -V_o \frac{R_{\pi}R_f + R_aR_f + R_aR_{\pi} + AR_aR_{\pi}}{AR_aR_{\pi}R_f}$$

The Close-Loop Gain, A_v

$$A_v = \frac{V_o}{V_{in}} = \frac{-AR_{\pi}R_f}{R_{\pi}R_f + R_aR_f + R_aR_{\pi} + AR_aR_{\pi}}$$



Close-Loop Gain

When the open loop gain is very large, the above equation become,

$$A_v \sim \frac{-R_f}{R_a}$$

Note : The close-loop gain now reduce to the same form as an ideal case

Input Impedance

Input Impedance can be regarded as,

$$R_{in} = R_a + R_\pi // R'$$

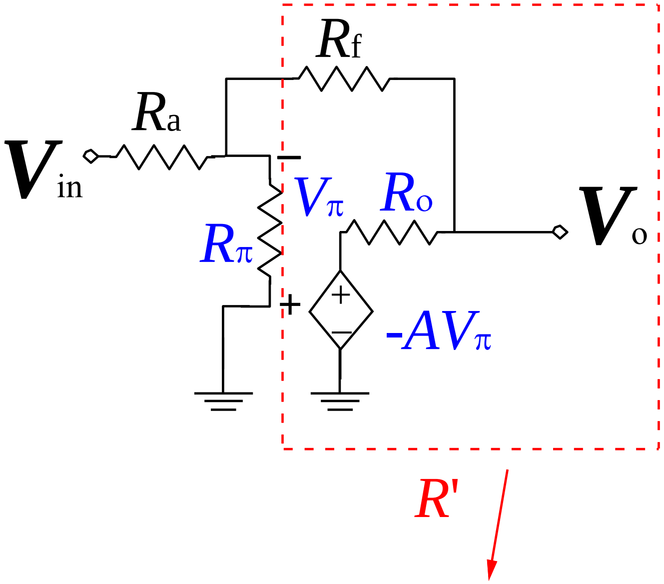
where R′ is the equivalent impedance of the red box circuit, that is

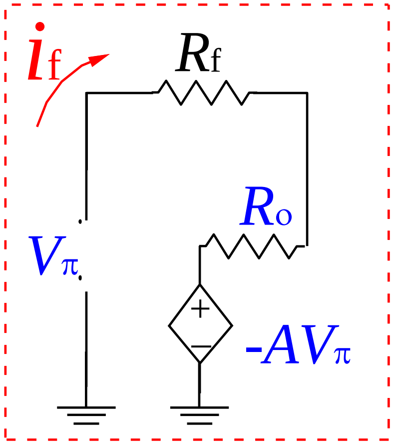
$$R' = \frac{V_\pi}{i_f}$$

However, with the below circuit,

$$V_\pi - (-AV_\pi) = i_f (R_f + R_o)$$

$$\Rightarrow R' = \frac{V_\pi}{i_f} = \frac{R_f + R_o}{1 + A}$$





Input Impedance

Finally, we find the input impedance as,

$$R_{in} = R_a + \left[\frac{1}{R_\pi} + \frac{1+A}{R_f + R_o} \right]^{-1} \quad \Rightarrow \quad R_{in} = R_a + \frac{R_\pi (R_f + R_o)}{R_f + R_o + (1+A) R_\pi}$$

Since, $R_f + R_o << (1+A) R_\pi$, R_{in} become,

$$R_{in} \sim R_a + \frac{(R_f + R_o)}{(1+A)}$$

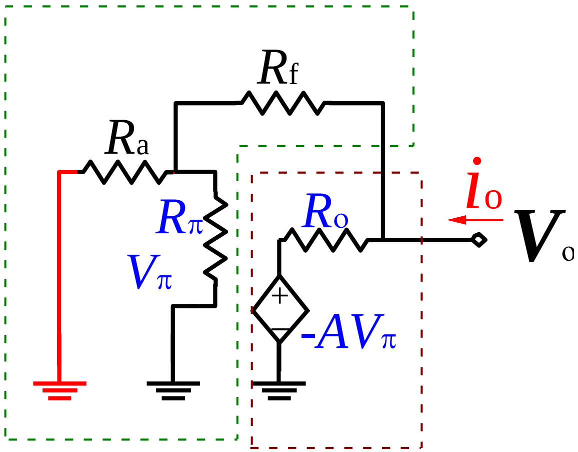
Again with $R_f + R_o << (1+A)$

$$R_{in} \sim R_a$$

Note: The op-amp can provide an impedance isolated from input to output

Output Impedance

Only source-free output impedance would be considered,
i.e. V_i is assumed to be 0



Firstly, with figure (a),

$$V_\pi = \frac{R_a // R_\pi}{R_f + R_a // R_\pi} V_o \Rightarrow V_\pi = \frac{R_a R_\pi}{R_a R_f + R_a R_\pi + R_f R_\pi} V_o$$

By using KCL, $i_o = i_1 + i_2$

$$i_o = \frac{V_o}{R_f + R_a // R_f} + \frac{V_o - (-AV_\pi)}{R_o}$$

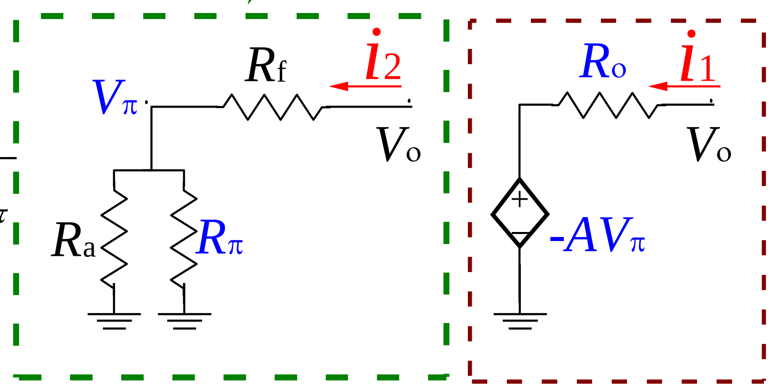
By substitute the equation from Fig. (a),

The output impedance, R_{out} is

$$\frac{V_o}{i_o} = \frac{R_o(R_a R_f + R_a R_\pi + R_f R_\pi)}{(1+R_o)(R_a R_f + R_a R_\pi + R_f R_\pi) + (1+A) R_a R_\pi}$$

$\therefore R_\pi$ and A comparably large,

$$R_{out} \sim \frac{R_o(R_a + R_f)}{AR_a}$$



(a) (b)