

ANTENNAS , WAVE PROPAGATION &TV ENGG

Lecture : Various Potentials used in
Antenna theory

Topics to be covered

- Various Potentials used in Antenna theory

Electric Scalar Potential V

➤ Electric Field is a field of force. If a body being acted upon by the force from one point to another, work will be done on or by the body.

Taking reference point as infinity, if test charge 'q' is moved from infinity along a radius line to a point 'P' at a distance 'R' from charge 'Q', work is done in moving test charge q against the force F & is given by

$$W = - \int_{\infty}^R F \cdot dr$$

$$\frac{Qq}{4\pi\epsilon r^2}$$



$$W = - \int_{\infty}^R \frac{Qq}{4\pi\epsilon r^2} dr$$

$$= - \frac{Qq}{4\pi\epsilon} \left[-\frac{1}{r} \right]_{\infty}^R = \frac{Qq}{4\pi\epsilon r}$$



➤ If test charge $q =$ Unit charge 1,

Work done on test charge per unit charge in moving from infinity to point P due to charge Q

Electric Potential

$$V = \frac{Q \times 1}{4\pi\epsilon_0 r} = \frac{Q}{4\pi\epsilon_0 r}$$

= Magnitude without direction



Electric Potential V is a "Scalar Potential" work;

Electric Field Intensity = Negative of Potential gradient at that point

$$E = - \nabla V \cdot \text{Grad } V \quad \dots\dots\dots 1$$



Defined as the work done on the test charge per unit charge in moving a charge from the infinity to the point Scalar Electric Potential : V (Unit = Volt = Joules/Coulomb)

$$W = - \int_{\infty}^R \vec{F} \cdot d\vec{r}$$



From Coulombs' law :

$$F = \frac{Qq}{4\pi\epsilon r^2} =$$

$$= \frac{Qq}{4\pi\epsilon R} \quad (\text{if test charge } q = 1 \text{ then})$$



Magnetic Vector Potential

creates
 $q \Rightarrow$ Electric Field

creates
 $I \cdot dl \Rightarrow$ Electric Field

(Current element : Vector)



Biot Savarts Law

$$dA = k \cdot \left(\frac{I dl}{r} \right)$$

where $k = \frac{\mu_0}{4\pi}$



- So Magnetic vector Potential \vec{A} due to the current flow in the entire circuit is obtained by

$$\int dA = \int \frac{\mu}{4\pi} \left(\frac{I dl}{r} \right)$$

$$A = \int \frac{\mu}{4\pi} \cdot \frac{I dl}{r}$$

Unit of \vec{A} is
 wb/m² or Tesla (T)



- If it is generalized, $Idl = Idv$ i.e. current flows throughout the volume.

$$A = \int \frac{\mu}{4\pi} \cdot \frac{J dv}{r^2}$$



Retarded Potentials:

- We have evaluated all the equation based on the basis of charges being fixed on position for 'V' and on the basis of constant charge velocities or constant current for A.
- We must take "time rate of change" into account in the dynamic case because all electric & magnetic effects are propagated with a velocity of 'c' – 3×10^8 m/s

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- The retarded scalar potentials that can be expressed in terms of retarded time $(t - r/c)$
- The expression for retarded scalar potential is given by

$$V(P, t) = \frac{1}{4\pi\epsilon_0} \int_V \frac{[\rho](t - r/c)}{r} dV.$$

- **Potential at point P, time t**



$$[\rho]_{(t-r/c)} = \text{Charge at earlier times } (t-r/c)$$

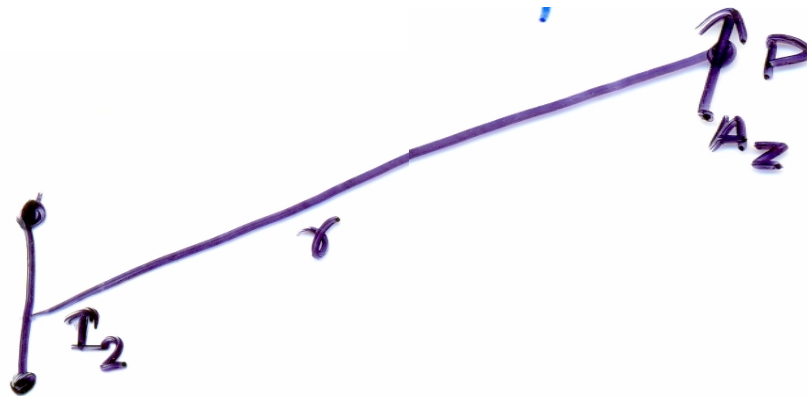
r/c = Retardation time is the time for the effect to be propagated the distance **r** at velocity **c**

- Similarly for \vec{A}

$$\vec{A}(P, t) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(t - r/c) \cdot d\vec{v}}{r}$$

Time varying potentials are called Retarded Potentials

Application in Radiation problems in which the distribution of the source is known approximately eg:



- For any filamentary current –

$$\vec{A} = \frac{\mu}{4\pi} \int \frac{I (t - \frac{r}{c}) dl}{r}$$



- For particular case when l is small in comparison with r

$$A_z = \frac{\mu}{4\pi} \cdot \frac{I_z(t - \frac{r}{c})}{r} l =$$

$$I_z = I_m \sin \omega t \quad \text{then}$$

$$A_z = \frac{\mu l I_m \sin \omega(t - \frac{r}{c})}{4\pi r}$$

Thus the value of electric field & A may be derived



Retarded Vector Potential

- Vector potential expression represents superposition's of potentials due to varying current elements $I \cdot dl$ at a distance pt P at a distance r .

But we made a assumption:

“ Time of propagation was ignored also called as Retardation Time”



- Hence instead of

$$I = I_m \sin \omega t$$

$$[I] = I_m \sin \omega \left(t - \frac{r}{c} \right)$$

$$= I_m \sin \left(\omega t - \frac{\omega}{c} \cdot r \right)$$

$$= I_m \sin (\omega t - \beta r)$$

where, $\sin (\omega t - \beta r)$ Is Travelling of spherical waves in radial direction

