Subject: Mathematics

Semester-III

1) Expand f(x) = $|x|$ $as\ a\ fourier\ series -\pi < x < \pi.$

2) Prove that $\sin(\alpha + n\theta) - e^{i\alpha}\sin n\theta = e^{-in\theta}\sin\alpha$

3) *The half range sine series for 1 in (0, π) is ………*

4) *If f(x) is odd function in (-l,l), then the Fourier co-efficient* a_n=……………

5) *Find the Fourier Sine Transform of* $e^{-|x|}$.

6) State Residue Theorem.

7) $Find\ the$ $\lim_{z\to(1-i)} \frac{(z^3-1)}{(z^2-1)}$

8) Show that log(6+8i)= log 10 + $i\ tan^{-1}\frac{4}{3}$

9) Find real & Imaginary parts of $i)e^{z^2}$ $ii)e^{e^z}$

   iii) tan(x+ i y)

10) Find the general value of log(-3)……………

11) Imaginary Part of $e^{(5+3i)^2}$ is ………………

12) The Pole of f(z)=$\frac{z^2+1}{z^3+1}$ are z=…………

13) What type of Singularity have the following functions:

    i) $\frac{e^{2z}}{(z-1)^4}$ ii) $ze^{\frac{1}{z^2}}$ iii) $\frac{1}{1-e^z}$

14) Expand $\frac{1}{z^2-3z+2}$ in the region |z|=1

15) Write Laurent's series for $\frac{1}{z^2-3z+2}$ when |z|>2.

16) Residue of f(z)=$\frac{\cos z}{z}$ at z=0 is…………….

17) What is the chance that a leap year should have fifty three Sundays ?

18) Define Poisson Distribution.

19) Define Student's-t-Distribution.

20) If A and B are two events such that P(A)=1/4, P(B) = 1/3 and P(A$\cup B$)=1/2. Show that A and B are independent events.

21) If a random variable has a Poisson Distribution such that

P(1)=P(2). Find mean of the distribution.

22) Intelligence test given of two groups of boys and Girls:

	Mean	S.D	Size
Girls	75	8	60
Boys	73	10	100

Examine if the difference between mean scores is significant.

23) In 256 sets of 12 tosses of a coin, in how many cases one expect 8 heads and 4 tails.

24) A box contain 9 tickets numbered 1 to 9 inclusive. If 3 tickets

are drawn from the box, one at a time, find the probability

they are alternatively either odd, even, odd or even, odd, even.

25) Using graphically (Using Corner Point Method) solving the following LPP

Maximize $Z=5x_1 + 3x_2$

Subject to the Constraints

$$3x_1 + 5x_2 \leq 15\ , \quad 5x_1 + 2x_2 \leq 10,$$

$$x_1, x_2 \geq 0$$

Long Questions

1. a)If $f(x)=\left(\frac{\pi-x}{2}\right)^2$ *in the interval* $0 < x < 2\pi$, show that

   i. $f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ hence obtain the following relations:

   ii. $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots .. = \frac{\pi^2}{6}$

   iii. $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots \ldots \ldots \ldots \ldots \ldots \ldots .. = \frac{\pi^2}{12}$

   iv. $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots .. = \frac{\pi^2}{8}$

b) Expand $f(x)= x\sin x$, in interval $(0,2\pi)$ as a Fourier Series.

c) Obtain the Fourier Series for the function $f(x)=x^2$,

$-\pi < x < \pi$. Hence show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots .. = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\cdots\ldots\ldots\ldots\ldots\ldots\ldots..=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}=\frac{\pi^2}{12}$$

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots..=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$$

2. a)Show that for $-\pi < x < \pi$,

$$\cos cx=\frac{\sin c\pi}{\pi}\left[\frac{1}{c}-\frac{2c\cos x}{c^2-1^2}+\frac{2c\cos 2x}{c^2-2^2}-\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\right]$$

Where c is non-integral. Hence deduce that

$$\pi cosec(c\pi)=\sum_{n=0}^{\infty}(-1)^n\left[\frac{1}{n+c}+\frac{1}{n+1-c}\right]$$

b) Find the Fourier series to represent e^x in the interval $-\pi\ and\ \pi$

c) Expand f(x) = $|cosx|\ as\ a\ fourier\ series - \pi < x < \pi.$

3. a) Obtain a fourier series to represent $e^{-ax}\ from\ x=-\pi\ to\ x=\pi$.hence derive series for $\frac{\pi}{sinh\pi}$

b) Obtain Fourier Series of the function

$$f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$$

c) Obtain the Half Range Cosine Series for

$$f(x)=\begin{cases}kx & for\quad 0\le x\le \frac{L}{2}\\ k(L-x) & for\quad \frac{L}{2}\le x\le L\end{cases}$$

Deduce the sum of the series $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ -----$

4. a) Express f(x)=x as a half range. Sine series 0<x<2
Cosine series in 0<x<2.

b) Find the Fourier series of

$$f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ x^2 & 0 \le x \le \pi \end{cases}$$

c) Find Fourier series for f(x)= $\begin{cases} 0, -\pi < x < 0 \\ \sin x \quad ,0 < x < \pi \end{cases}$

5. a) Find the half-range series for the function f(x)= $(x-1)^2$
In the interval 0$< x < 1$ and show that

$$(i) \qquad \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots.. = \frac{\pi^2}{6}$$

$$\text{(ii)} \qquad \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots \ldots\ldots\ldots\ldots\ldots\ldots.. = \frac{\pi^2}{12}$$

$$(iii) \quad \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots.. = \frac{\pi^2}{8}$$

b) Find a series of cosines of multiples of x which will represent $x\sin x$ in interval (0,π) and show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7}$ - ………. = $\frac{\pi-2}{4}$.

c) Expand f(x)= $\pi\ x$ from x=-c to c as a Fourier Series

6. a) Prove that

(i) $\overline{\sin z} = \sin \overline{z}$ (ii) $\overline{\tan z} = \tan \overline{z}$ (iii) $\overline{\cos z} = \cos \overline{z}$

d) Show that $\log_e \frac{3-i}{3+i} = 2i(n\pi - \tan^{-1}\frac{1}{3})$

7. a) Prove that $\sqrt{i}^{\sqrt{i}} = e^{\frac{-\pi}{4\sqrt{2}}} \left(\cos \frac{\pi}{4\sqrt{2}} + i \sin \frac{\pi}{4\sqrt{2}}\right)$

b) If u=log tan($\frac{\pi}{4} + \frac{\theta}{2}$) , then prove that

a. $\tanh \frac{u}{2} = \tan \frac{\theta}{2}$
b. $\cosh u = \sec \theta$

c) If $\tan(\theta + i\varphi) = \tan\alpha + i\, sec\alpha$ show

$$e^{2\varphi} = \mp\cot\frac{\alpha}{2} \text{ and } 2\theta = \left(n + \frac{1}{2}\right)\pi + \alpha$$

d) If C tan(x+iy) = A+iB, Prove that $\tan 2x = \frac{2CA}{C^2 - A^2 - B^2}$

e) If cosh x =sec θ, Prove that $tanh^2\left(\frac{x}{2}\right) = tan^2(\frac{\theta}{2})$

8. a) If sin[log(A+iB)]=x+iy, show that $\frac{x^2}{sin^2u} - \frac{y^2}{cos^2u} = 1$

Where $A^2 + B^2 = e^{2u}$

b) Solve tan z = $e^{i\alpha}$, where α is real.

C) *Show that the function*

i. f(z)= $\begin{cases} \frac{Im(Z)}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not continuous at z =0

9. a) If x+i y = cosh (u+ i y) show that

$$i)\ \frac{x^2}{cosh^2u} + \frac{y^2}{sinh^2u} = 1$$

$$ii)\ x^2sec^2v - y^2cosec^2v = 1$$

b) If $\tan(\theta + i\varphi) = \tan\alpha + i\, sec\alpha$ show

$$e^{2\varphi} = \mp\cot\frac{\alpha}{2} \text{ and } 2\theta = \left(n + \frac{1}{2}\right)\pi + \alpha$$

c) Reduce $\tan(\theta + i\emptyset) = (cos\alpha + i\, sin\,\alpha) = e^{i\alpha}$ to the form a+ib. Hence show that

$$tan^{-1}\left(e^{i\theta}\right) = \frac{n\pi}{2} + \frac{\pi}{4}\ and\ \emptyset = \frac{1}{2}\log\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

d) *Show that the function*

a. f(z)=$f(x) = \begin{cases} \frac{Im(Z)}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not continuous at z =0

e) Find the value of f(i) so that the function

a. f(z)= $\frac{i\,z^3\ -1}{z-i}$ $\quad is\ not\ continuous\ at\ z = i$

10. a) If $f(z) = \frac{yx^3\,(y-ix)}{y^2+x^6}$ $\ ,z \neq 0\ \ \&\ f(0) = 0\ ,prove\ that$

$\frac{f(z)-f(0)}{z}$ → 0 as z→ 0 along any radius vector but not as z→ 0

In any manner.

b) Examine the nature of the function

$$f(z) = \begin{cases} \frac{x^2y^5(x+iy)}{x^4+y^{10}}\ ,\ z \neq 0 \\ 0,\ z = 0 \end{cases}$$

In a region including the origin.

c) Determine the Analytic function whose

real part is : $e^x[(x^2 - y^2)cosy - 2xy\ siny]$

imaginary part : $e^{-x}\,(xcosy + ysiny)$

d) If $f(z) = u + iv$ is an analytic function ,find f(z) if

a. $u - v = e^x(cosy - siny)$

11. a) Show that the function $v(x, y) = \ln(\,x^2 + y^2) + x - 2y$

Is harmonic . find its conjugate harmonic function u(x,y) and the corresponding analytic function f(z)

b) If f(z) is an analytic function of z , prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|R\ f(z)|^2 = 2|f'(z)|^2$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|\ f(z)|^2 = 4|f'(z)|^2$$

c) Evaluate $\int_0^{1+i}(x^2 - iy)$ dz along the paths y = x

d) Show that *Evaluate the integral* $\oint_c \frac{(\cos\pi z^2+\sin\pi z^2)dz}{(z-2)(z-1)^2}$

a. c: $|z| = 3$ by Cauchy's integral formula.

e) $\oint_c \frac{z\,dz}{(z-1)(z-2)^2}$ c: $|z-2| = \frac{1}{2}$ by Cauchy's integral formula.

12. *a) State Residue Theorem.*

b) Expand the following function in Laurent's series and Taylor's series

$\frac{1}{z(z-1)(z-2),}, for\ |z| > 2$

c) Prove that the Necessary and Sufficient conditions for the function w=f(z)=u+iv to be analytic in a region R, are

continuous function of x and y in the region R.

$i\ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

$ii)\ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

13. a) Drive Cauchy-Riemann equations in Polar form Hence deduce that $(\frac{\partial^2 x}{\partial r^2})+\frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}(\frac{\partial^2 u}{\partial \theta^2})=0$

OR

Prove that Cauchy-Remann Equations in polar form

b) Determine the Analytic function whose real part is $\frac{\sin 2x}{cosh2y-cos2x}$

c) If f(z) is an analytic function with constant modulus, show that f(z) is constant.

14. a)Prove that $\int_C \frac{1}{z} dz = -\pi i \; and \; \pi i \; according$ as C is the semi-circular arc |z|=1 from 1 above or below the real axis.

b) State and prove Cauchy's Integral Formula.

15. Use Cauchy integral formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} \, dz$, where C is the circle |z|=2

16. a) Evaluate $\oint_C \frac{sin^2 z}{(z-\frac{\pi}{6})^3} \, dz$ where C is the circle |z|=1

b) Evaluate using Cauchy integral formula $\oint_C \frac{e^{3z}}{(z-In\;2)^4} \, dz$, where C is the square with vertices at ±1,±i

c) Evaluate $\int_0^{1+i}(x^2 - iy)$ dz along the paths y = x

17. *a) Find the regular function whose imaginary part is* $\frac{x-y}{x^2+y^2}$

b) Use Cauchy integral formula to evaluate $\oint_C \frac{\log z}{(z-1)^3} \, dz$, where C is the circle |z-1|=$\frac{1}{2}$

c) Expand $\frac{1}{(z^2+1)(z^2+2)}$ as a Laurent's series valid for

a) 0<|z|<1 b)1<|z|<$\sqrt{2}$ c) |z|<$\sqrt{2}$

18. a) Show that $|z+1|<1, z^{-2}=1+\sum_{n=1}^{\infty}(n+1)(z+1)^n$

b) *Expand the following functions in Laurent's Series or Taylor's Series*

i) $\frac{e^z}{(z-1)^2}$ *about z=1*

ii) $\frac{1-\cos z}{z^3}$ *about z=1*

iii) $\frac{e^{2z}}{(z-1)^3}$, *about the Singularity z=1*

c) Expand $\frac{7z-2}{z(z+1)(z-2),}$, when

i) 0$< |z+1| < 1$

ii) 1<|z+1|<3 iii) |z+1|<3

19.a)Find the sum of the residues of the function $f(z) = \frac{\sin(z)}{z\cos(z)}$ at its poles inside the circle |z|=2

b) Evaluate $\oint_C \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle |z|=3

c) *Solve the integral equation* $\int_0^\infty f(x)cos\, sx\, dx = e^{-s}$

20.a) Evaluate $\oint_C \frac{1}{z^2+9} dz$ where C is

i) |z+3i|=2 ii) |z|=5

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2a\, sin\theta+a^2}$, 0<a<1

c) Evaluate $\int_0^\infty \frac{x^2}{x^6+1} dx$ using complex Integration C is the circle |z-1|=$\frac{1}{2}$

d) Find the Fourier sine and cosine transforms of x^{n-1}, n>0.

21.a) The contents of Urn I,II and III are as follows 1 white, 2 black and 3 red balls, 2 white, 1 black and 1 red ball, and 4 white , 5 Black and 3 red balls drawn. one Urn is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from I,II or III?

b) The probability that a man aged 50 years will die with in a year is 0.01125. what is the probability that out of 12 such men, at least 11 will reach their fifty first birthday?

c) Fit a Binomial distribution to the following frequency distribution:

X:	0	1	2	3	4	5	6
f:	13	25	52	58	32	16	4

22.a) Is the function defined as follows a density function?

$$f(x)= \begin{cases} e^{-x}, & x \geq 0 \\ 0 & otherwise \end{cases}$$

if so, find $P(1 \leq X \leq 2)$

b) In a Lottery, m tickets are drawn at a time out of n tickets numbered from 1 to n. find the expected value of the sum of the numbers on the tickets drawn.

c) A cubical die is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the die can't be regarded as an unbiased one and find the extreme limits between which the probability of a throw of 3 or 4 lies.

23. a) The means of a simple samples of sizes 1000 and 2000 are 67.5 and 68.0 cm respectively. Can the sample be regarded as drawn from the same population of S.D. 2.5 cm?

b) A filling machine is expected to fill 5 kg. of powder into bags. A sample of 10 bags gave the following weights. 4.7,4.9, 5.0,5.1,5.4,5.2,4.6,5.1,4.6 and 4.7. test whether the machine is working properly.

24. a) A housewife wishes to mix two types of foods X & Y in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B . Food X costs RS. 60 per Kg and Food Y costs Rs. 80 per Kg. Food X contains 3 units per Kg. of vitamin A and 5 units per kg. of vitamin B. While Food Y contains 4 units per kg. of vitamin A And 2 units per kg of vitamin B. Formulate the above problem as an LPP to minimize the cost of mixture.

b) A Company Produce two type of model M_1 and M_2. Each M_1 model require 4 hours of grinding and 2 hours of polishing where as each M_2 model requires 2 hours of grinding and 5 hours of polishing. The company has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works for 60 hours a week. Profit on an M_1 model is Rs. 3 and on an M_2 model is Rs. 4. Whatever is produce in a week is sold in the market. How should the company allocate its production capacity to the two types of models so that it may make the maximum profit in a week?

25. a) Formulate the problem as an LPP.

Solve the LPP by simplex method:

Maximize $z = 2x_1 + 5x_2$

Subject to the constraints

1. $x_1 + 4x_2 \leq 24$, $3x_1 + x_2 \leq 21$, $x_1 + x_2 \leq 9$
2. $x_1, x_2 \geq 0$

b) Solve the LPP by dual simplex method:

Minimize $z = 3x_1 + x_2$

Subject to the constraints:

$x_1 + x_2 \geq 1$, $2x_1 + 3x_2 \geq 2$

$x_1 + 2x_2 \geq 20$, $x_1, x_2 \geq 0$

c) Obtain the Dual of

Maximize $z = 5x_1 + 4x_2 + 3x_3$

Subject to the constraints

$3x_1 + 2x_2 + x_3 \leq 10$, $2x_1 + x_2 + 2x_3 \leq 12$,

$x_1 + x_2 + 3x_3 \leq 15$

$x_1, x_2, x_3 \geq 0$