Subject: Mathematics

1) Expand f(x) = $|x|$ $as\ a\ fourier\ series\ -\pi < x < \pi.$

2) Express f(x)=x as a Fourier Series in the interval $-\pi < x < \pi$.

3) *If f(x) =* $x^2, -\pi < x < \pi$ *, find fourier coefficient* a_n

4) *The half range sine series for 1 in (0,* π*) is ………*

5) *If f(x) is odd function in (-l,l), then the Fourier co-efficient* a_n=……………

6) *Find the Fourier Sine Transform of* $e^{-|x|}$.

7) Find the Fourier Sine transform of 1/x.

8) Real part of $e^{5+\frac{1}{2}i\pi}$ is ………

9) $Find\ the$ $\lim_{z\to(1-i)} \frac{(z^3-1)}{(z^2-1)}$

10) Show that log(6+8i)= log 10 + $i\ tan^{-1}\frac{4}{3}$

11) Find real & Imaginary parts of $i) e^{z^2}$ $ii) e^{e^z}$

12) Prove that

    a. (i) $\overline{sinz} = \sin\bar{z}$ (ii) $\overline{\tan z} = \tan\bar{z}$ (iii) $\overline{cosz} = \cos\bar{z}$

13) Find the general value of log(-3)……………

14) Imaginary Part of $e^{(5+3i)^2}$ is ………………

15) The Pole of f(z)=$\frac{z^2+1}{z^3+1}$ are z=…………

16) Expand $\frac{1}{z^2-3z+2}$ in the region |z|=1

17) Write Laurent's series for $\frac{1}{z^2-3z+2}$ when |z|>2.

18) Taylor's series expansion of $\frac{1}{z-2}$ in |z|<1 is………………

19) Residue of f(z)=$\frac{\cos z}{z}$ at z=0 is…………….

20) If f(z) has a simple pole at z=a, then Res.{f(z),a}=………..

21) If f(z) has a pole of order m at z=a, then

Res {f(z),} = ………………….

22) What is the chance that a leap year should have fifty three Sundays ?

23) Define Poisson Distribution.

24) If A and B are two events such that P(A)=1/4, P(B) = 1/3 and P(A∪ B)=1/2. Show that A and B are independent events.

25) If a random variable has a Poisson Distribution such that

P(1)=P(2). Find mean of the distribution.

26) A Card is drawn from a well-shuffled pack of playing cards. What is the probability that it is either a spade or a ace?

27) Intelligence test given of two groups of boys and Girls:

	Mean	S.D	Size

Girls	75	8	60
Boys	73	10	100

Examine if the difference between mean scores is significant.

28) In 256 sets of 12 tosses of a coin, in how many cases one expect 8 heads and 4 tails.

29) A card is drawn from a well-shuffled pack of playing cards.

What is the probability that it is either a spade or an ace?

30) A box contain 9 tickets numbered 1 to 9 inclusive. If 3 tickets are drawn from the box, one at a time, find the probability they are alternatively either odd, even, odd or even, odd, even.

31) Using graphical method, solving the following LPP

Maximize Z=$2x_1 + 3x_2$

$x_1 - x_2 \leq 2$, $x_1 + x_2 \geq 4$,

$x_1, x_2 \geq 0$

Long Questions

32) If f(x)= $\left(\frac{\pi - x}{2}\right)^2$ $in\ the\ interval\ 0 < x < 2\pi$, show that

f(x) = $\frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{cosnx}{n^2}$ hence obtain the following relations:

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots.. = \frac{\pi^2}{6}$$

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots \ldots\ldots\ldots\ldots\ldots\ldots.. = \frac{\pi^2}{12}$$

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots.. = \frac{\pi^2}{8}$$

33) Expand f(x) = $|cosx|$ $as\ a\ fourier\ series - \pi < x < \pi.$

34) *Expand* $f(x) = |sinx|$ *as a Fourier series in the interval,* $-\pi < x < \pi$

35) Obtain a fourier series to represent e^{-ax} $from\ x = -\pi\ to\ x = \pi$.hence derive series for $\frac{\pi}{sinh\pi}$

Prove that

$$coswx = \frac{2wsinw\pi}{\pi}\left(\frac{1}{2w^2} + \frac{cosx}{1^2 - w^2} - \frac{cos2x}{2^2 - w^2} + \ldots\ldots\right) in\ the\ interval(-\pi, \pi)$$

Where w is a non integral value.

36) Find the fourier series of the function

$$f(x) = \begin{cases} x^2, & 0 \le x \le \pi \\ -x^2, & -\pi \le x \le 0 \end{cases}$$

37) Find fourier series for $f(x)=\begin{cases} 0, & -\pi < x < 0 \\ sinx, & 0 < x < \pi \end{cases}$

38) Obtain the Fourier series for the function

39) $F(x)=\begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$

40) Find the Fourier of the function

$f(x) = \begin{cases} x, & -1 \le x \le 0 \\ x+2, & 0 \le x \le 1 \end{cases}$ *where* $f(x+2) = f(x)$

41) Find the half-range series for the function f(x)=$(x-1)^2$

In the interval 0$< x < 1$ and show that

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots.. = \frac{\pi^2}{6}$

(ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots \ldots\ldots\ldots\ldots\ldots\ldots.. = \frac{\pi^2}{12}$

(iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots.. = \frac{\pi^2}{8}$

42) Find the half range sine and cosine series of the function

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < x < \pi \end{cases}$$

43) Find a series of cosines of multiples of x which will represent $xsinx$ in interval (0,π) and show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7}$ - ………. = $\frac{\pi-2}{4}$.

44) Draw the graph of wave forms & find fourier series for all wave forms.

45) Expand f(x)= $xsinx$, in interval (0,2π) as a Fourier Series.

46) Find the Fourier sine and cosine transforms of x^{n-1}, n>0.

47) Solve the integral equation $\int_0^\infty f(x) \cos sx \, dx = e^{-s}$

48) If α and β are the imaginary cube roots of unity, prove that

$$\alpha e^{\alpha x} + \beta e^{\beta x} = -e^{\frac{-x}{2}}(\cos\frac{\sqrt{3}}{2}x + \sqrt{3}\sin\frac{\sqrt{3}}{2}x)$$

49) Show that $\log_e \frac{3-i}{3+i} = 2i(n\pi - \tan^{-1}\frac{1}{3})$

50) Prove that $\sqrt{i^{\sqrt{i}}} = e^{\frac{-\pi}{4\sqrt{2}}}(\cos\frac{\pi}{4\sqrt{2}} + i\sin\frac{\pi}{4\sqrt{2}})$

51) If $u = \log\tan(\frac{\pi}{4} + \frac{\theta}{2})$, then prove that

(i) $\tanh\frac{u}{2} = \tan\frac{\theta}{2}$

(ii) $\cosh u = \sec\theta$

52) If $\tan(\theta + i\varphi) = \tan\alpha + i\sec\alpha$ show

$e^{2\varphi} = \mp\cot\frac{\alpha}{2}$ and $2\theta = \left(n + \frac{1}{2}\right)\pi + \alpha$

53) *Show that the function*

$$f(z) = f(x) = \begin{cases} \frac{Im(Z)}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$ is not continuous at z =0

54) Find the value of f(i) so that the function

$f(z) = \frac{i z^3 - 1}{z - i}$ *is not continuous at* $z = i$

55) If $f(z) = \frac{yx^3(y - ix)}{y^2 + x^6}$, $z \neq 0$ & $f(0) = 0$, *prove that*

$\frac{f(z) - f(0)}{z} \to 0$ as $z \to 0$ along any radius vector but not as $z \to 0$

In any manner.

56) Examine the nature of the function

$$f(z) = \begin{cases} \dfrac{x^2y^5(x+iy)}{x^4+y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

In a region including the origin.

57) Determine the Analytic function whose real part is

(i) $e^x[(x^2-y^2)cosy - 2xy\ siny]$

(ii) $\log\sqrt{(x^2+y^2)}$

58) Determine the Regular function whose imaginary part is

(i) $e^{-x}\,(xcosy + ysiny)$

(ii) $\cos y \sinh x$

59) If $f(z) = u + iv$ is an analytic function ,find f(z) if

$$u - v = e^x(cosy - siny)$$

60) Show that the function $v(x,y) = \ln(x^2+y^2) + x - 2y$

Is harmonic . find its conjugate harmonic function u(x,y) and the corresponding analytic function f(z)

61) If f(z) is an analytic function of z , prove that

(i) $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|R\ f(z)|^2 = 2|f'(z)|^2$

(ii) $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|\ f(z)|^2 = 4|f'(z)|^2$

62) Evaluate $\int_0^{1+i}(x^2 - iy)$ dz along the paths y = x

63) *Evaluate the integral* $\oint_c \frac{(\cos\pi z^2 + \sin\pi z^2)dz}{(z-2)(z-1)^2}$

c: $|z| = 3$

64) *Evaluate the integral* $\oint_c \frac{(1-2z)dz}{z(z-2)(z-1)}$ c: $|z| = \frac{1}{2}$ by Cauchy 's integral formula.

65) $\oint_c \frac{z\,dz}{(z-1)(z-2)^2}$ c: $|z-2| = \frac{1}{2}$ by Cauchy's integral formula.

66) State Residue Theorem.

67) Expand the following function in Laurent's series and Taylor's series

$\frac{1}{z(z-1)(z-2),}$ $, for\ |z| > 2$

68) Prove that the Necessary and Sufficient conditions for the function w=f(z)=u+iv to be analytic in a region R, are

are continuous function of x and y in the region R.

$i\ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

$ii)\ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

69) Drive Cauchy-Riemann equations in Polar form Hence deduce that $(\frac{\partial^2 x}{\partial r^2})+\frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}(\frac{\partial^2 u}{\partial \theta^2})=0$

OR

Prove that Cauchy-Remann Equations in polar form

70) Determine the Analytic function whose real part is $\frac{\sin 2x}{cosh2y-cos2x}$

71) Evaluate the integral by Cauchy integral formula $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the Circle |z|=3/2.

72) If the potential function is log (x^2+y^2), find the flux function and complex potential function.

73) If f(z) is an analytic function with constant modulus, show that f(z) is constant.

74) Prove that $\int_C \frac{1}{z} dz = -\pi i \; and \; \pi i \; according$ as C is the semi-circular arc |z|=1 from 1 above or below the real axis.

75) State and prove Cauchy's Integral Formula.

76) Evaluate $\oint_C \frac{3z^2+z}{z^2-1} dz$, where C is the Circle |z-1|=1

77) Use Cauchy integral formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} \, dz$, where C is the circle |z|=2

78) If $f(\xi) = \oint_C \frac{4z^2+z+5}{z-\xi}$dz,where C is the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$,find f(1),f(i),f'(-1) and f''(-i).

79) Evaluate $\oint_C \frac{sin^2 z}{(z-\frac{\pi}{6})^3} \, dz$ where C is the circle |z|=1

80) Evaluate using Cauchy integral formula $\oint_C \frac{e^{3z}}{(z-In\,2)^4} \, dz$, where C is the square with vertices at ±1,±i

81) Expand $\frac{1}{(z^2-3z+2)}$ in the region

a) |z|<1 b) 1<|z|<2 c)|z|>2

(d) 0<|z-1|<1

82) Expand $f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region

a. |z|<1 b) 1<|z|<4 c) |z|>4

83) Find the sum of the residues of the function $f(z) = \frac{\sin(z)}{\cos(z)}$ at its poles inside the circle |z|=2

84) Expand $\frac{1-\cos z}{z^3}$ about z=0.

85) Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2a\sin\theta+a^2}$, 0<a<1

86) Evaluate $\int_0^{\infty} \frac{dx}{(1+x^6)}\ dx$ using complex Integration.

87) Evaluate $\int_0^{\infty} \frac{x^2dx}{(1+x)^4}\ dx$ using complex Integration

88) Evaluate $\int_0^{1+i}(x^2-iy)$ dz along the paths y = x

89) Find the regular function whose imaginary part is $\frac{x-y}{x^2+y^2}$

90) Show that $|z+1|<1, z^{-2}=1+\sum_{n=1}^{\infty}(n+1)(z+1)n$

91) Use Cauchy integral formula to evaluate $\oint_C \frac{\log z}{(z-1)^3}\ dz$, where C is the circle $|z-1|=\frac{1}{2}$

92) The contents of Urn I,II and III are as follows 1 white, 2 black and 3 red balls, 2 white, 1 black and 1 red ball, and 4 white ,

5 Black and 3 red balls drawn. They happen to be white and red. What is the probability that they come from I,II or III?

93) Compute the variance of the probability distribution of the number of doublets in four throws of a pair of die.

94) The probability that a man aged 50 years will die with in a year is 0.01125. what is the probability that out of 12 such men, at least 11 will reach their fifty first birthday?

95) Fit a Binomial distribution to the following frequency distribution:

X:	0	1	2	3	4	5	6
f:	13	25	52	58	32	16	4

96) Compute the variance of the probability distribution of the number of doublets in four throws of a pair of dice.

97) Is the function defined as follows a density function?

$$f(x)= \begin{cases} e^{-x}, & x \geq 0 \\ 0 & otherwise \end{cases}$$

if so, find P(1≤X≤2)

98) In a Lottery, m tickets are drawn at a time out of n tickets numbered from 1 to n. find the expected value of the sum of the numbers on the tickets drawn.

99) A cubical die is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the die can't be regarded as an unbiased one and find the extreme limits between which the probability of a throw of 3 or 4 lies.

100) The means of a simple samples of sizes 1000 and 2000 are 67.5 and 68.0 cm respectively. Can the sample be regarded as drawn from the same population of S.D. 2.5 cm?

101) Define Student's-t-Distribution.

102) A filling machine is expected to fill 5 kg. of powder into bags. A sample of 10 bags gave the following weights. 4.7,4.9, 5.0,5.1,5.4,5.2,4.6,5.1,4.6 and 4.7. test whether the machine is working properly.

103) A housewife wishes to mix two types of foods X & Y in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B . Food X costs RS. 60 per Kg and Food Y costs Rs. 80 per Kg. Food X contains 3 units per Kg. of vitamin A and 5 units per kg. of vitamin B. While Food Y contains 4 units per kg. of vitamin A And 2 units per kg of vitamin B. Formulate the above problem as an LPP to minimize the cost of mixture.

104) A Company Produce two type of model M_1 and M_2. Each M_1 model require 4 hours of grinding and 2 hours of polishing where as each M_2 model requires 2 hours of grinding and 5 hours of

polishing. The company has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works for 60 hours a week. Profit on an M_1 model is Rs. 3 and on an M_2 model is Rs. 4. Whatever is produce in a week is sold in the market. How should the company allocate its production capacity to the two types of models so that it may make the maximum profit in a week? Formulate the problem as an LPP.

105) *Solve the LPP by Graphical method :*

Minimize $z = 20x+10y$

Subject to the constraints

$$x + 2y \le 40 \quad , \quad 3x + y \ge 30 \ ,$$
$$4x+3y \ge 60$$
$$x, y \ge 0$$

106) *Solve the LPP by simplex method:*

Maximize $z = 5x_1 + 4x_2 + 3x_3$

Subject to the constraints

$$3x_1 + 2x_2 + x_3 \le 10 \quad , \quad 2x_1 + x_2 + 2x_3 \le 12,$$
$$x_1 + x_2 + 3x_3 \le 15$$
$$x_1, x_2, x_3 \ge 0$$

107) *Solve the LPP by dual simplex method:*

Minimize $z = 2x_1 + 2x_2 + 4x_3$

Subject to the constraints:

$$2x_1 + 3x_2 + 5x_3 \geq 2,$$

$$3x_1 + x_2 + 7x_3 \leq 3, \quad x_1 + 4x_2 + 6x_3 \leq 5 \quad\quad x_1, x_2, x_3 \geq 0$$

108) *Obtain the Dual of*

Maximize z = $5x_1 + 4x_2 + 3x_3$

Subject to the constraints

$$3x_1 + 2x_2 + x_3 \leq 10 \quad , \quad\quad 2x_1 + x_2 + 2x_3 \leq 12,$$

$$x_1 + x_2 + 3x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0$$